Potential fields for navigation

EECS 367
Intro. to Autonomous Robotics

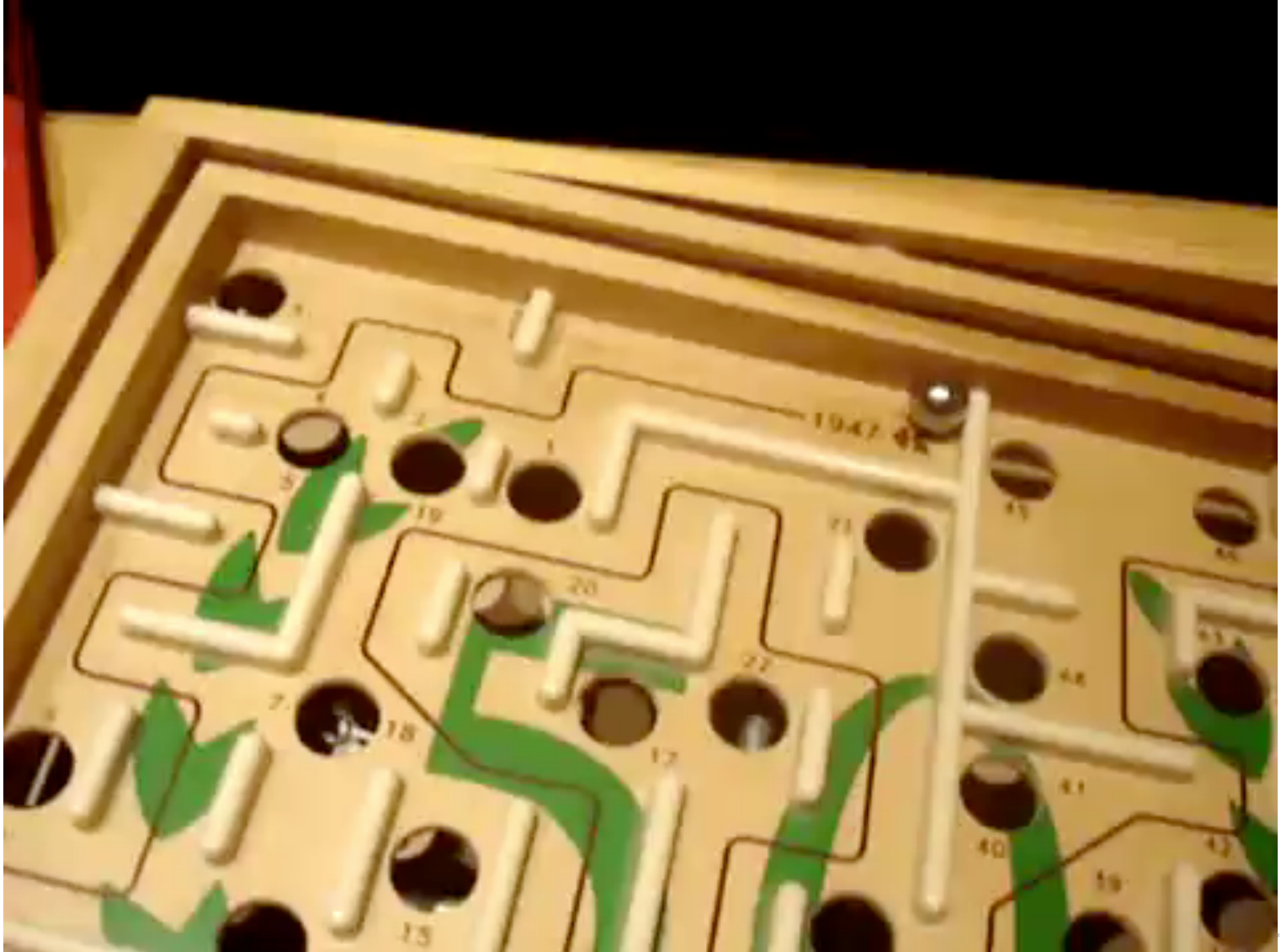
ROB 320
Robot Operating Systems

Winter 2022

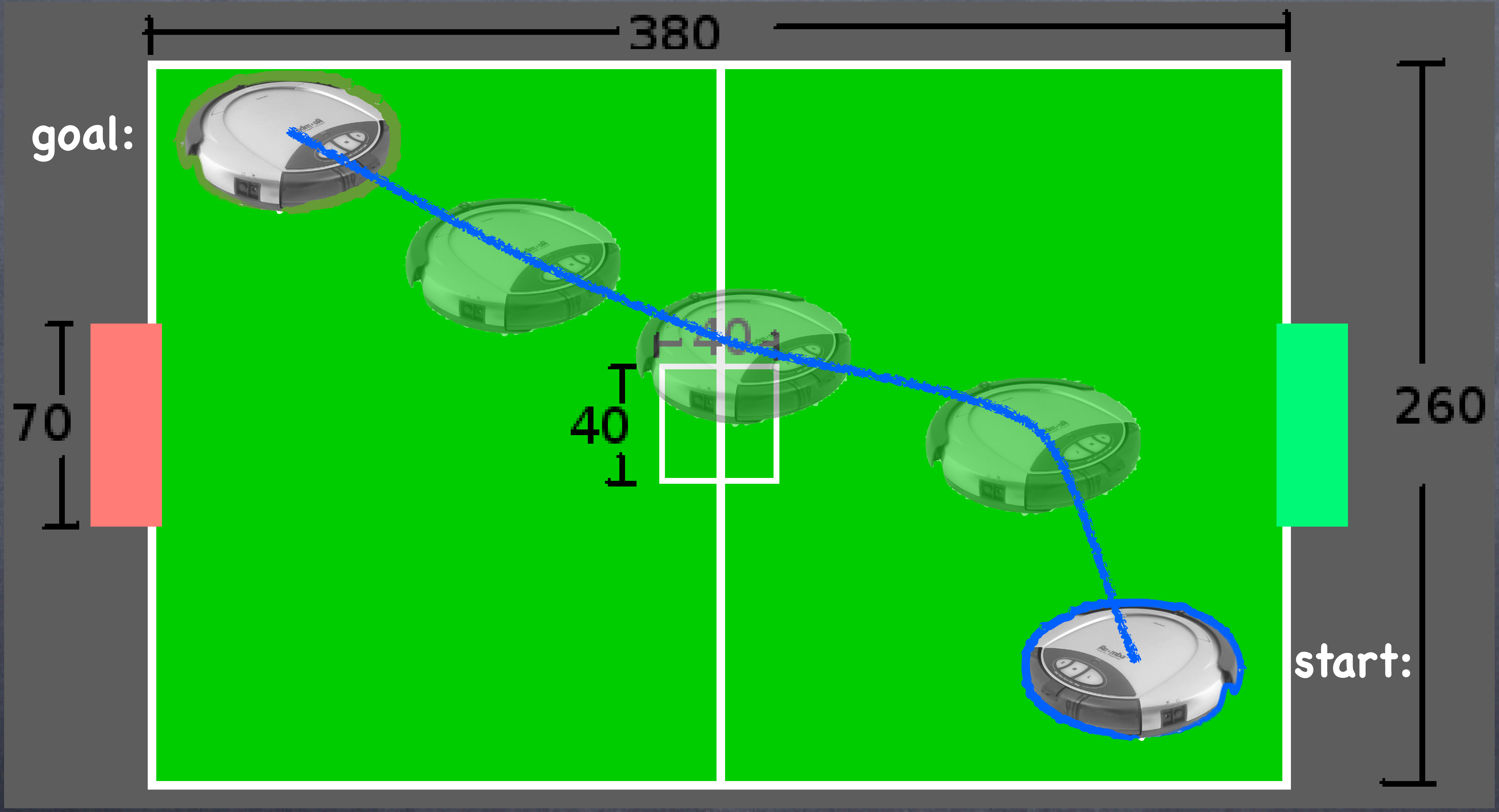
autorob.org

Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- Graph Search (fixed graph)
  - Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first
- Sampling-based Search (build graph):
  - Probabilistic Road Maps, Rapidly-exploring Random Trees
- **Optimization and local search:**
  - **Gradient descent, Potential fields, Simulated annealing, Wavefront**

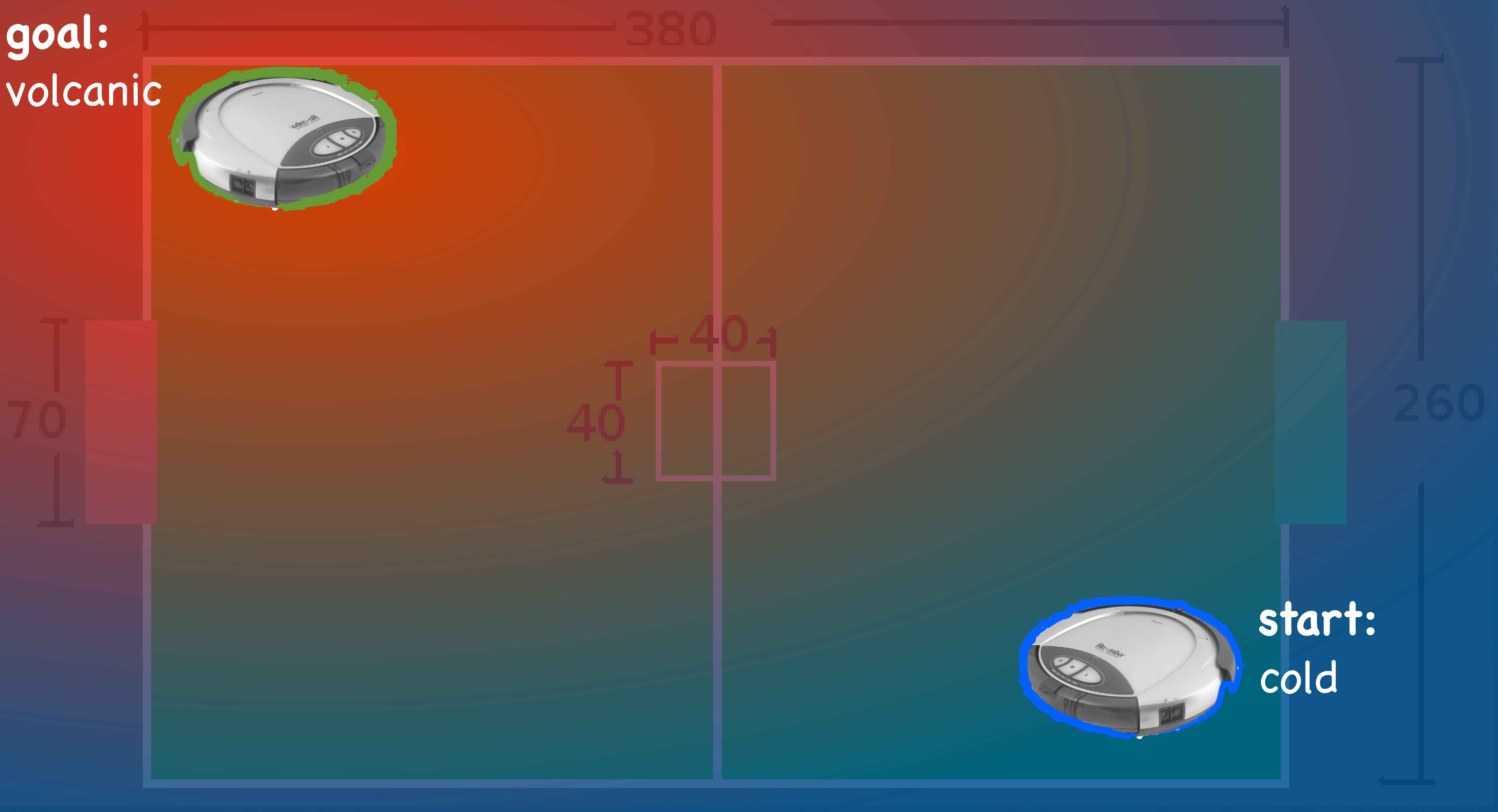


Navigation (again)



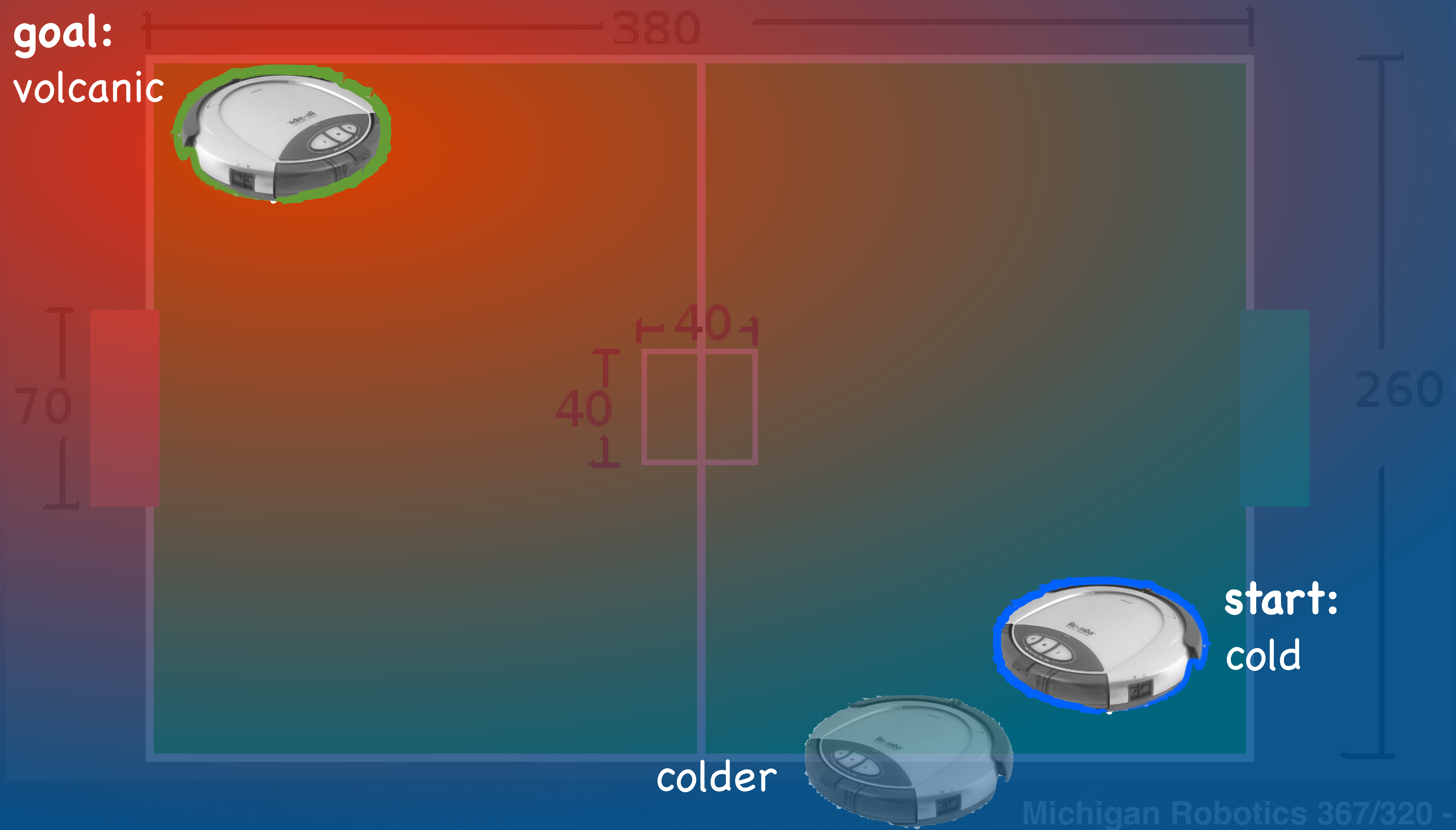
Potential field

(like a game of "warmer-colder")



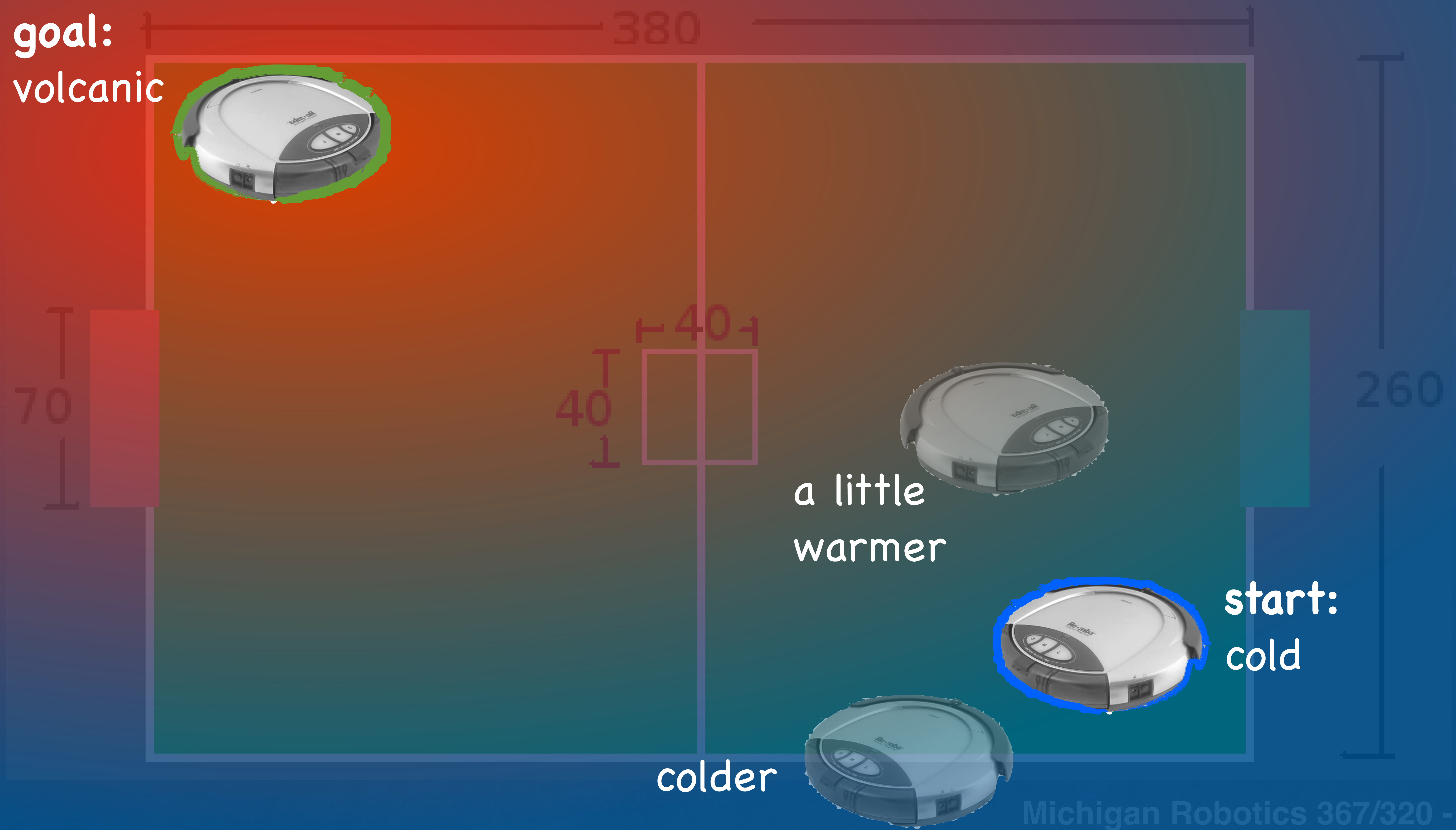
Potential field

(like a game of "warmer-colder")



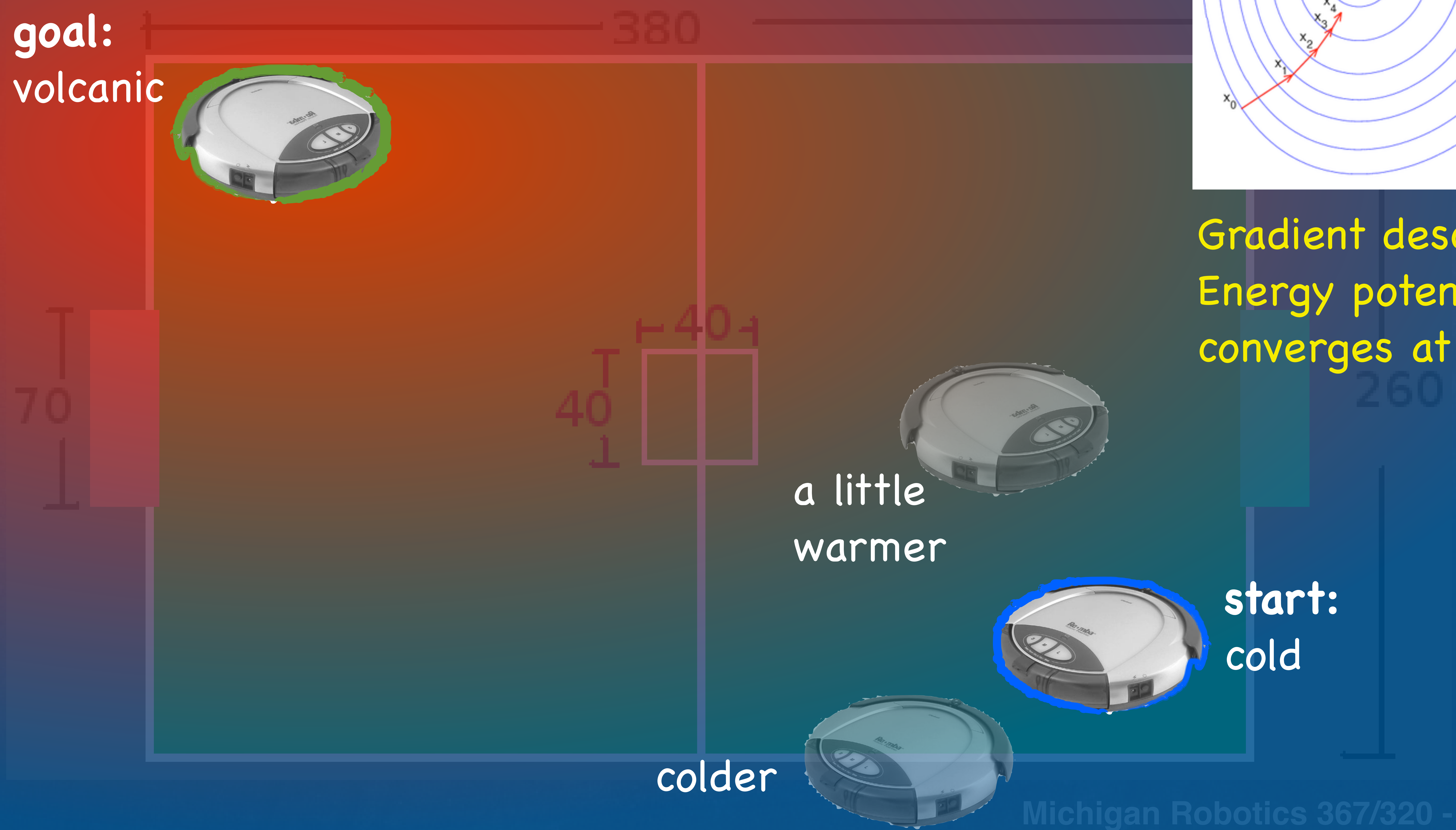
Potential field

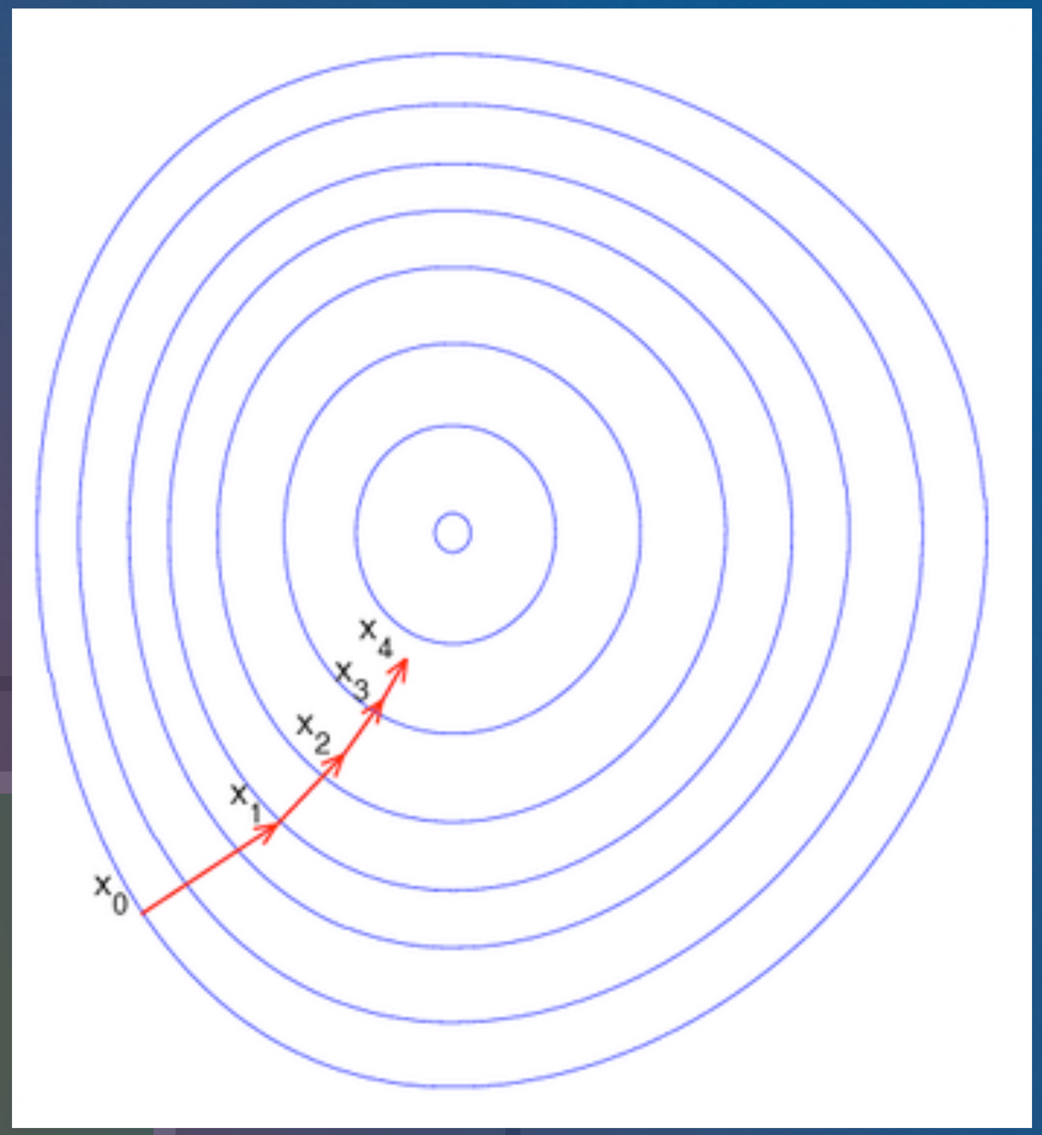
(like a game of "warmer-colder")



Potential field

(like a game of "warmer-colder")

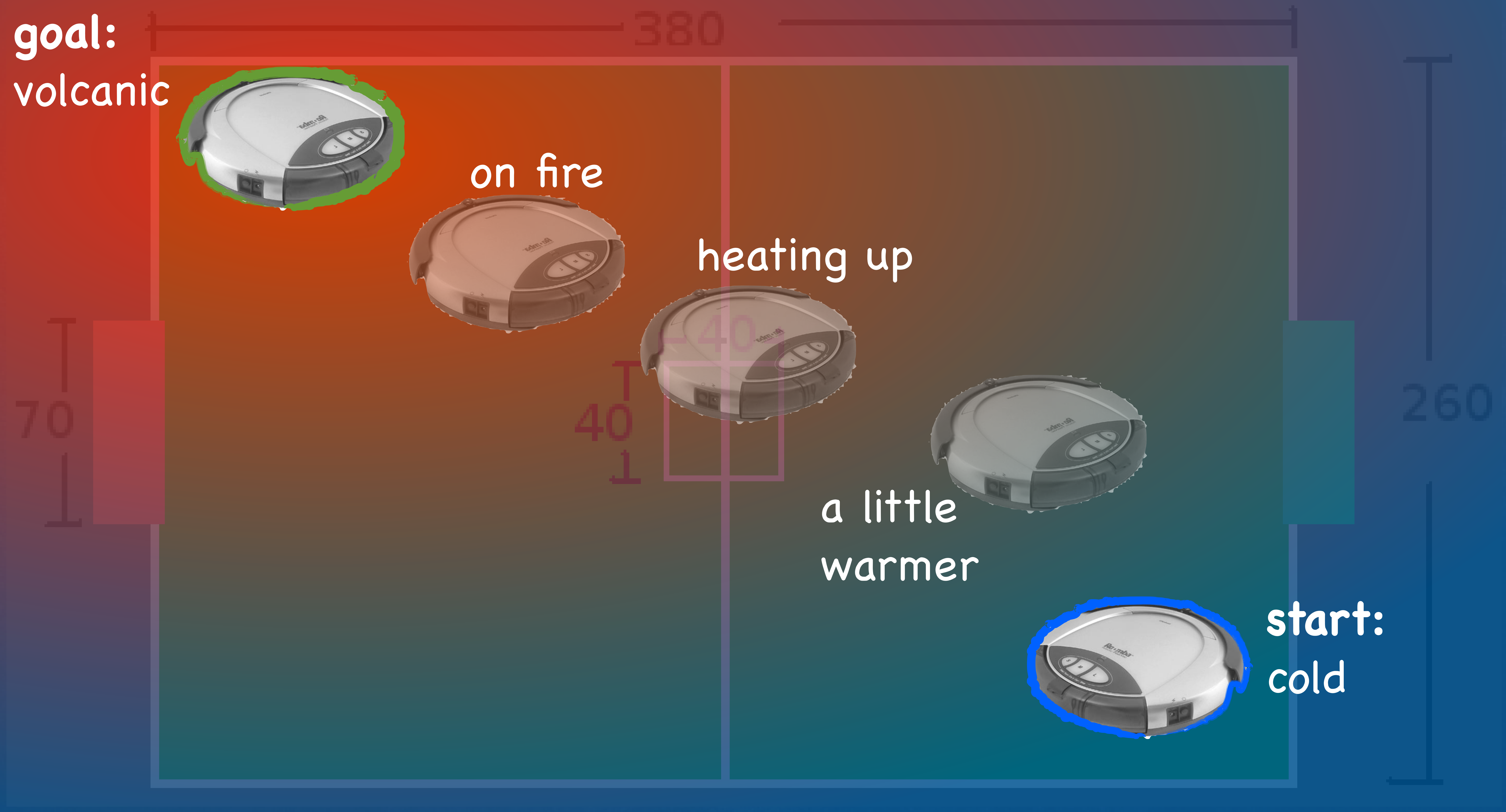




Gradient descent:
Energy potential
converges at goal

Potential field

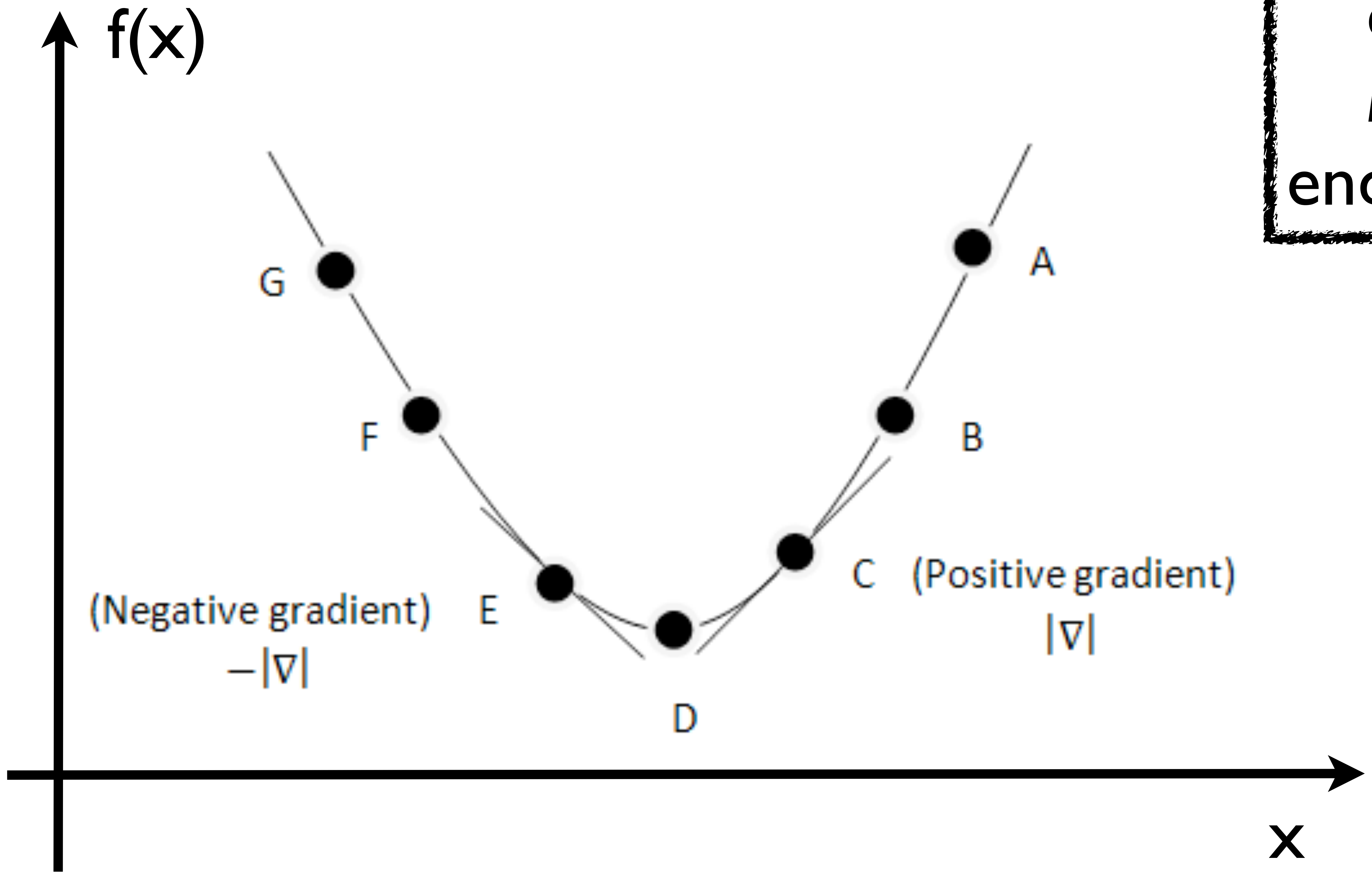
(like a game of "warmer-colder")



How do we define a potential field?

Potential Field

- A potential field is a differentiable function *U(q)* that maps configurations to scalar "energy" value
- At any *q*, gradient $\nabla U(q)$ is the vector that maximally increases *U*
- At goal q_{goal}, energy is minimized such that $\nabla U(q_{goal}) = 0$
- Navigation by descending field $-\nabla U(q)$ *to goal*



Gradient Descent Algorithm:

$q_{path}[0] \leftarrow q_{start}$

$i \leftarrow 0$

while ($|| \nabla U(q[i]) || > \varepsilon$)

  $q_{path}[i+1] \leftarrow q_{path}[i] - \alpha \nabla U(q_{path}[i])$

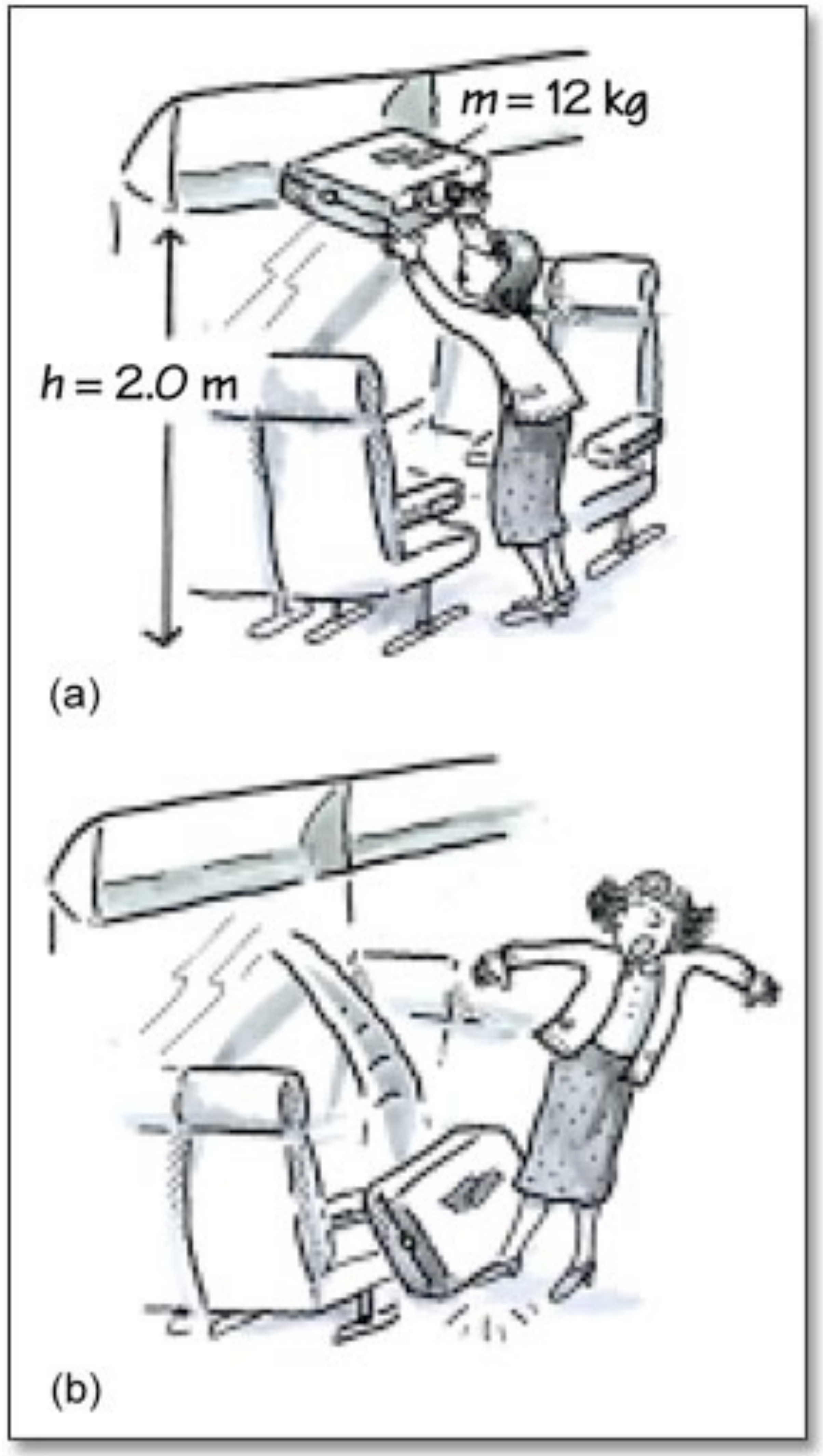
  $i \leftarrow i+1$

end

Derivative assumed to be direction of steepest ascent away from goal

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \gamma_n \boxed{\nabla F(\mathbf{x}_n)}$$

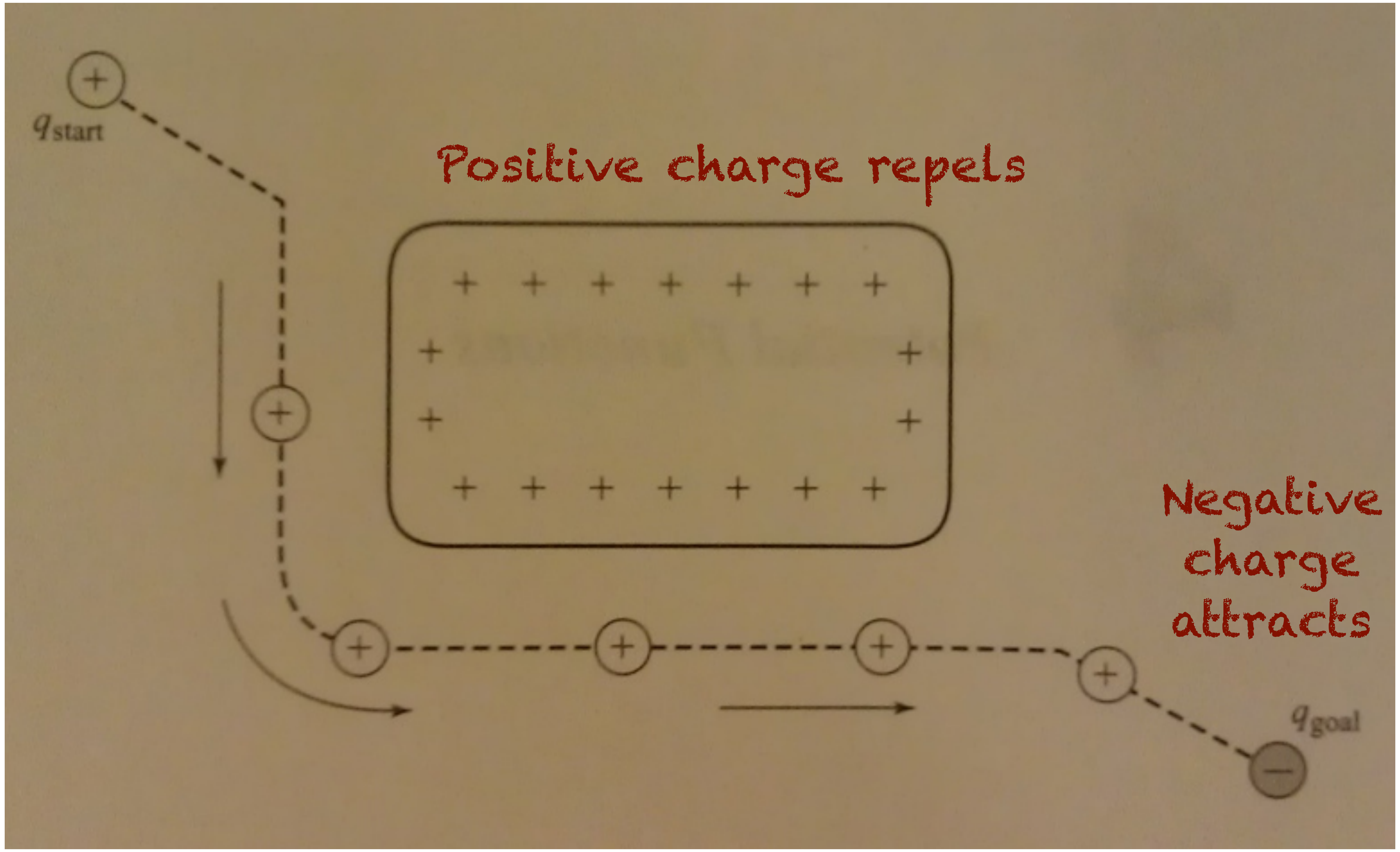
Potential Energy



- Energy stored in a physical system
- Kinetic motion caused by system moving to lower energy state
- For objects acting only w.r.t. gravity
  - potential_energy = mass*height*gravity

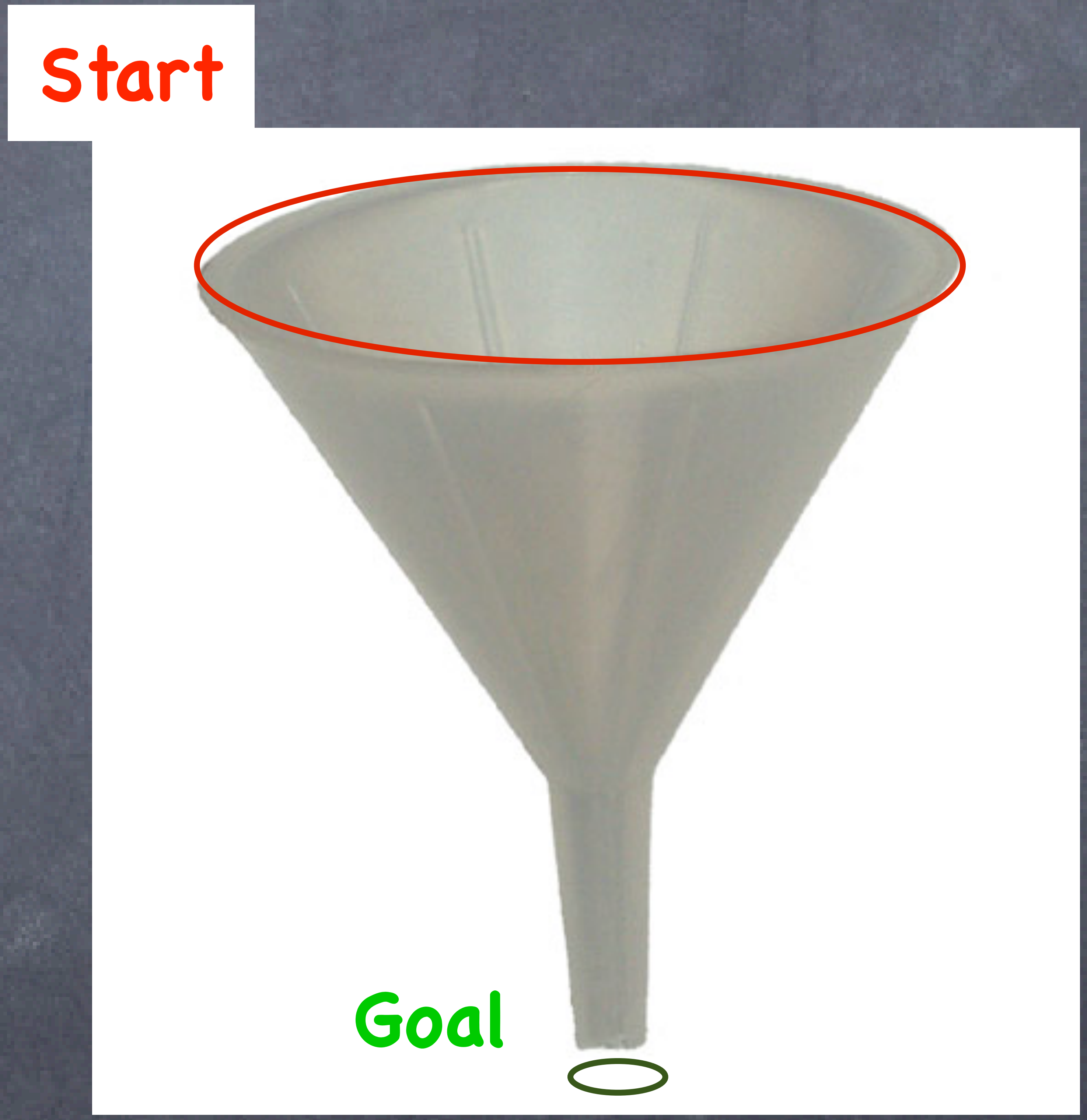
Charged Particle Example

Positively charged particle follows potential energy to goal



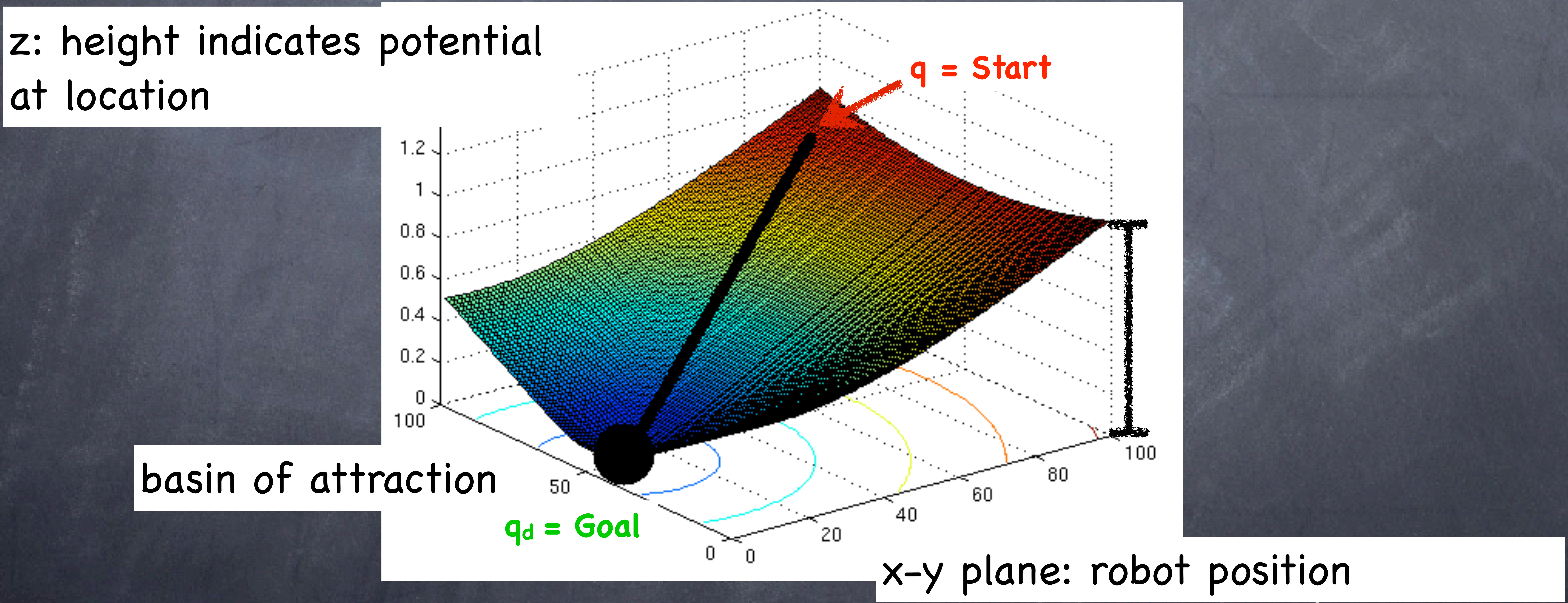
Convergent Potentials

let's call these "attractor landscapes"

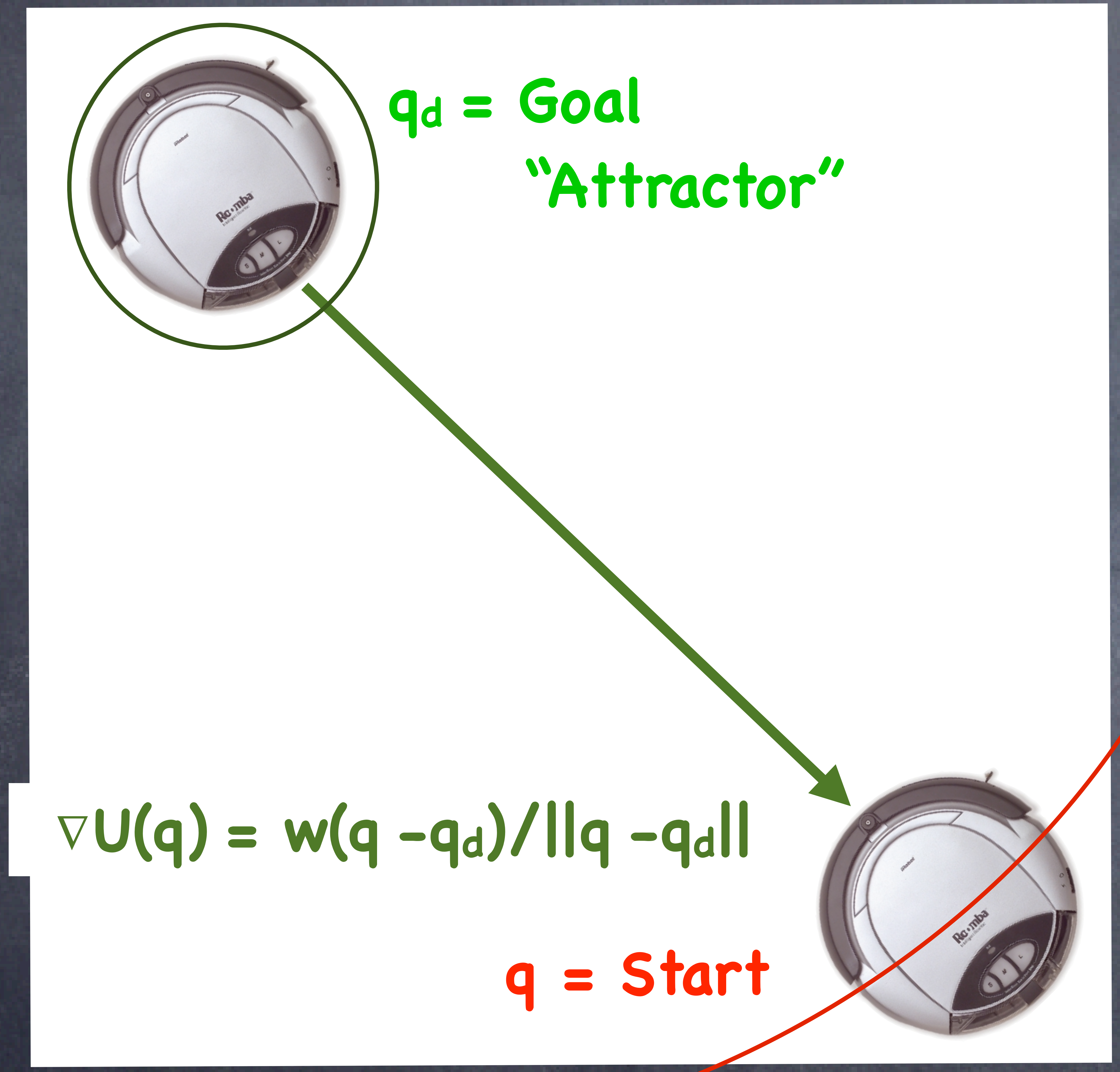


basin of attraction

2D potential navigation



"Cone" Attractor

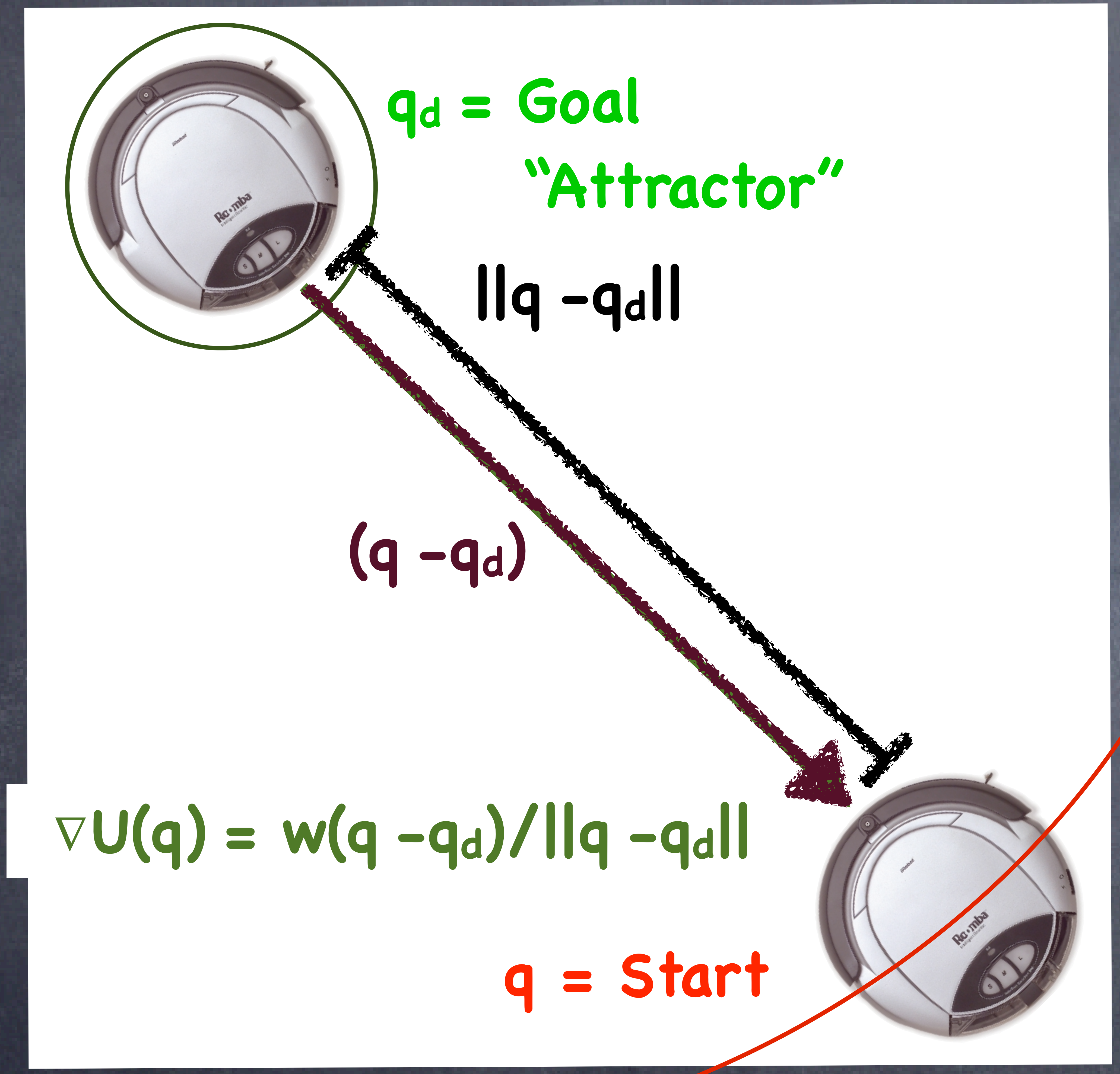


top view

w: weight
$(q - q_d)$: direction
$\|q - q_d\|$: distance

side view

"Cone" Attractor

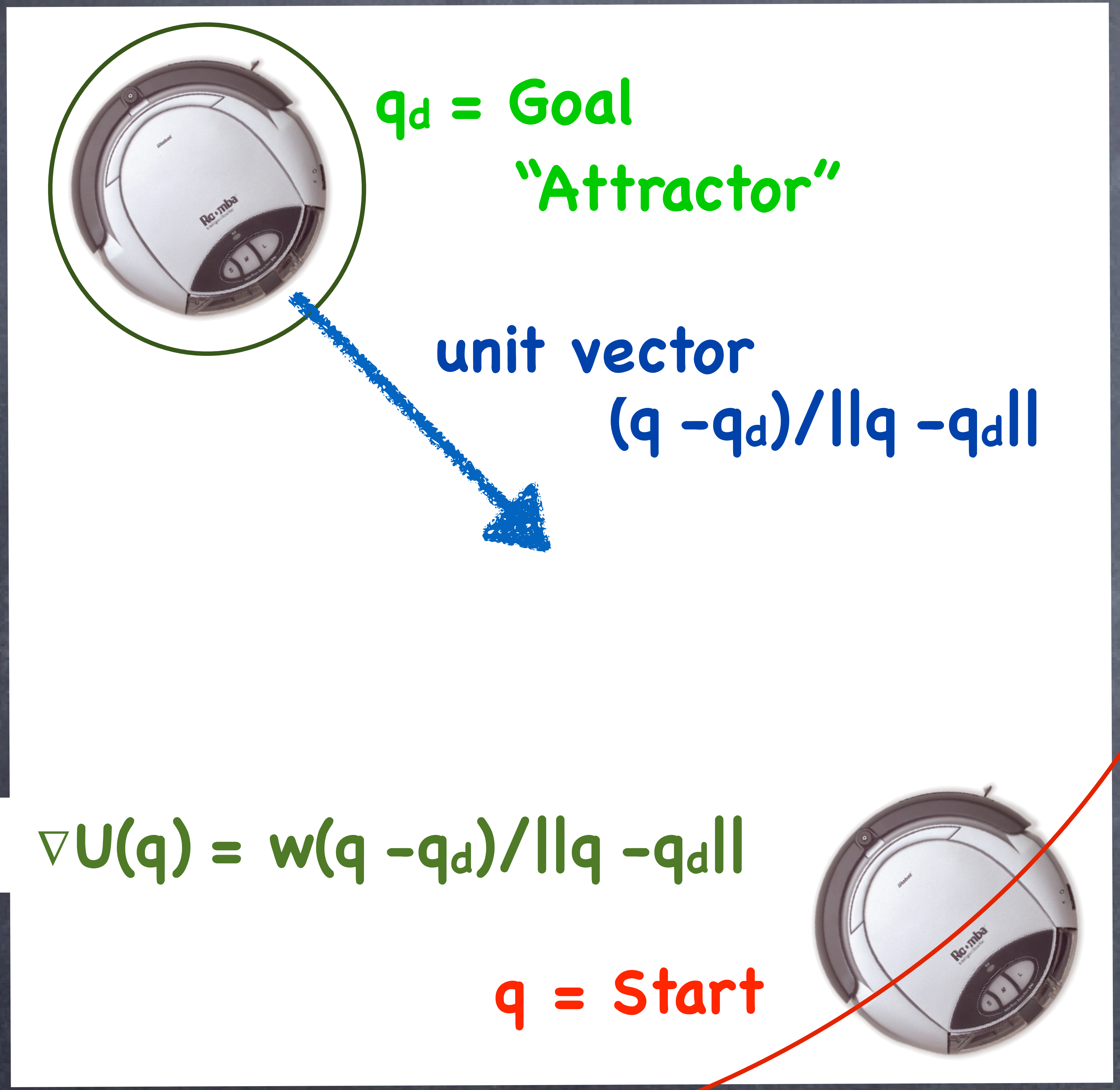


top view

w: weight
$(q - q_d)$: direction
$\|q - q_d\|$: distance

side view

"Cone" Attractor



$\nabla U(q) = w(q - q_d)/\|q - q_d\|$

top view

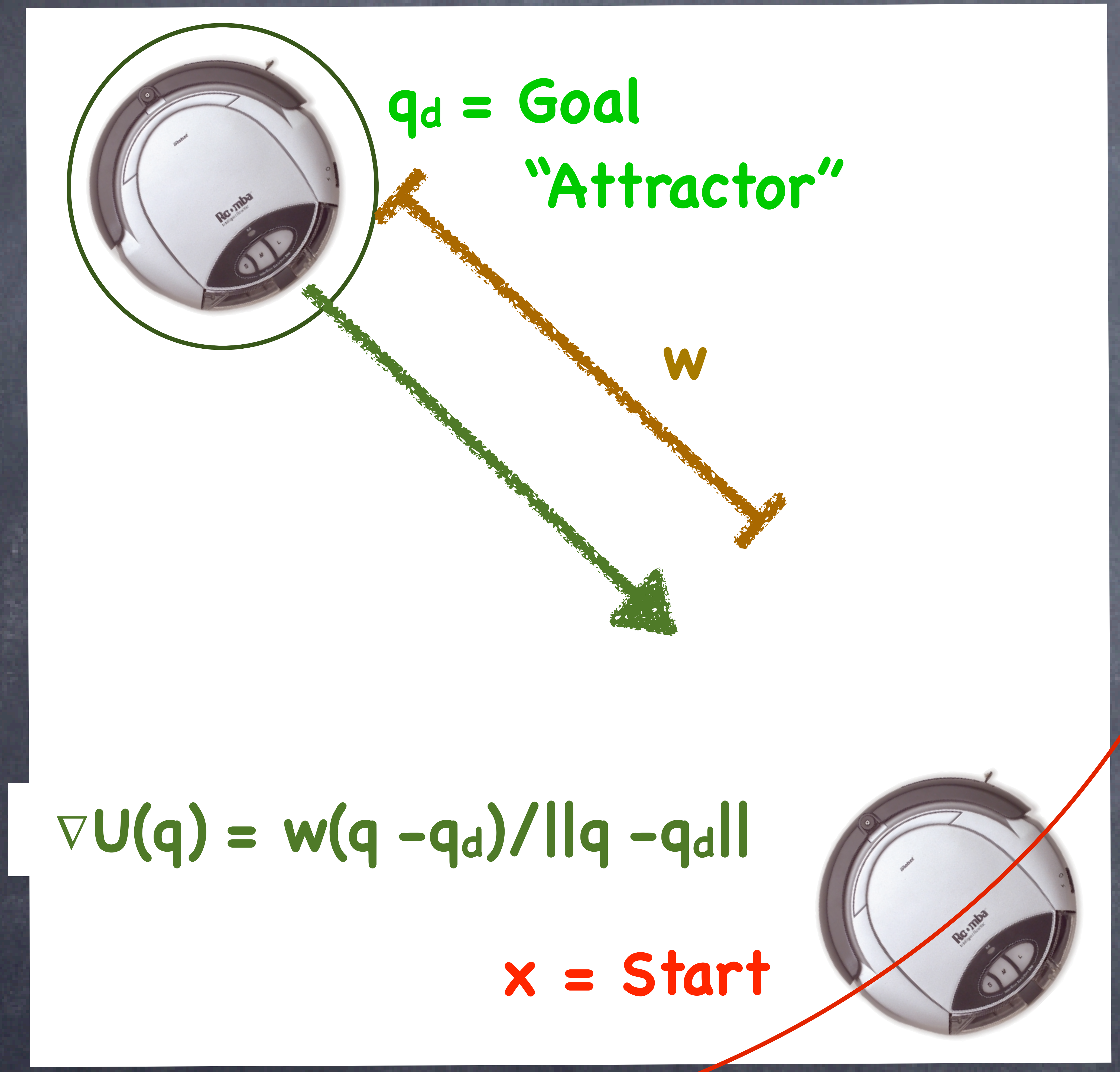
w: weight

$(q_d - q)$: direction

$\|q_d - q\|$: distance

side view

"Cone" Attractor



top view

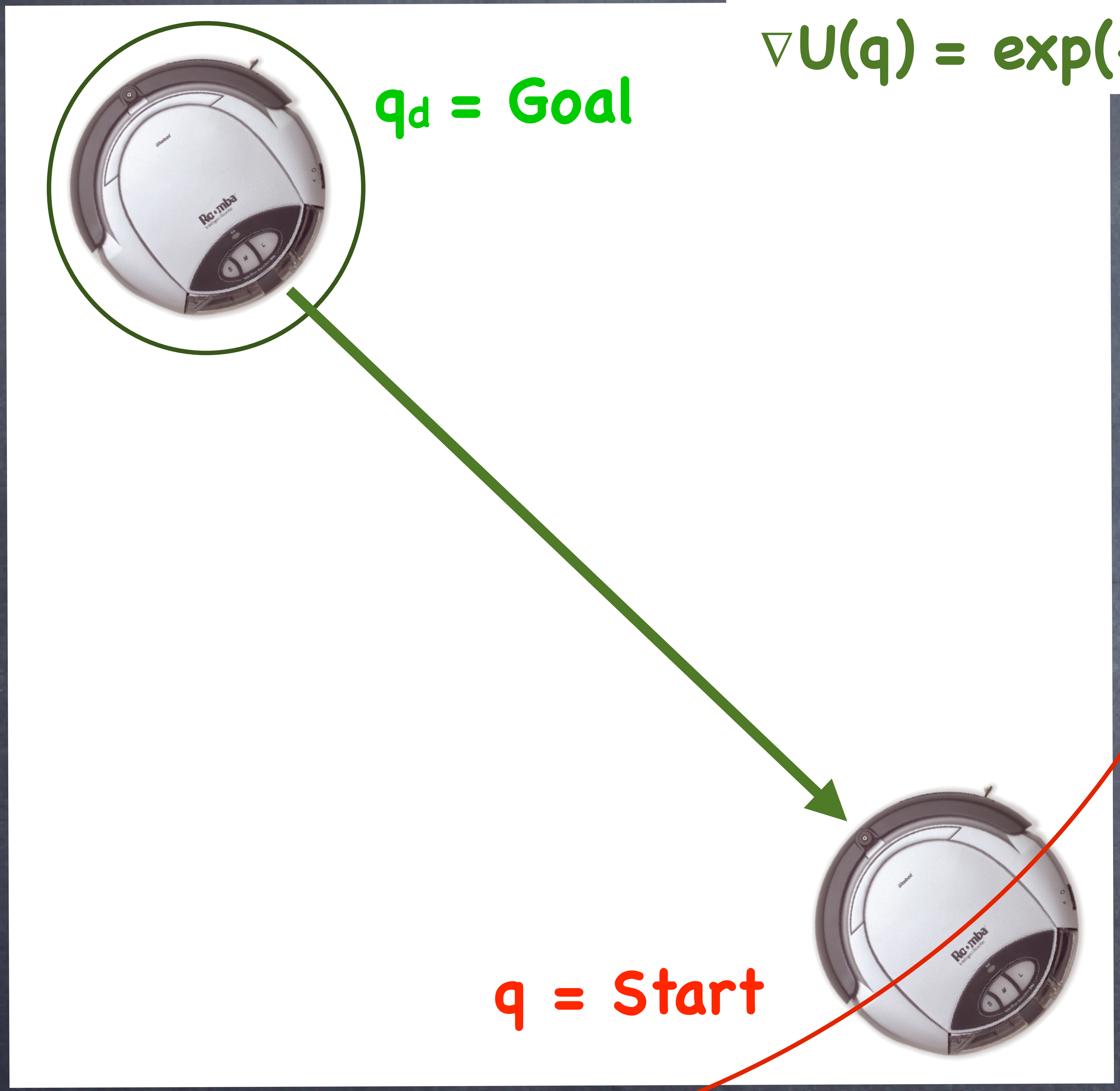
w: weight (< 1)
$(q - q_d)$: direction
$||q - q_d||$: distance

side view

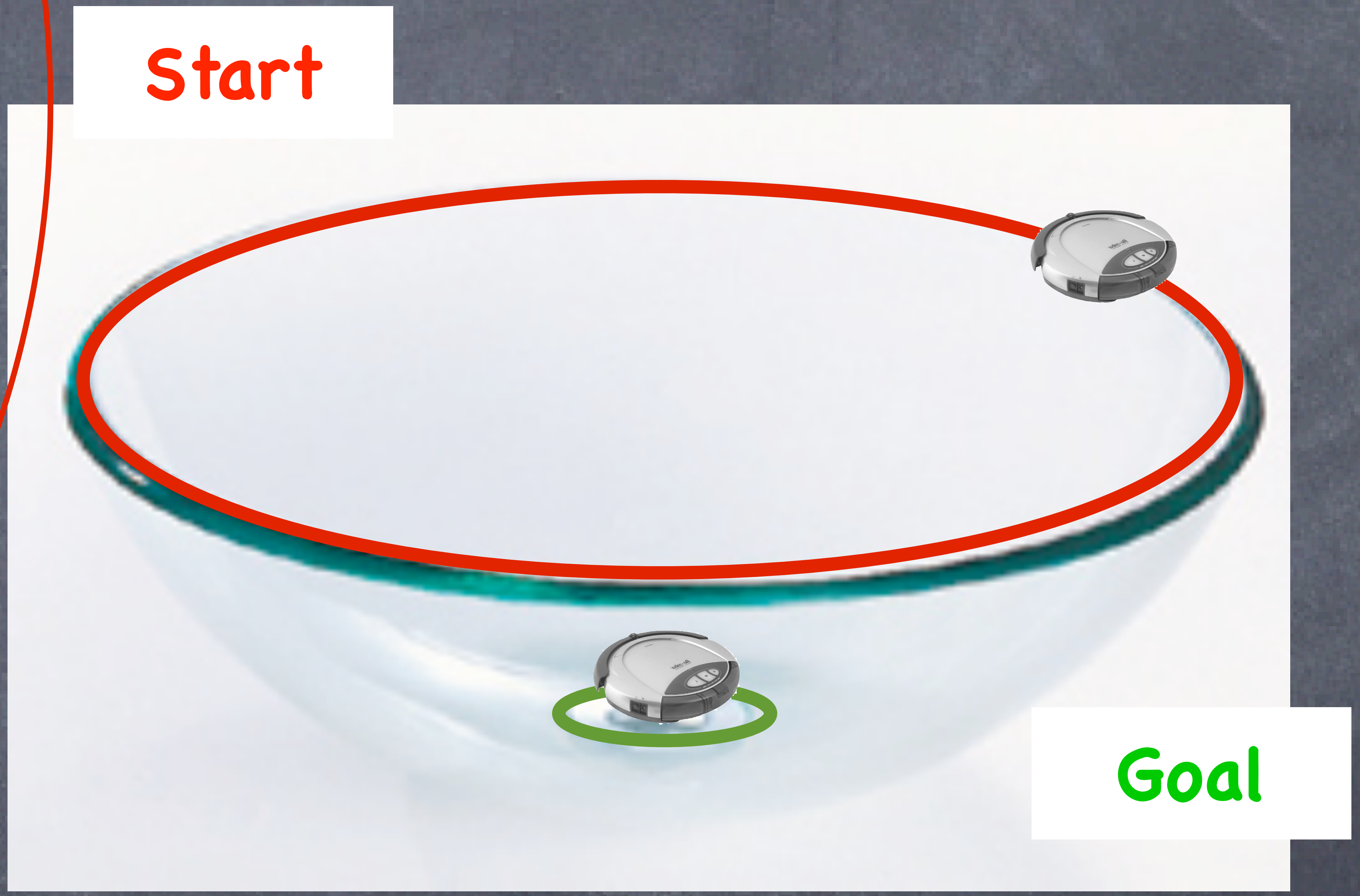
Can we modulate the
range of a potential field?

"Bowl" Attractor

$\nabla U(q) = \exp(-\|q - q_d\|/w)\,(q - q_d)$



top view

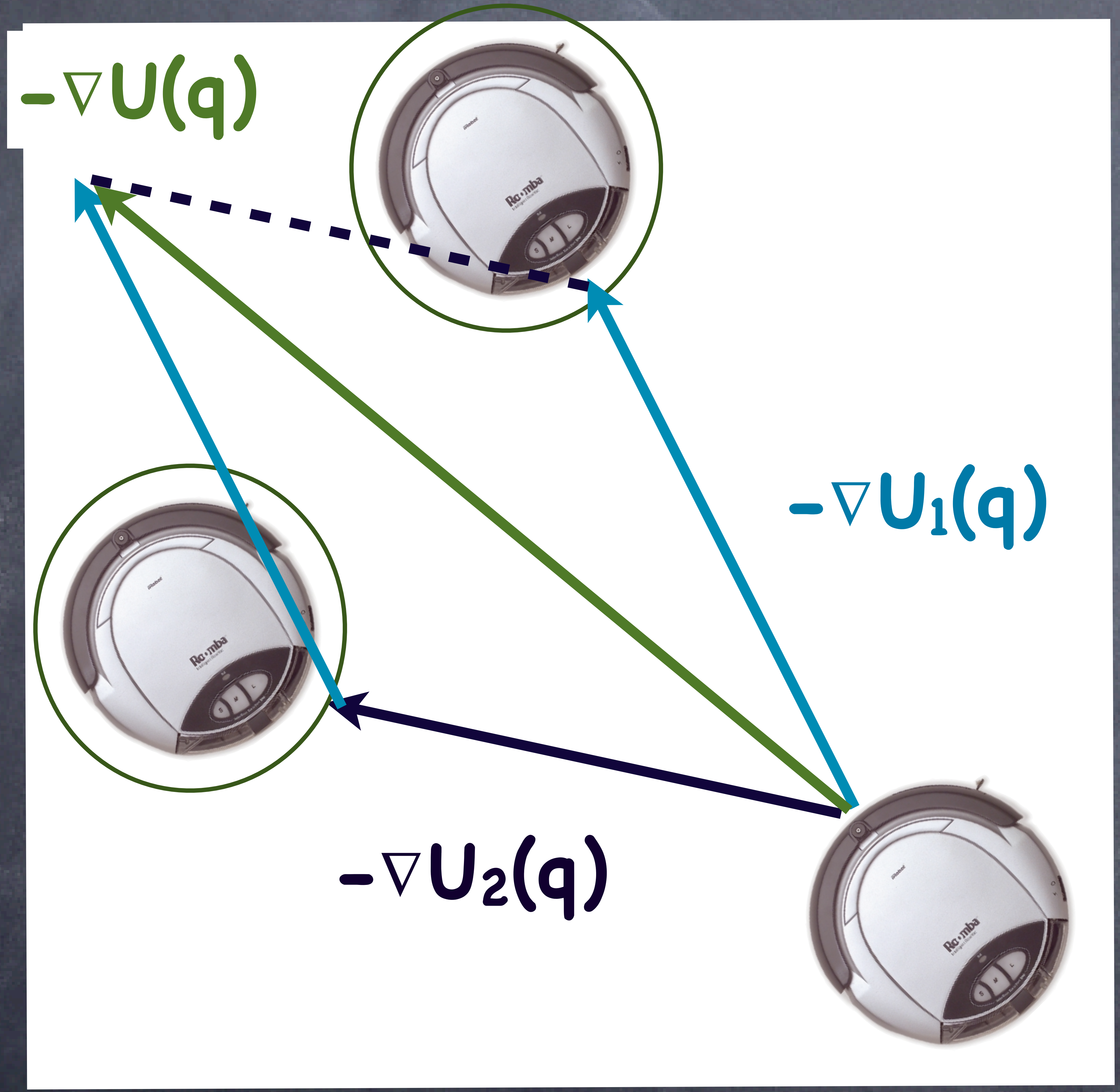


side view


desmos.com/calculator
Untitled Graph
desmos
Create Account
or
Sign In
$e^{-\frac{(x^2)}{10}}$
$e^{-\frac{(x^2)}{4}}$
$e^{-\frac{(x^2)}{2}}$
$e^{-\frac{(x^2)}{1}}$
$e^{-\frac{(x^2)}{0.5}}$
$e^{-\frac{(x^2)}{0.25}}$
$\frac{(x^2)}{0.1}$
exp(-d/w)
weights the influence of a potential
exp(-d/.1)
exp(-d/0.5)
exp(-d/1)
exp(-d/2)
exp(-d/10)
x-axis is d = distance
-5
-4
-3
-2
-1
0
1
2
3
4
2
1
-1
-2

Can we combine
multiple potentials?

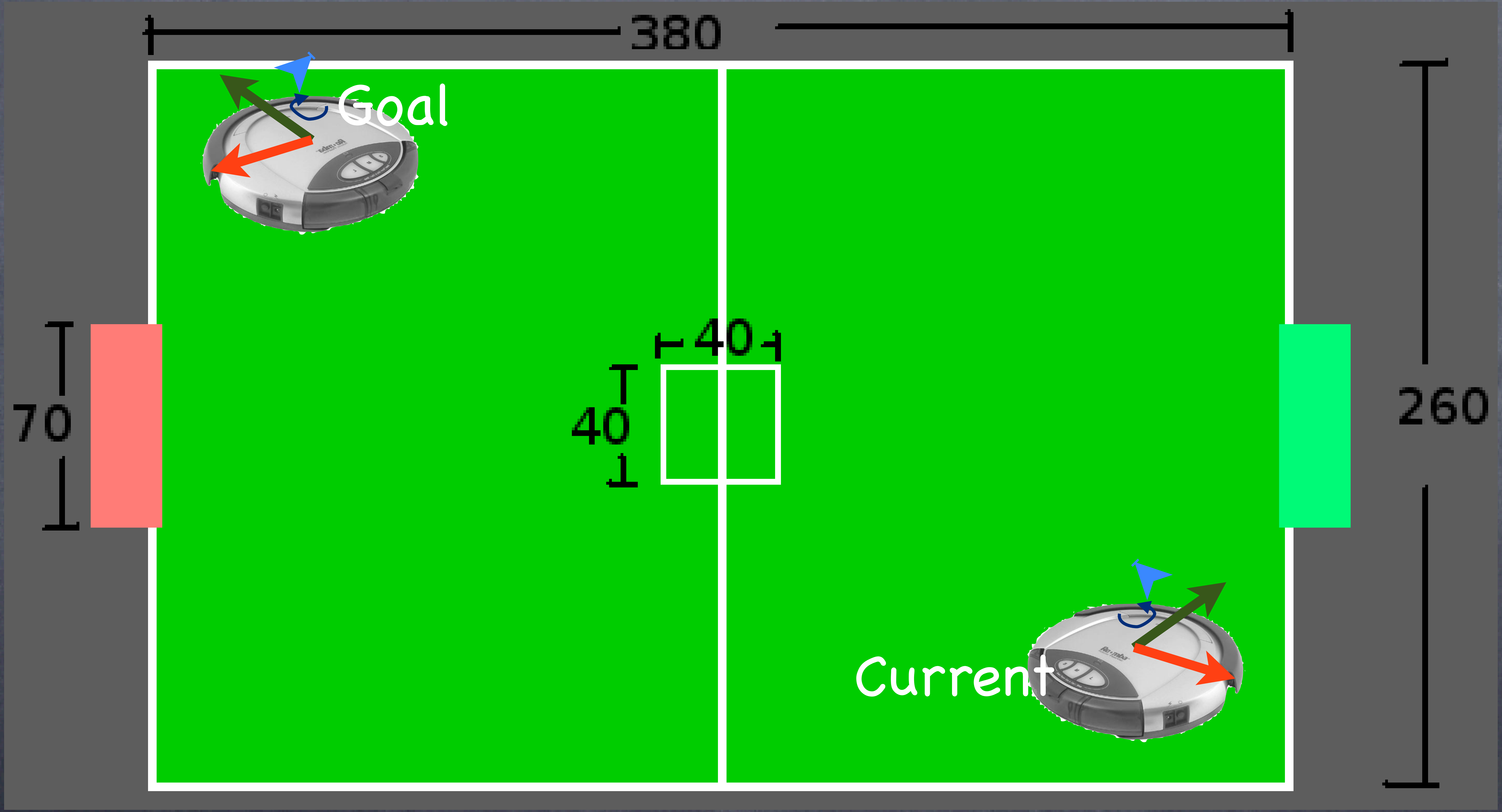
Multiple potentials



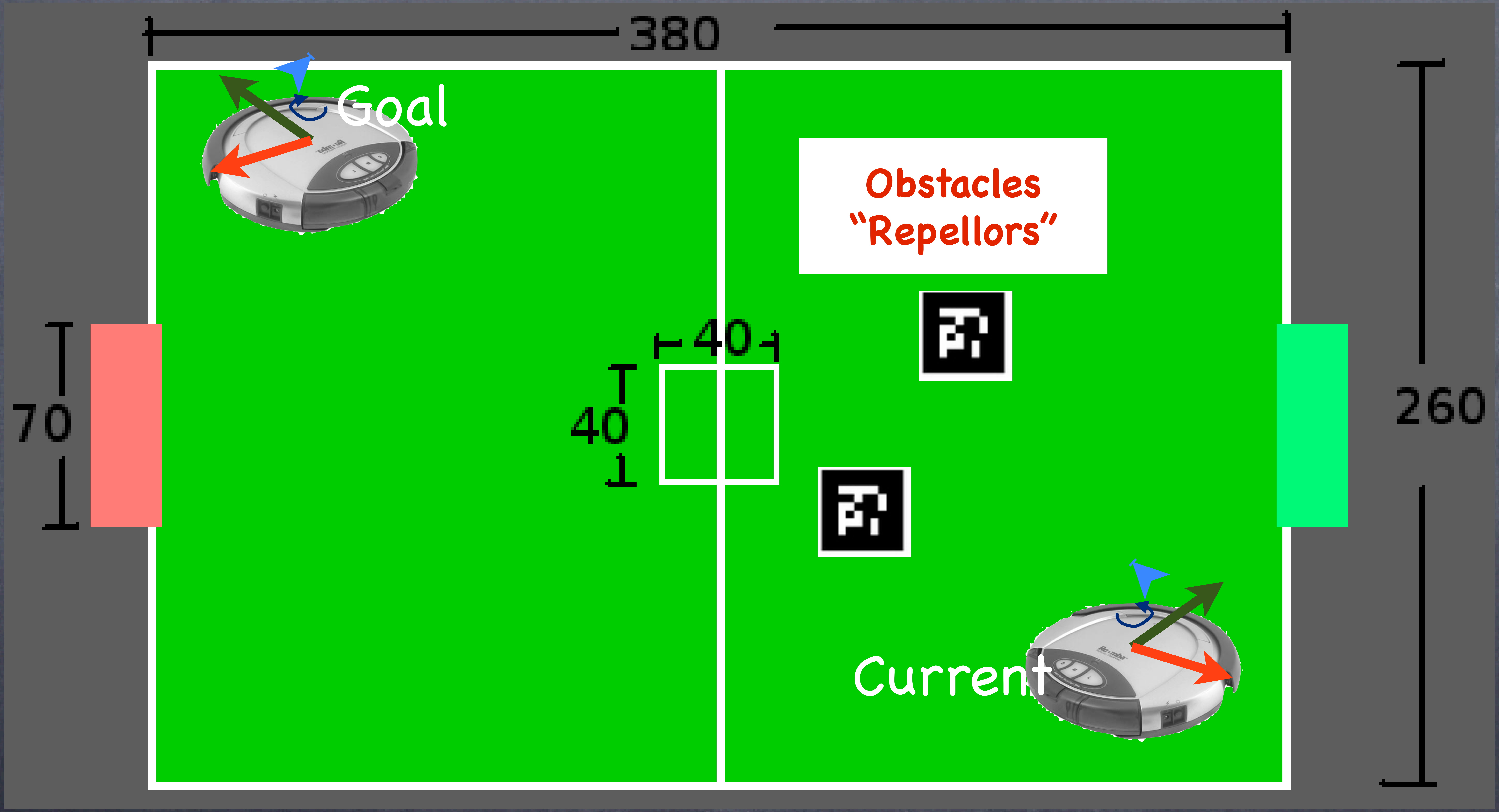
- Output of potential field is a vector
- Combine multiple potentials through vector summation

$$U(q) = \Sigma_i U_i(q)$$

describe performance for this case

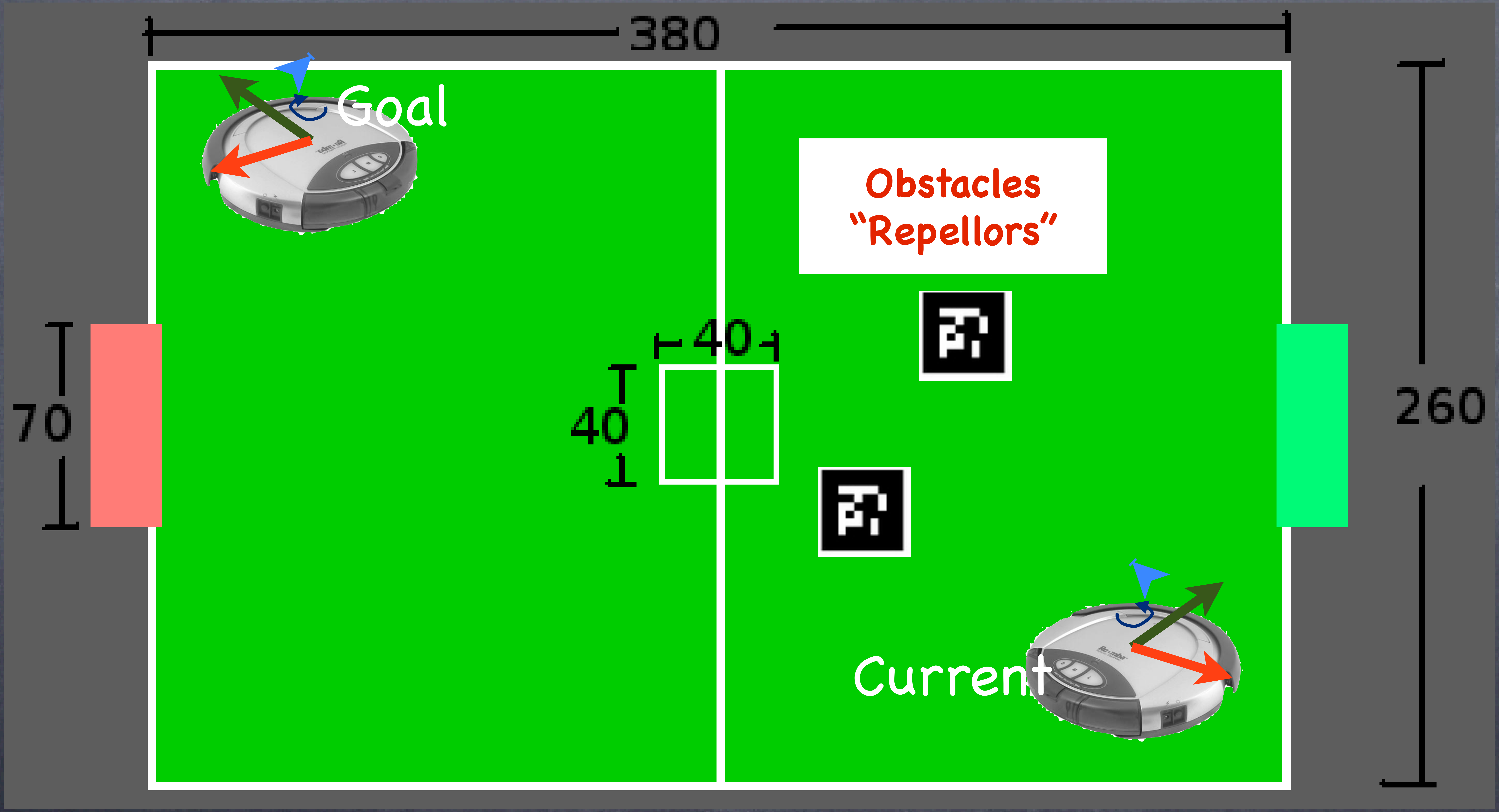


describe performance for this case



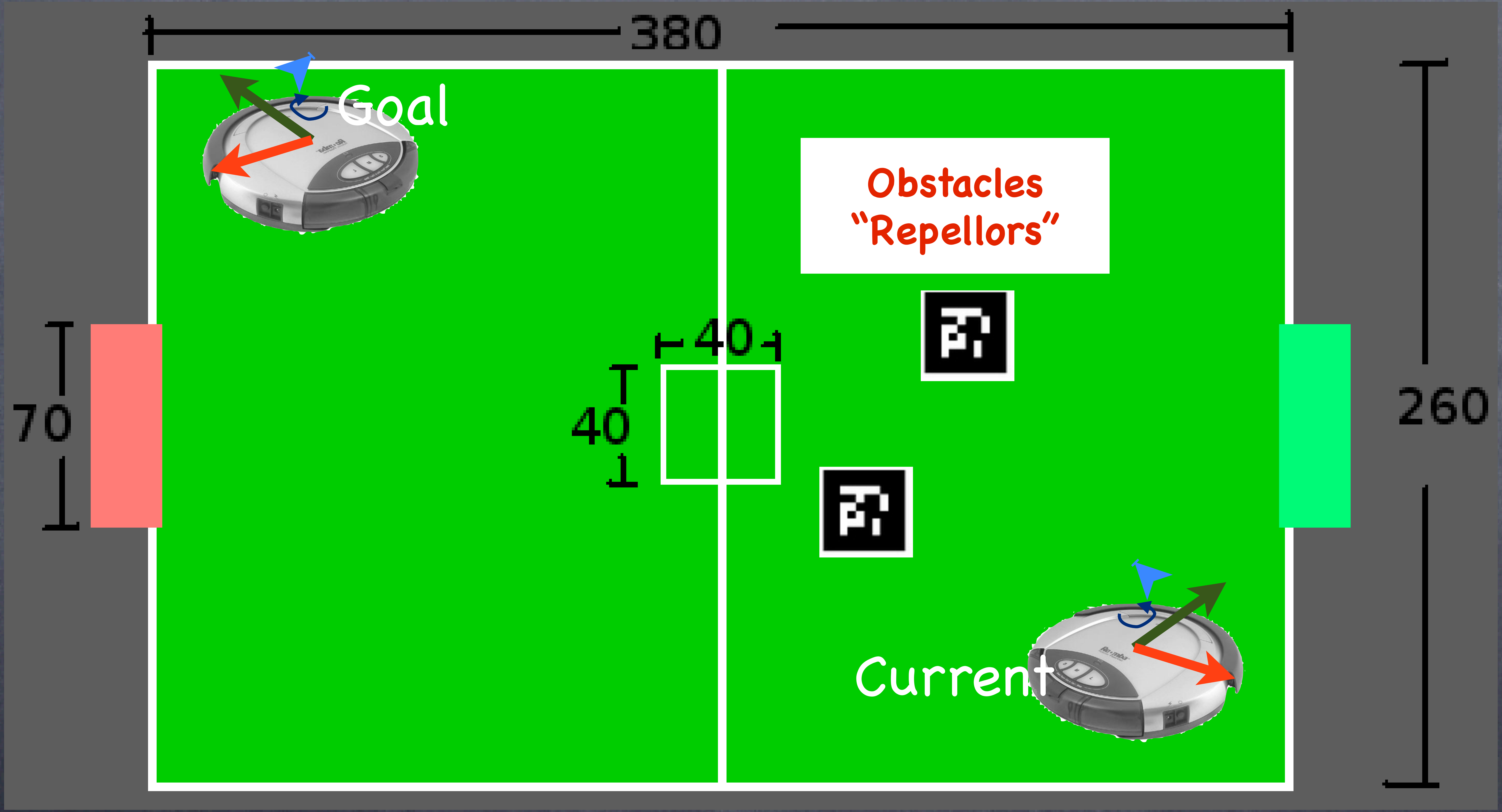
describe performance for this case

how do we deal with repellors?



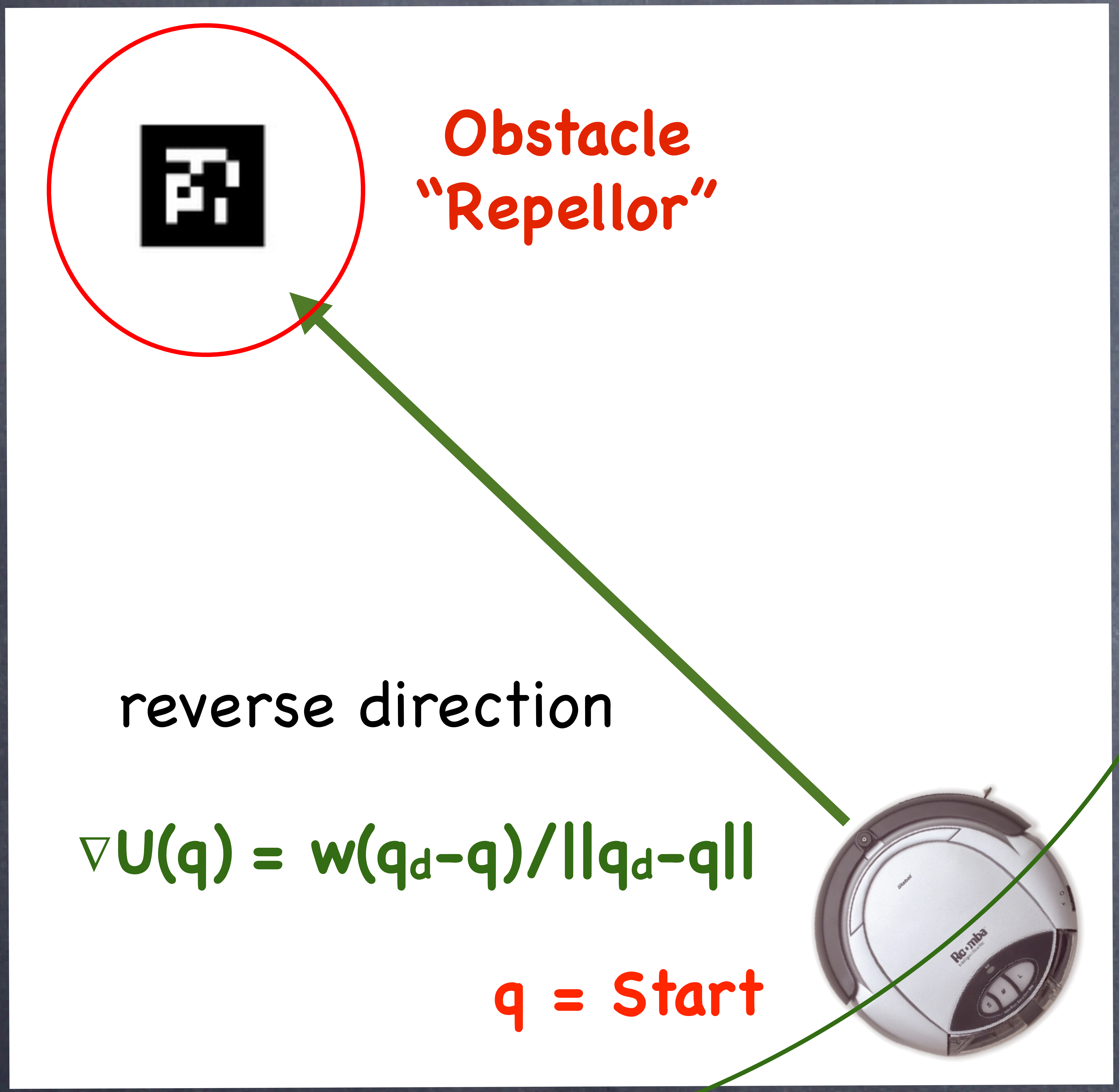
add sum of repulsive potentials

$$U(q) = U_{attracts}(q) + U_{repellors}(q)$$



"Cone" Repellor

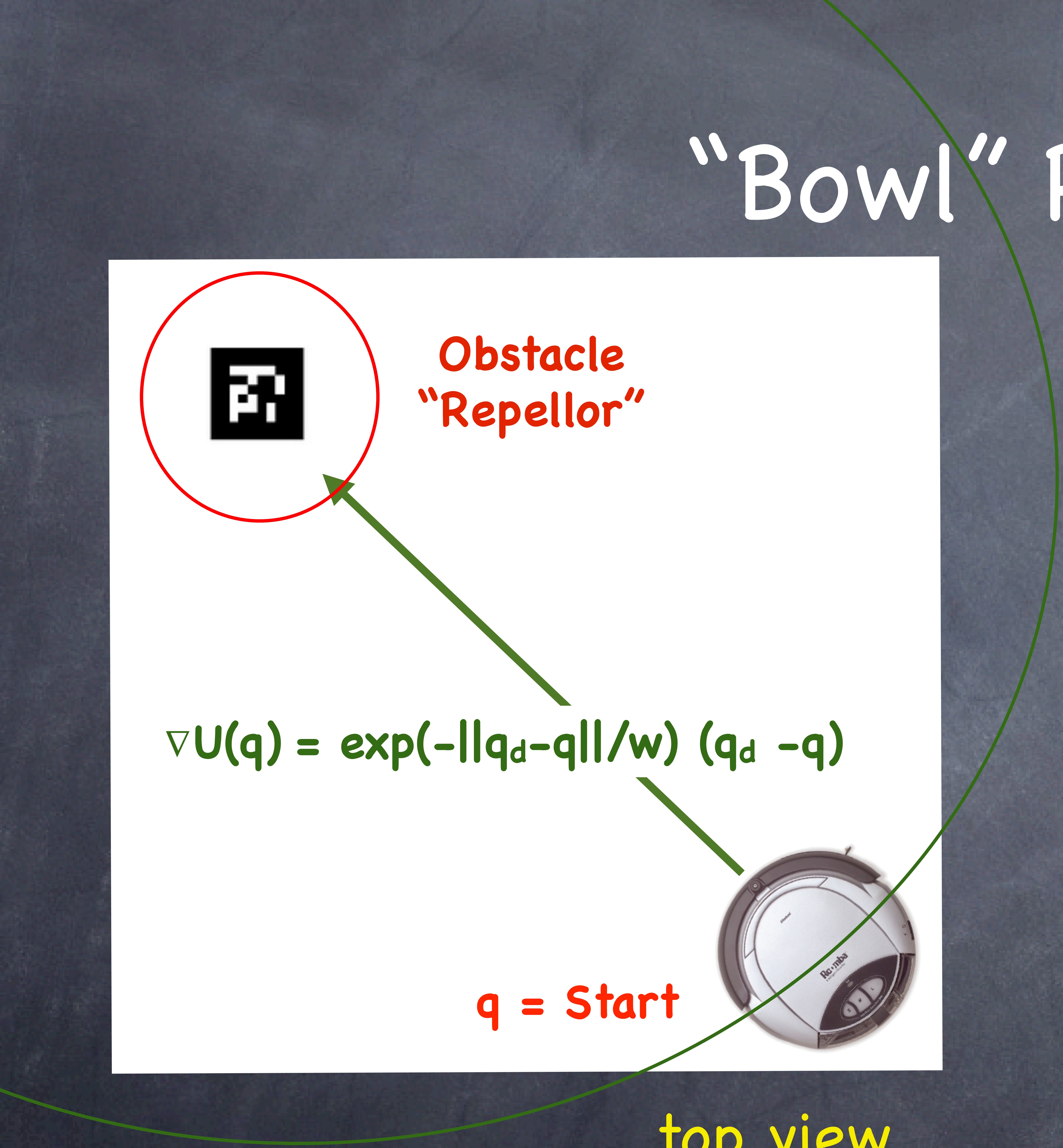
potential problems?



top view

side view

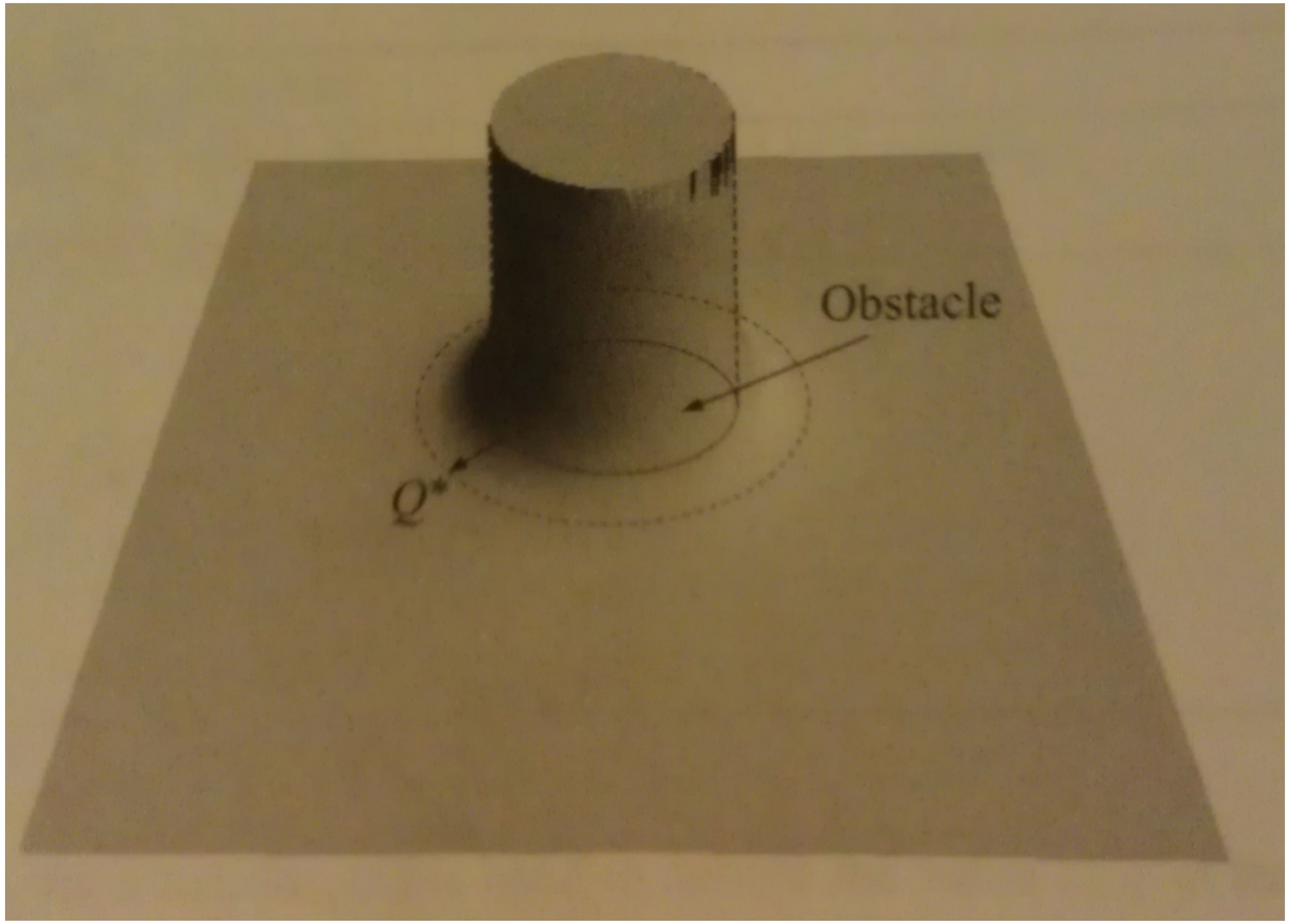
"Bowl" Repellor



top view

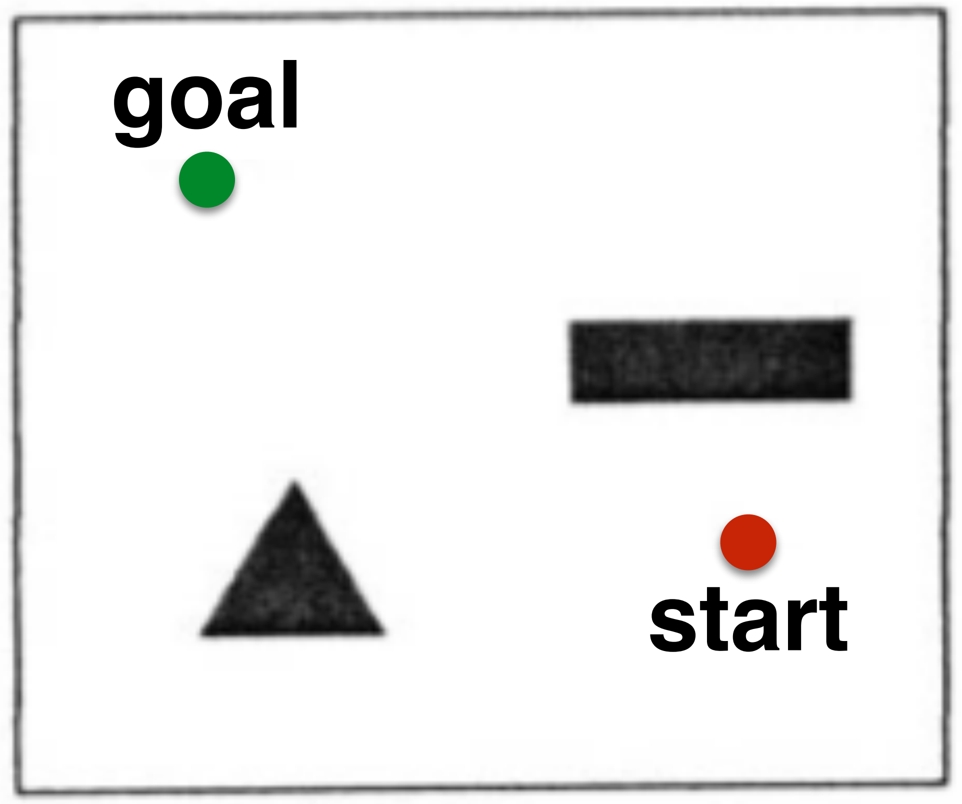
side view

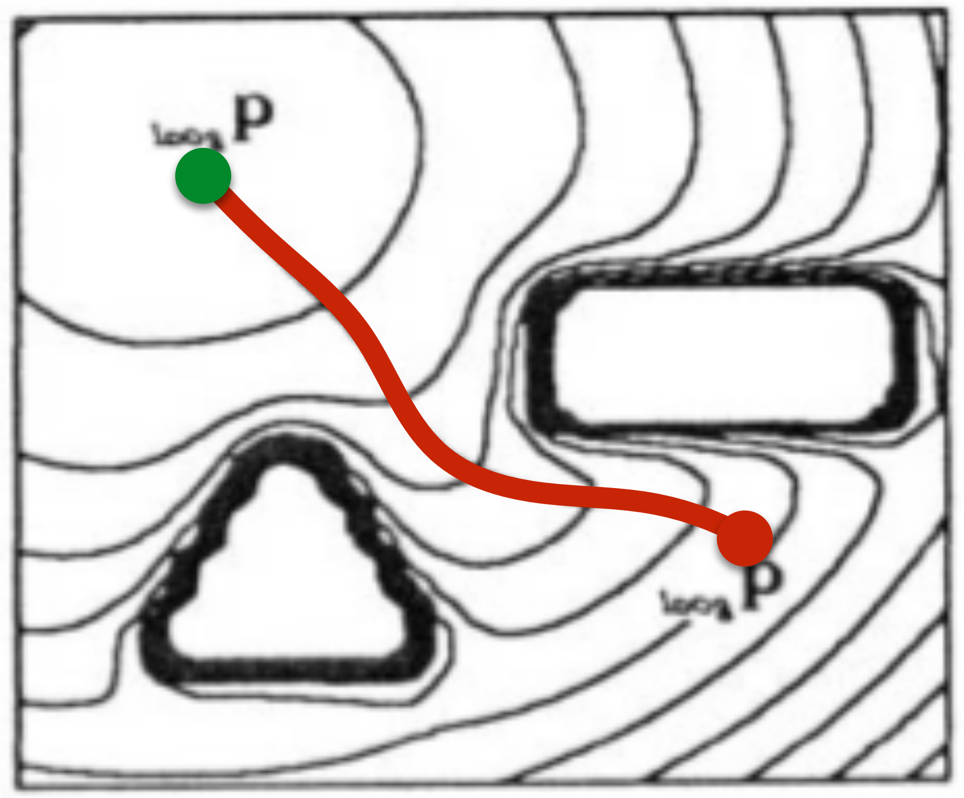
repellor should only have local influence,
repelling only around boundary improves path



2 Obstacle example

configuration space





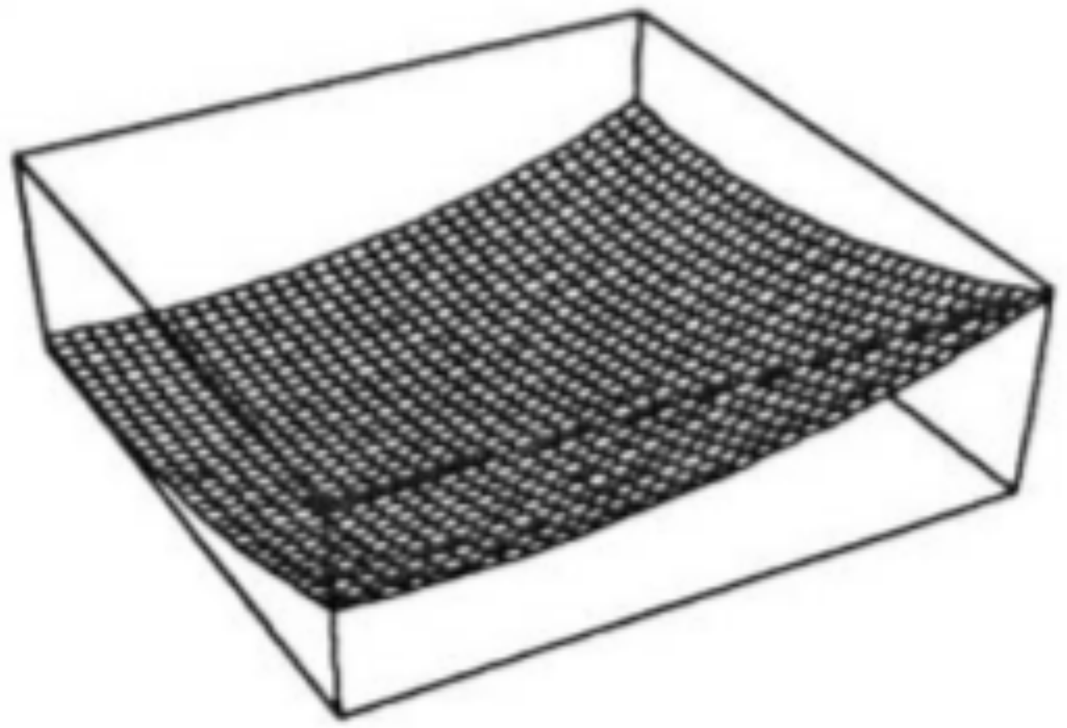
path from descent on gradient field

resulting gradient field

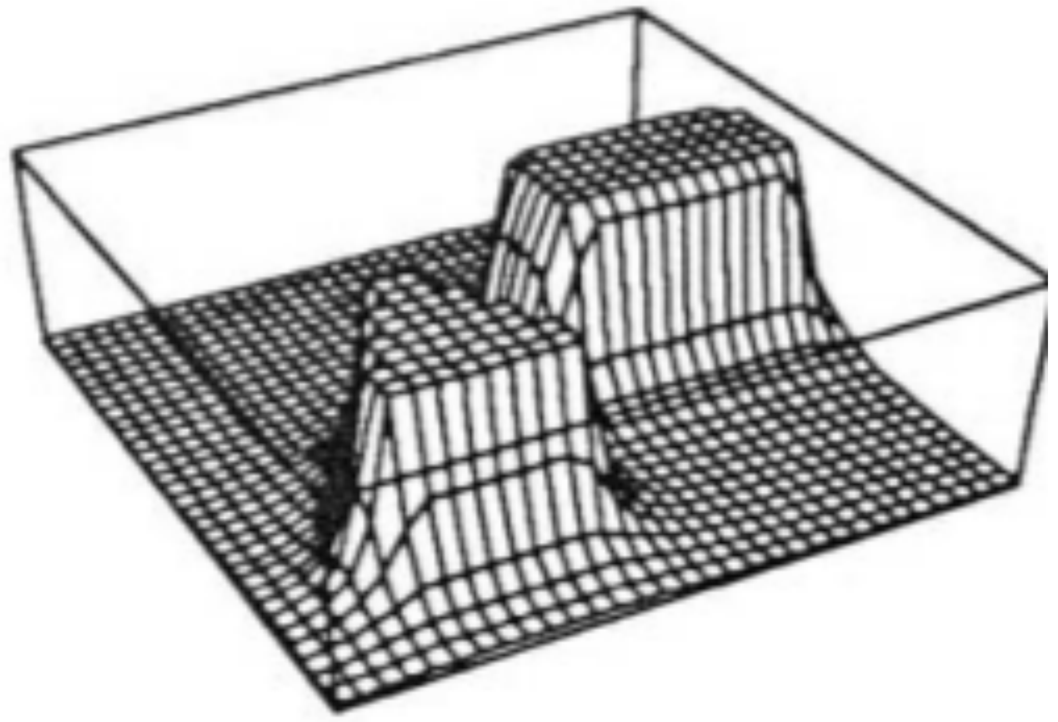
attractor field

repellor fields

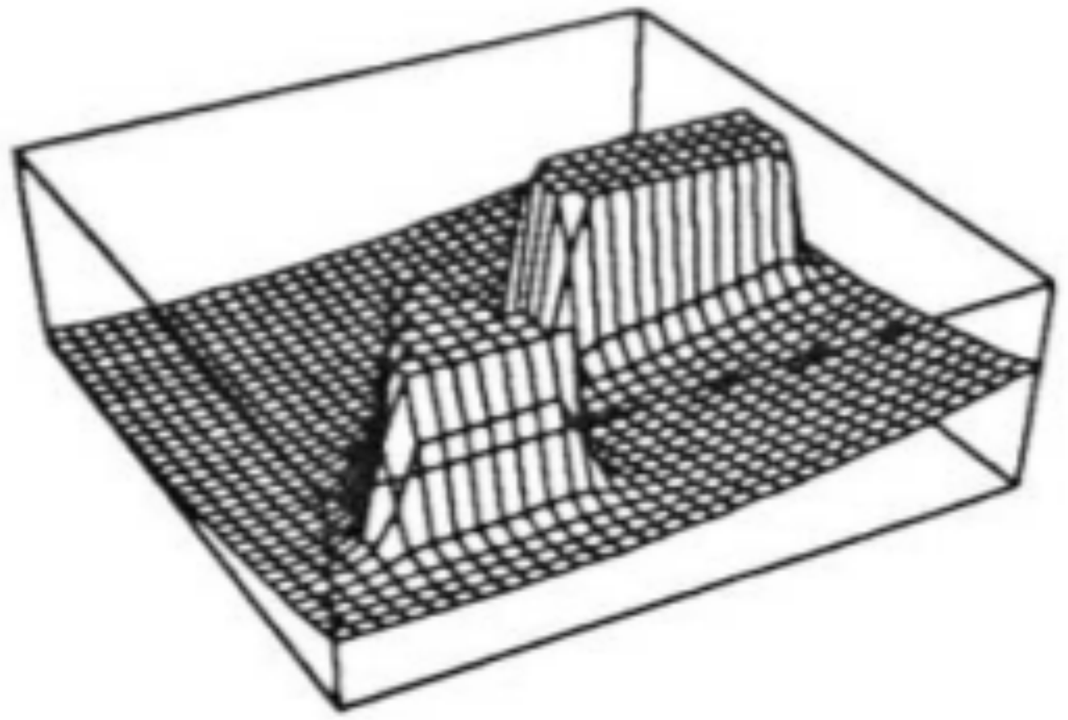
combined potential

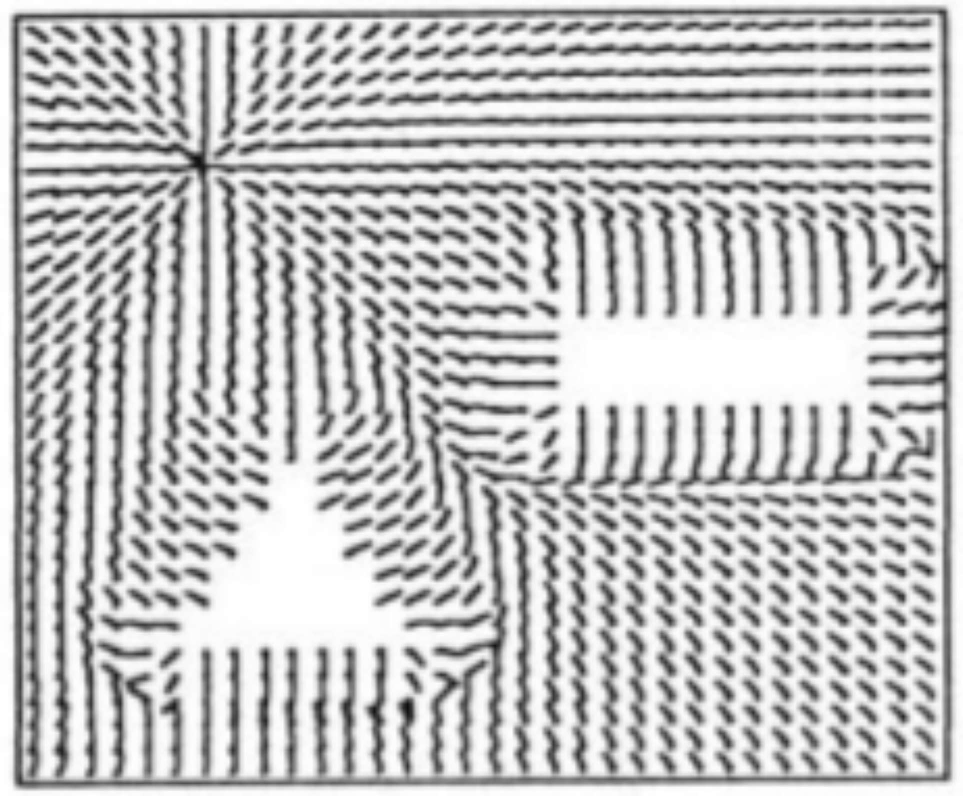


+

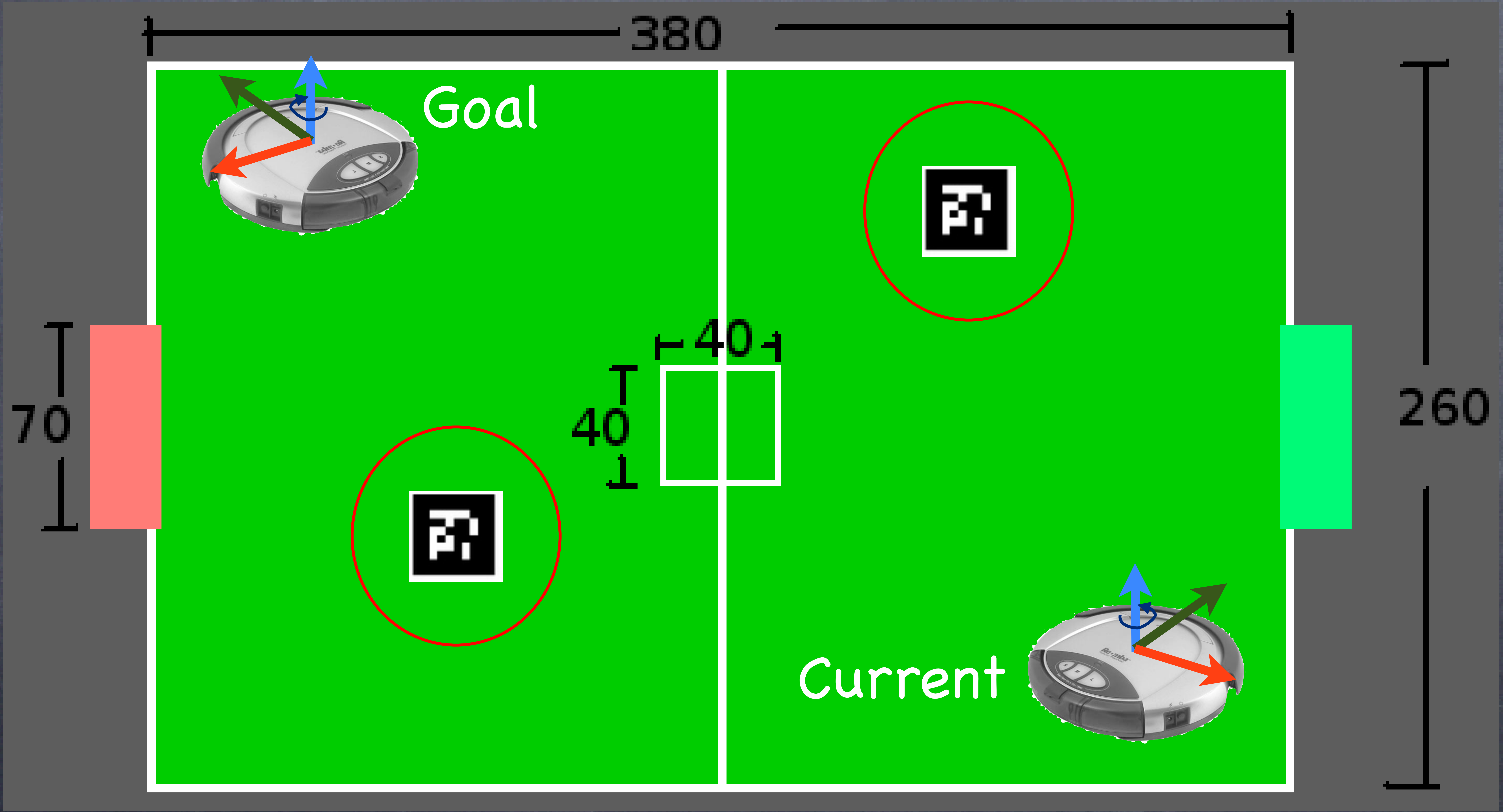


=

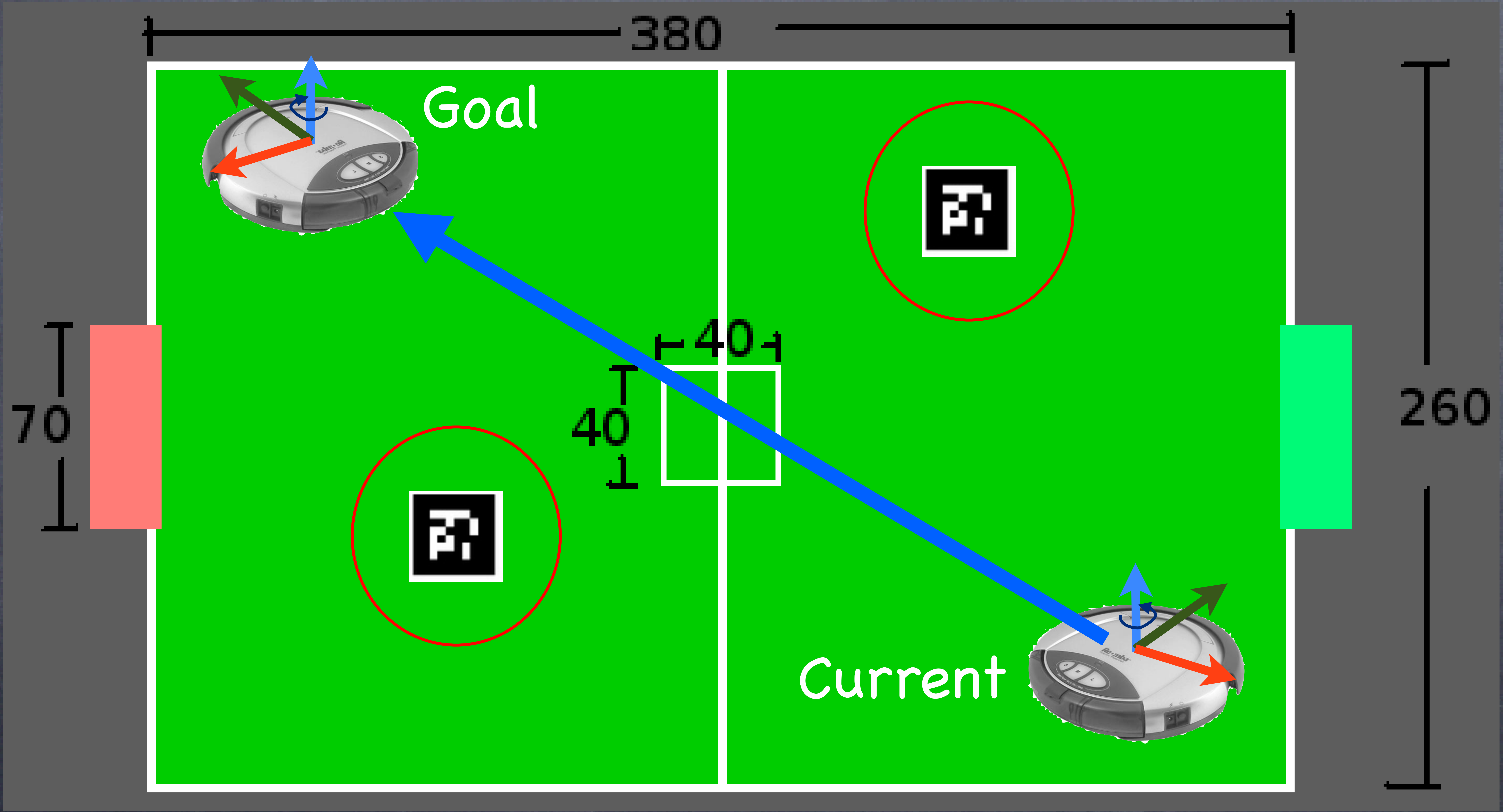




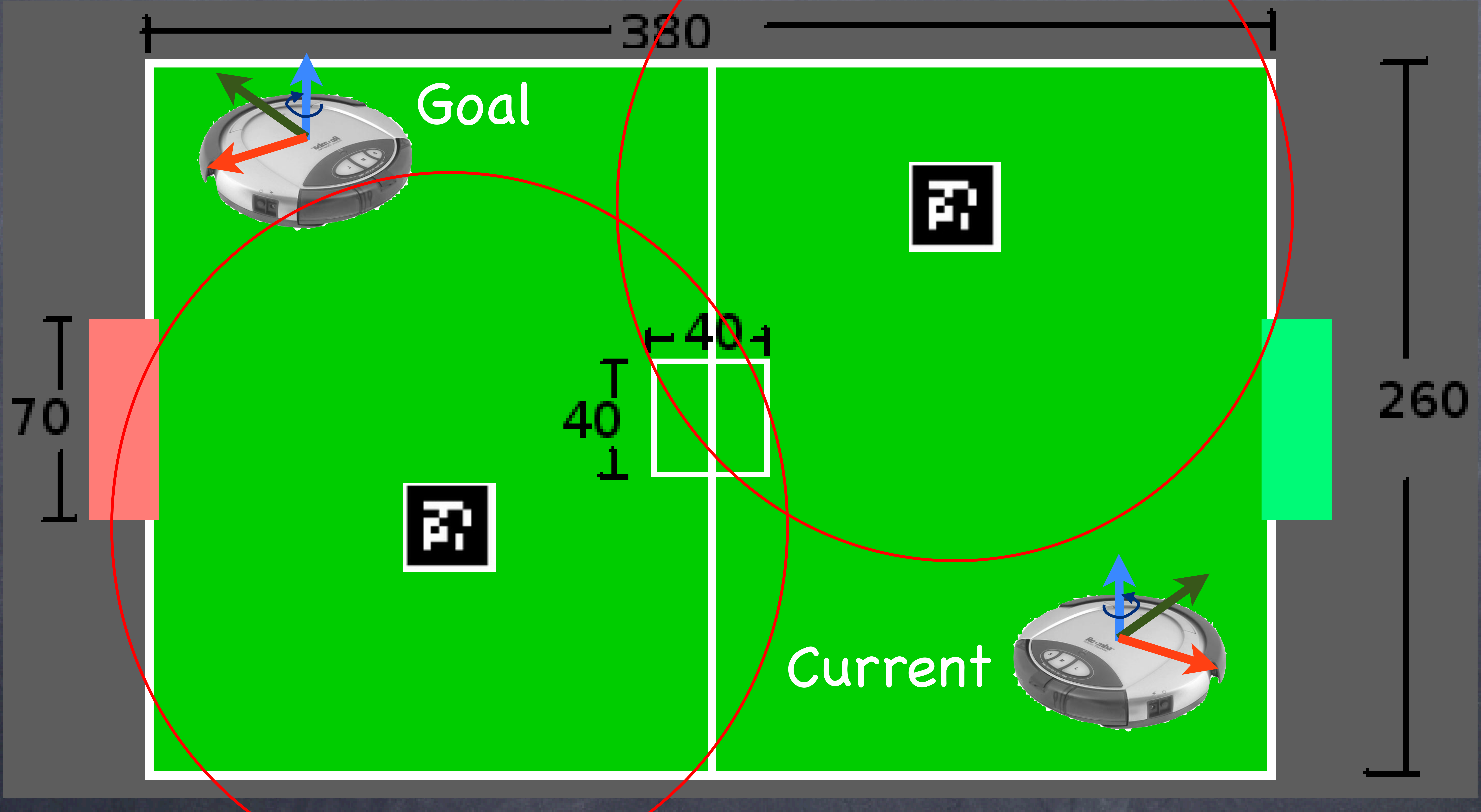
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



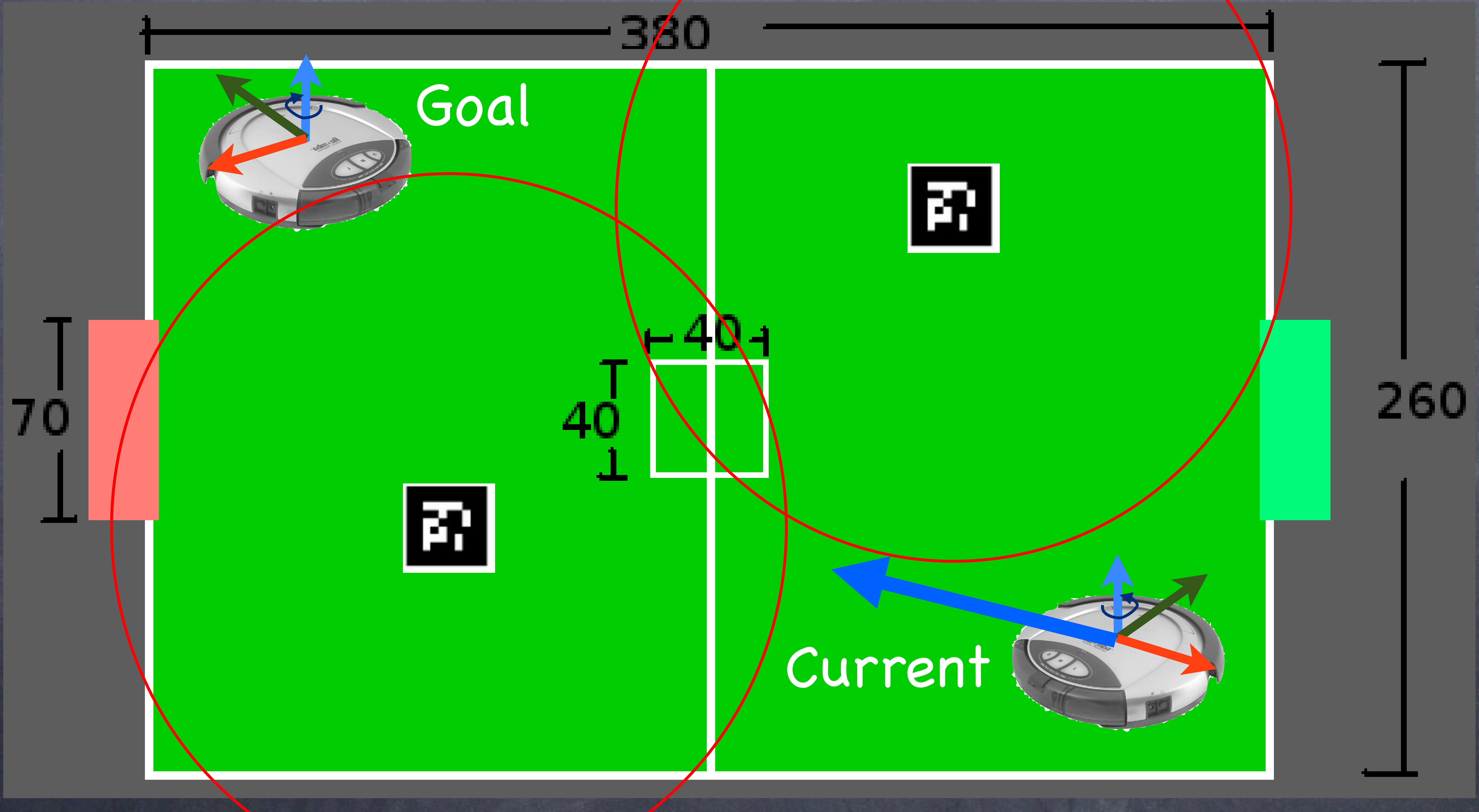
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



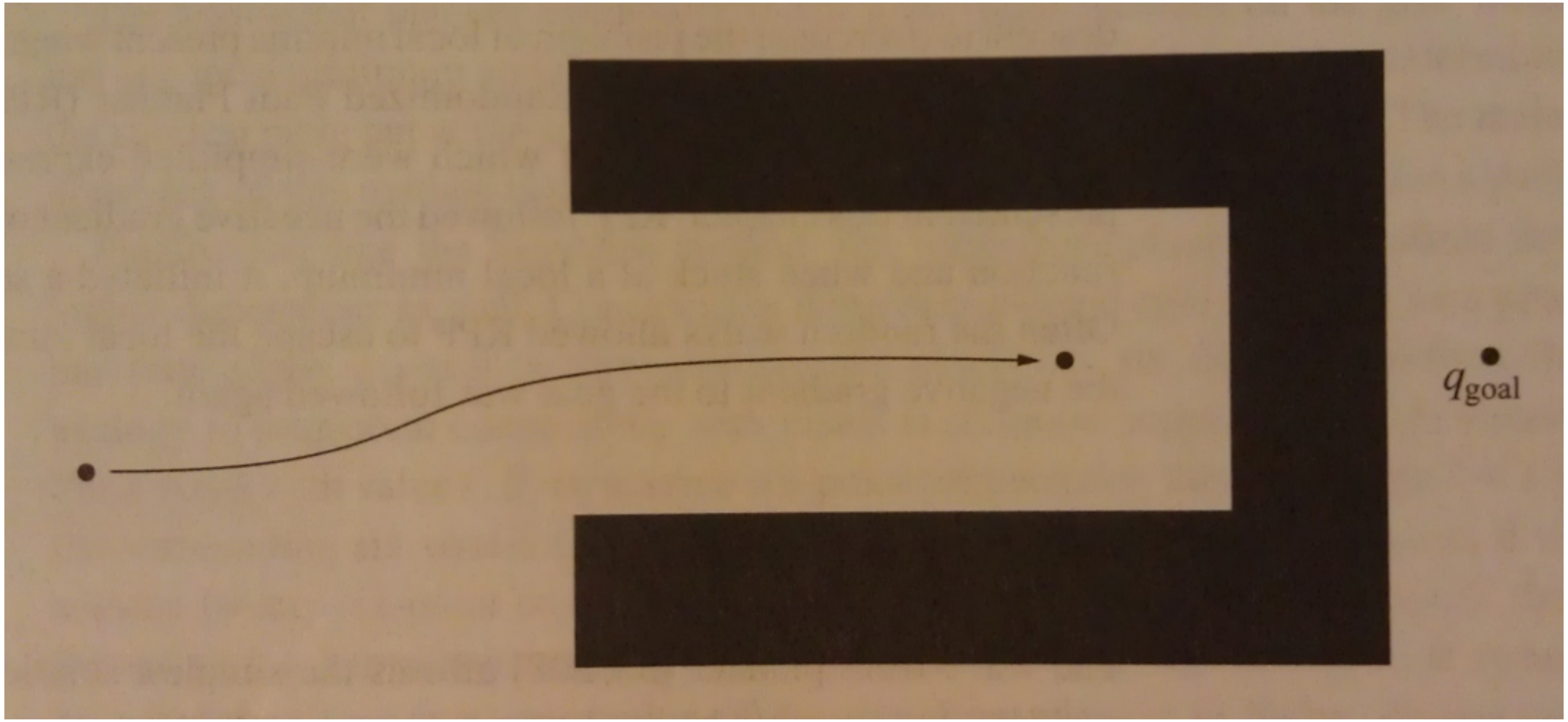
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



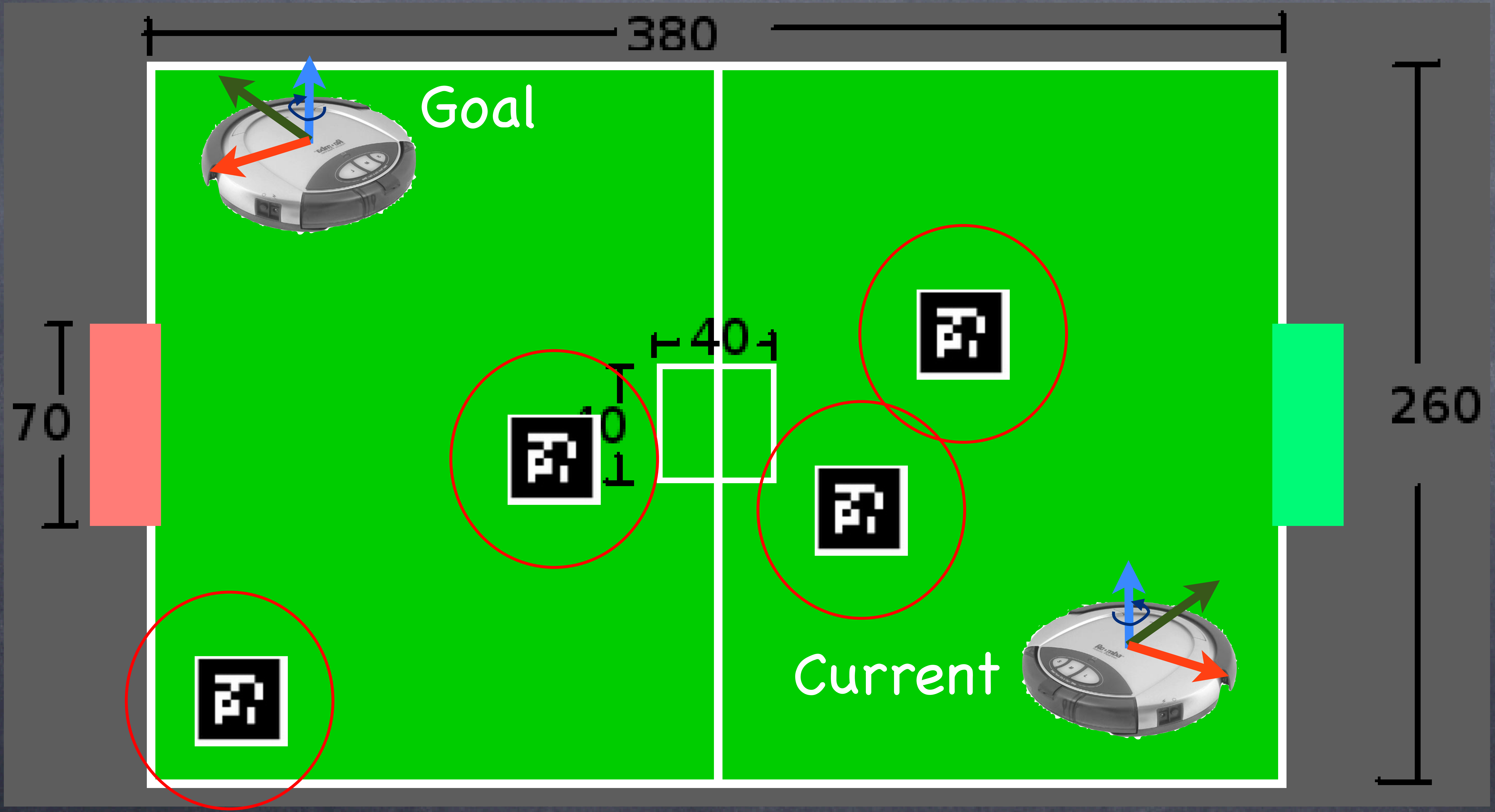
describe performance for this case
with cone attractor to goal and bowl repellors
with limited weight



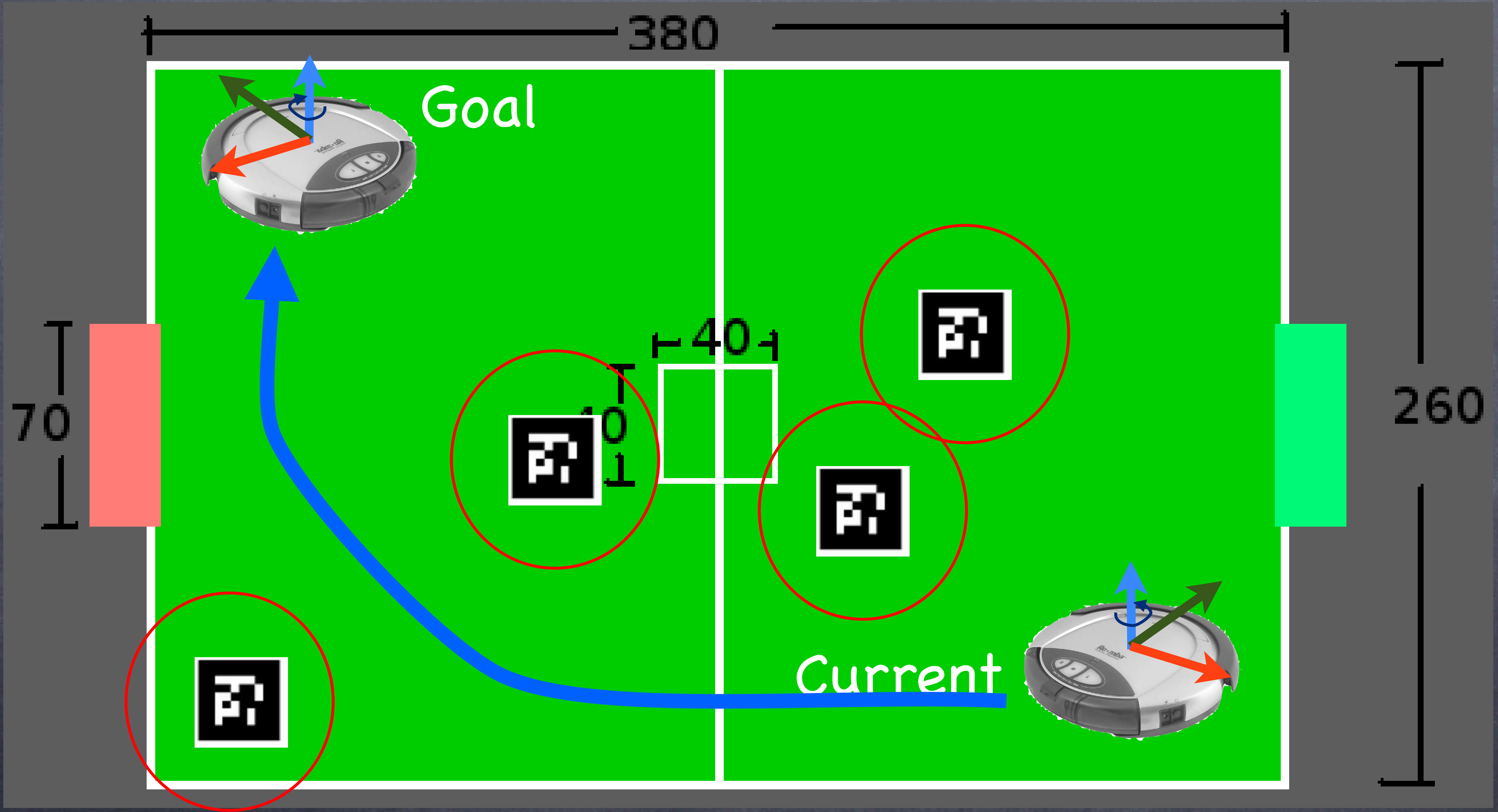
Local Minima



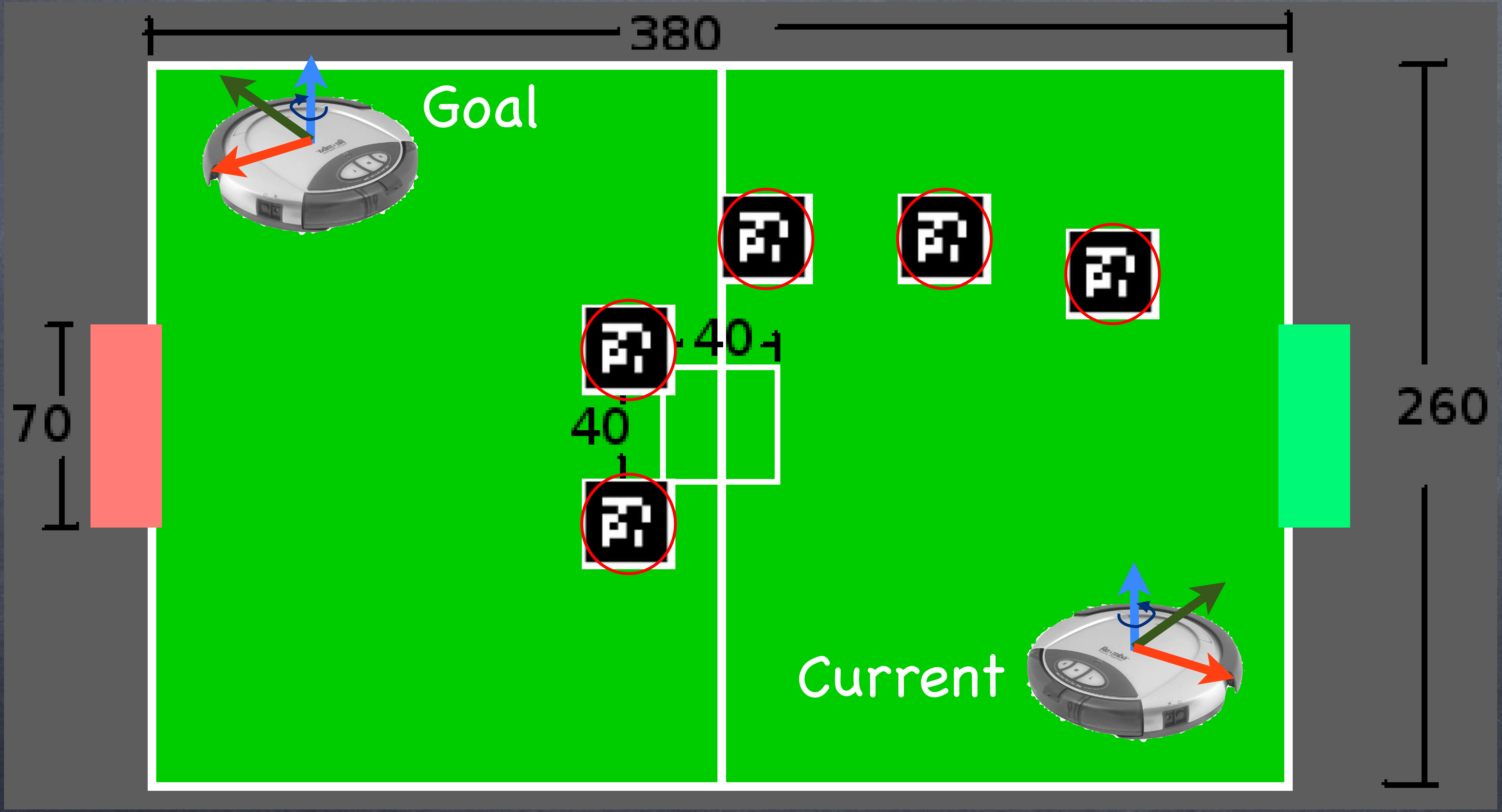
describe performance for this case



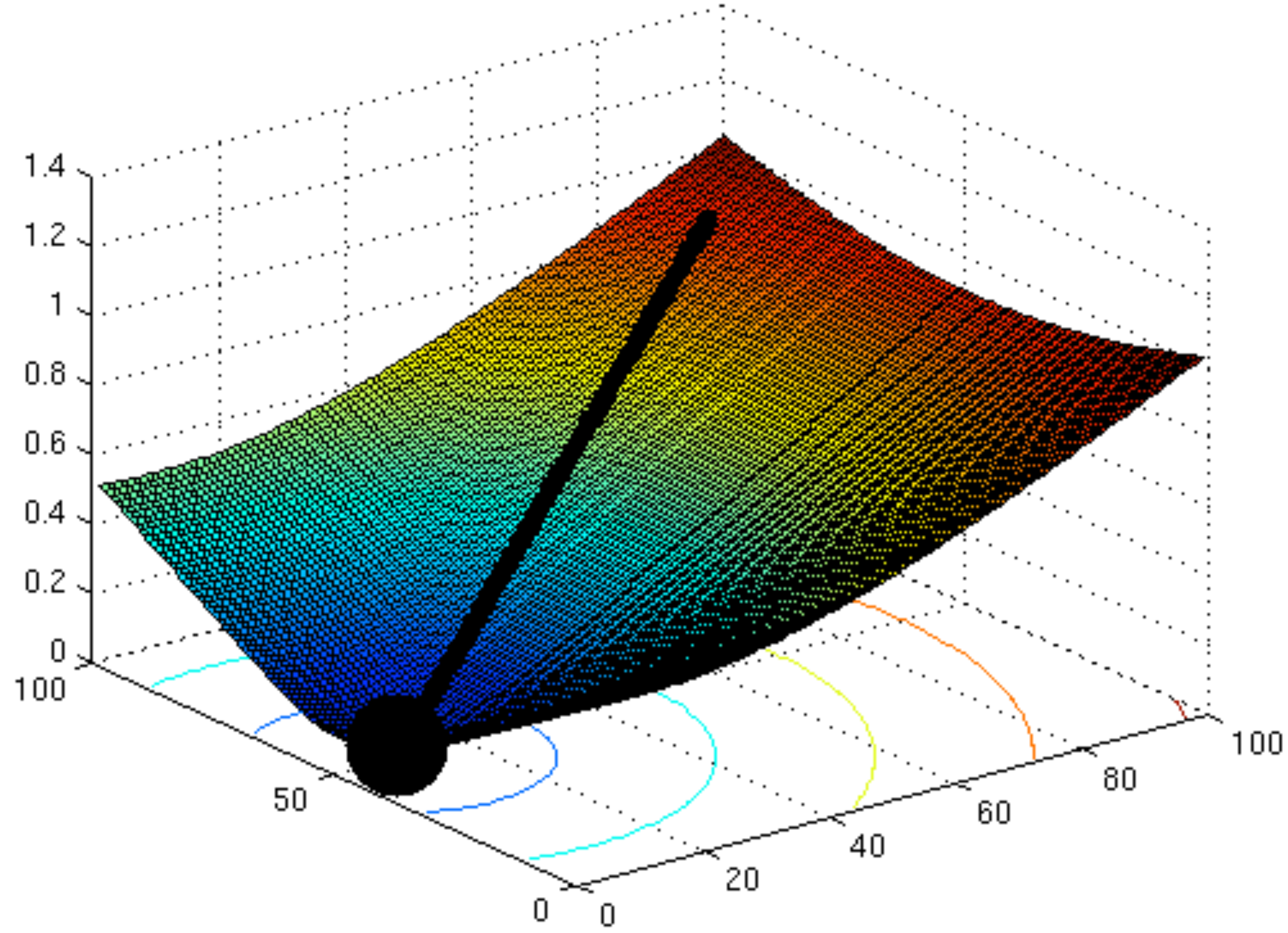
describe performance for this case



describe performance for this case

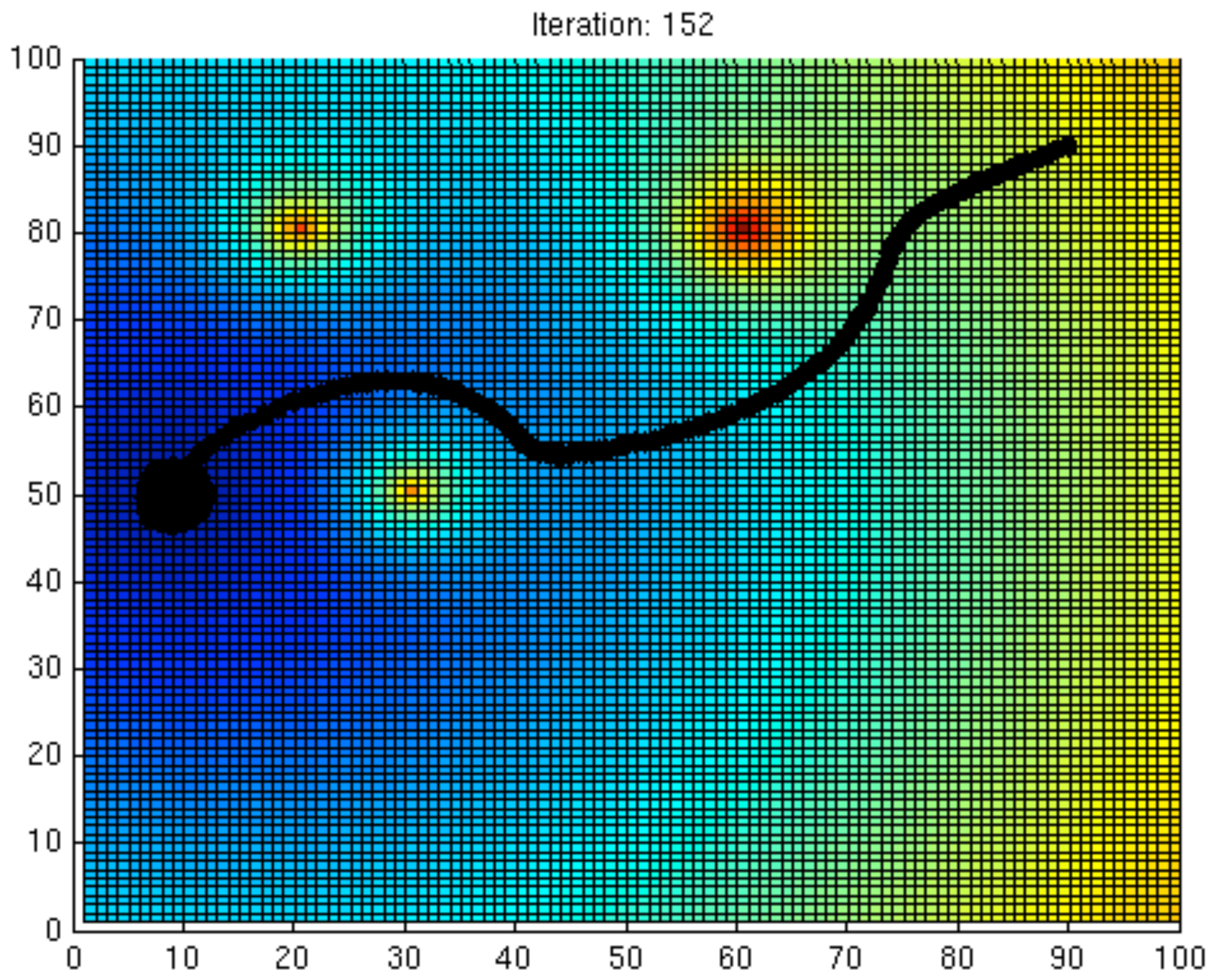


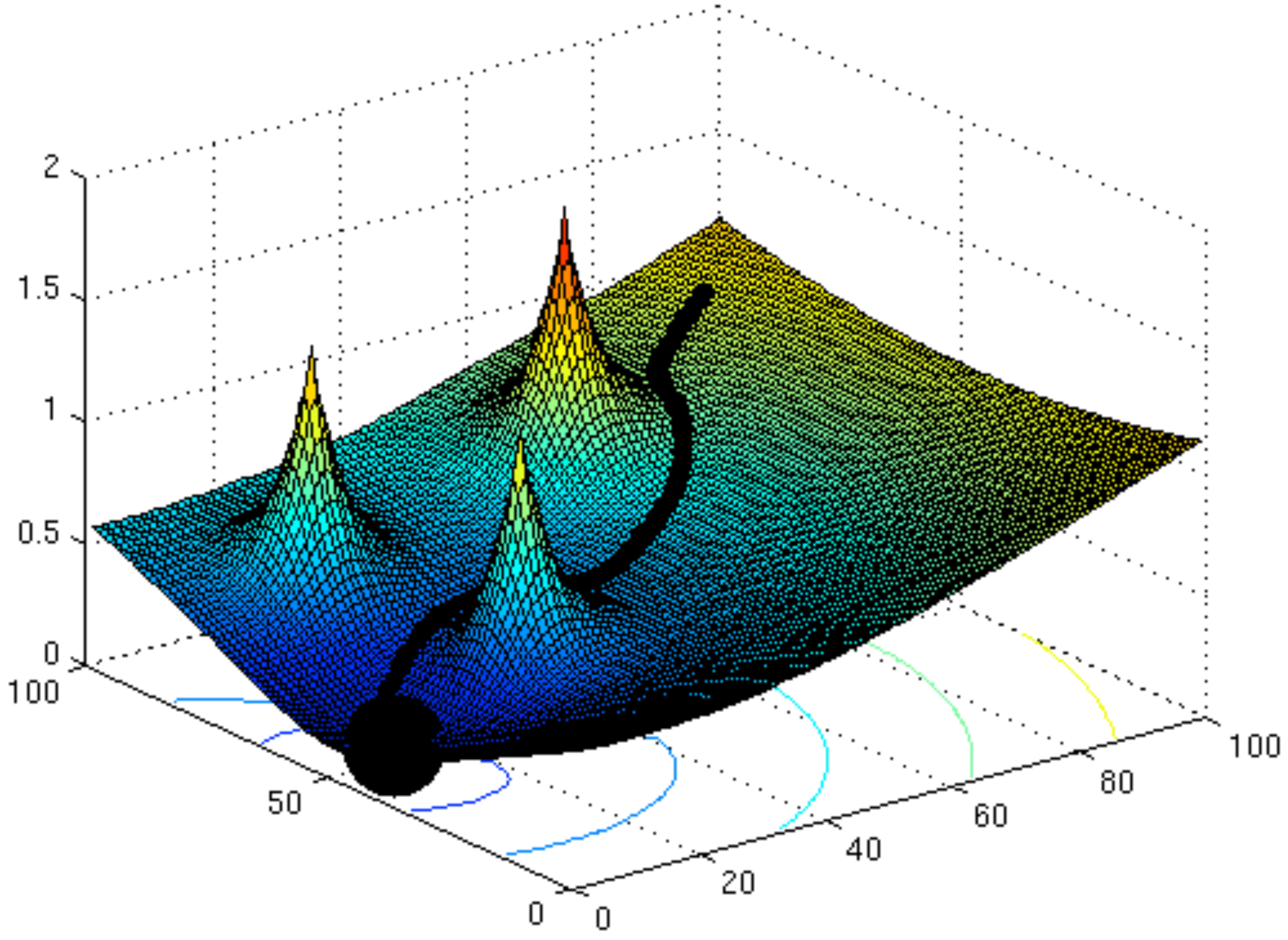
matlab example



pfield.m [1 5 8 12]

matlab example

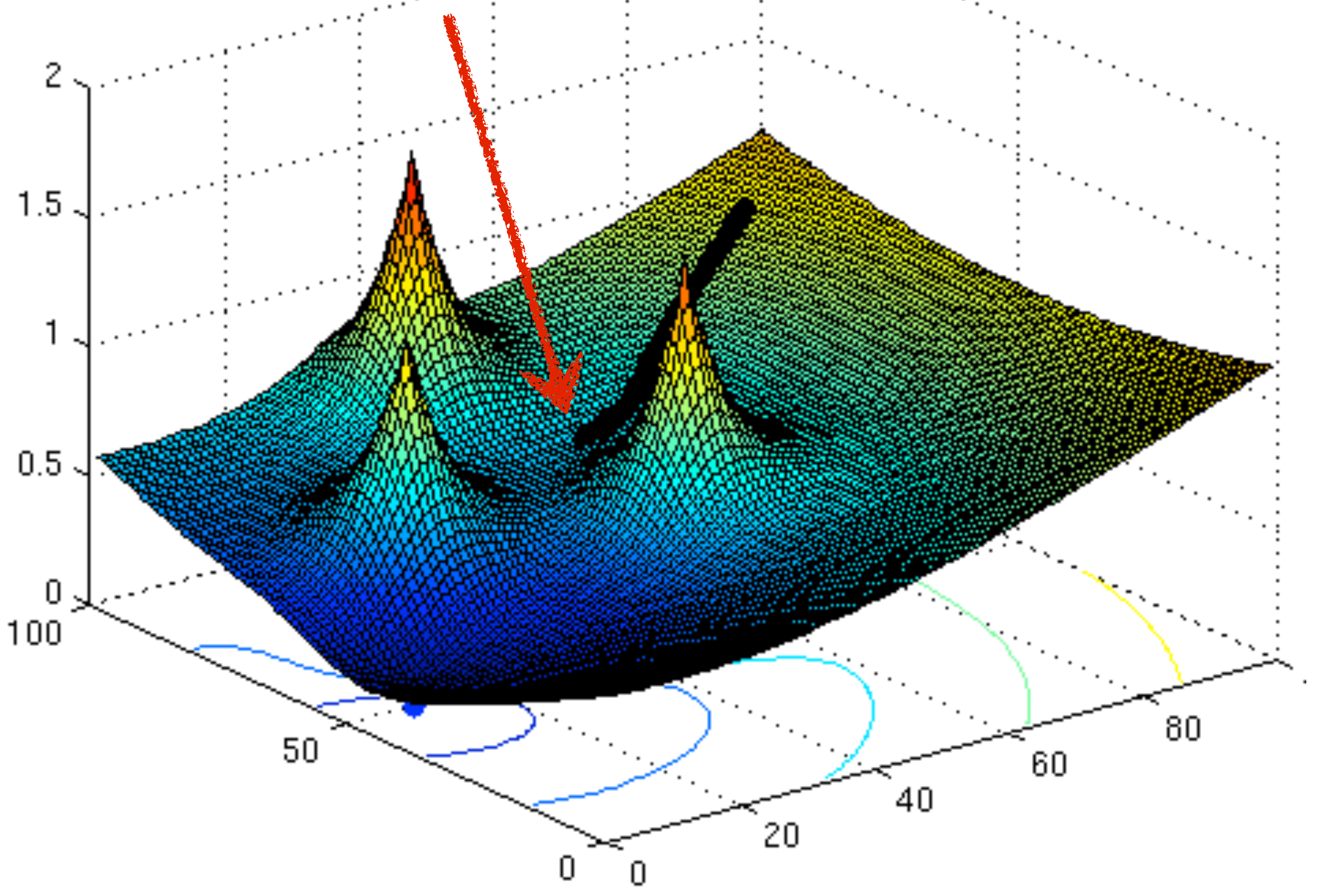




pfield.m [1 5 8 12]

matlab example

How to address local minima?



pfield.m [1 5 8 12]

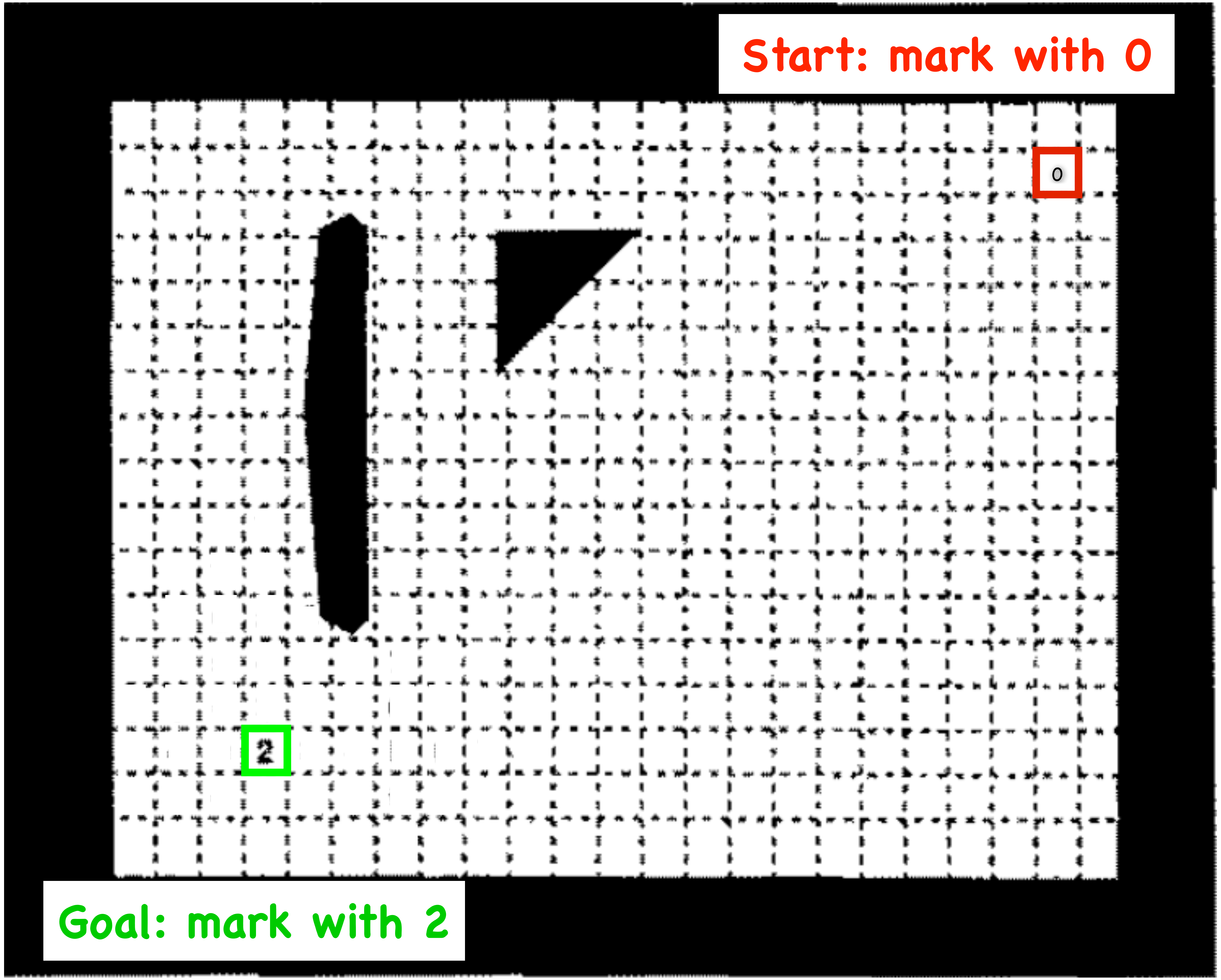
How can we get out of local minima?

How can we get out of local minima?

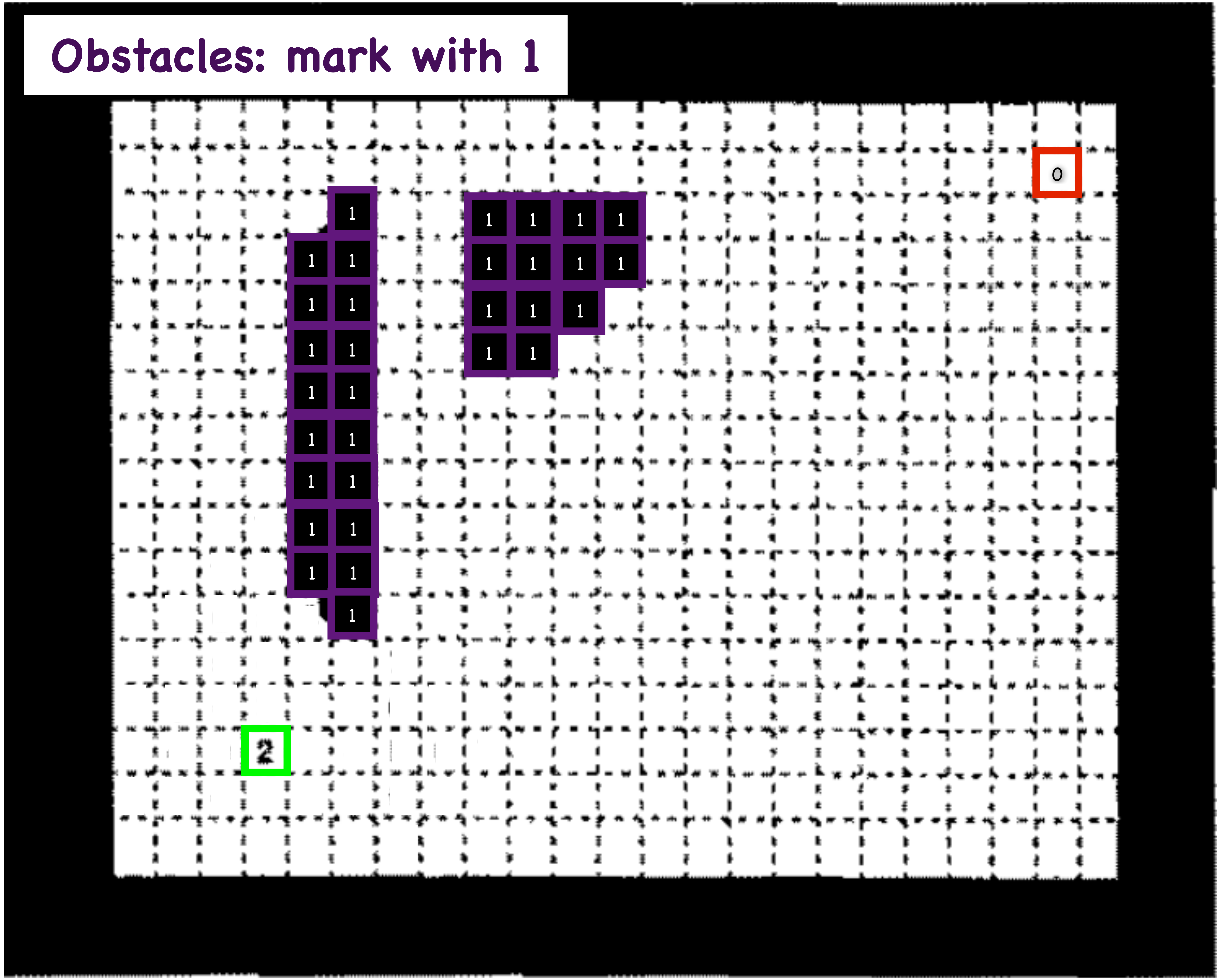
Go back to planning.

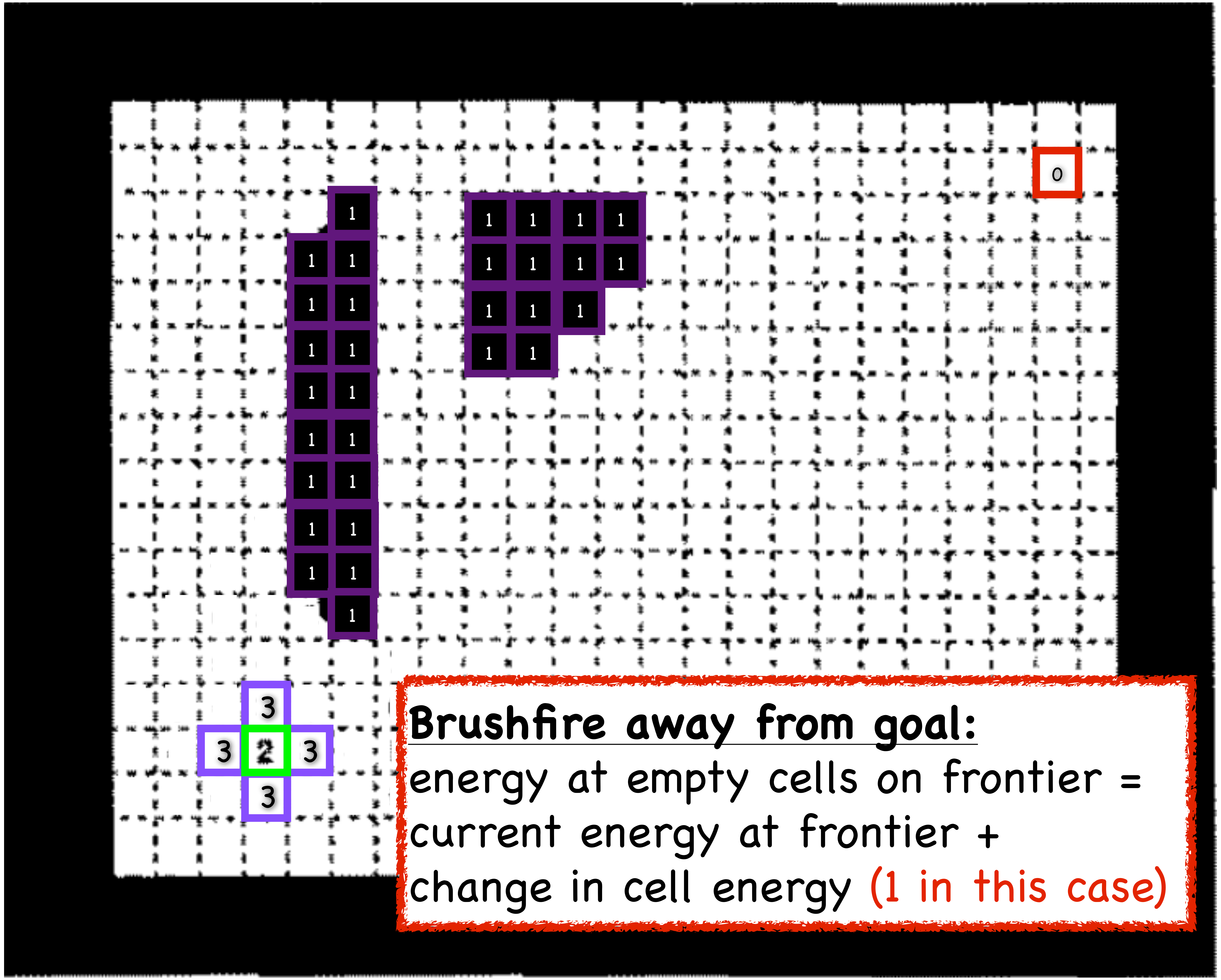
Wavefront Planning

- Discretize potential field into grid
    - Cells store cost to goal with respect to potential field
    - Computed by Brushfire algorithm (essentially BFS)
- Grid search to find navigation path to goal


Start: mark with 0
0
2
Goal: mark with 2

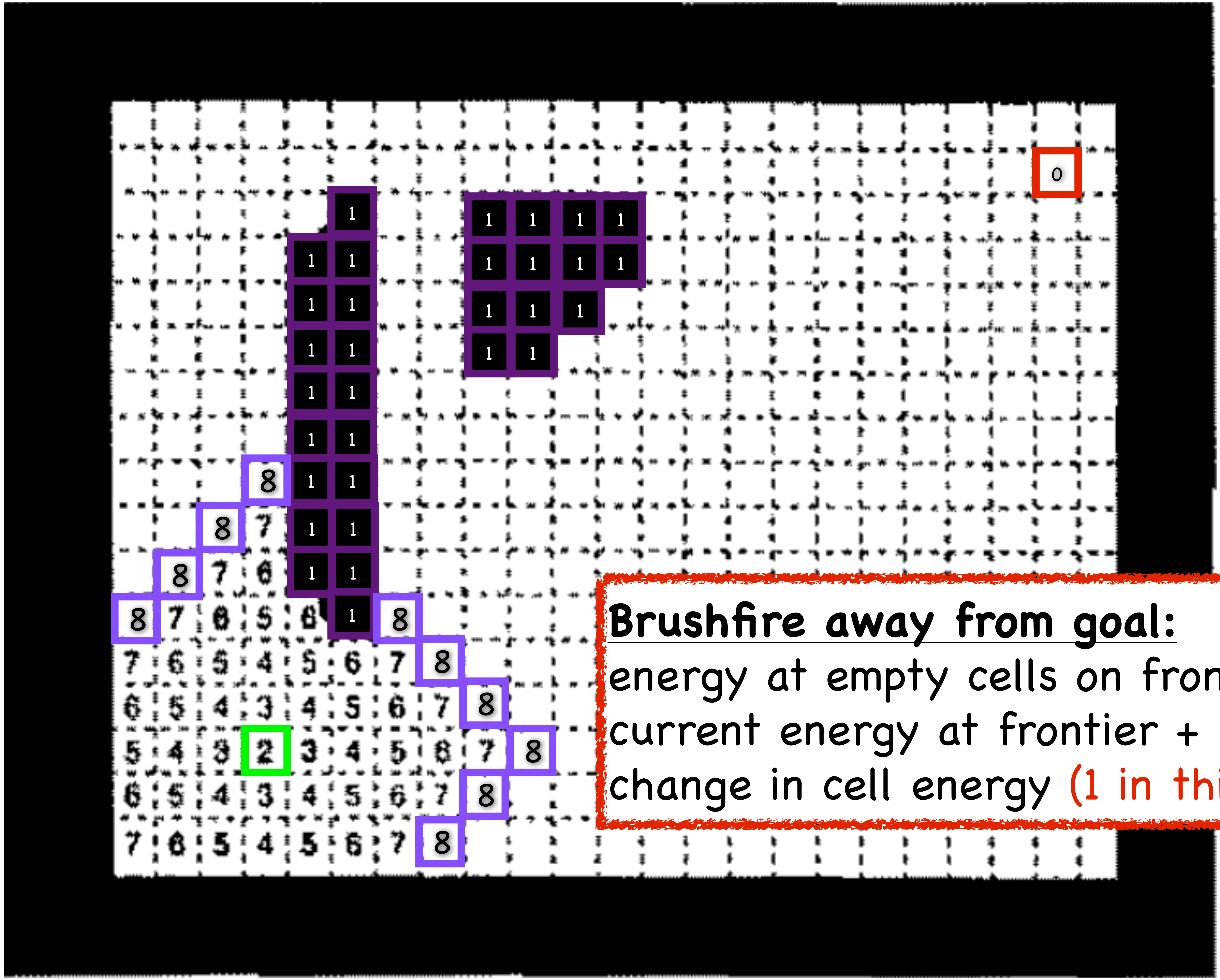
Obstacles: mark with 1





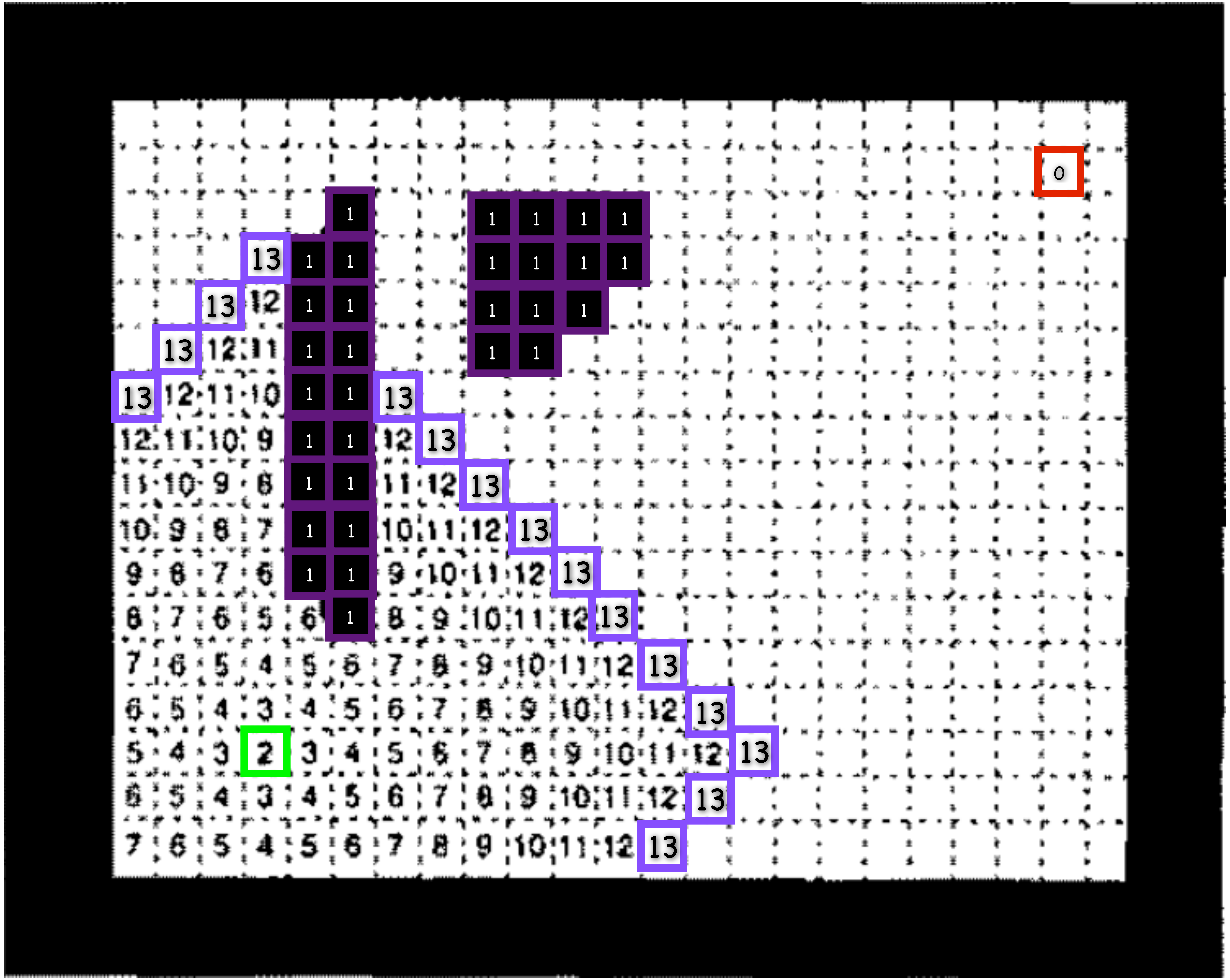
Brushfire away from goal:

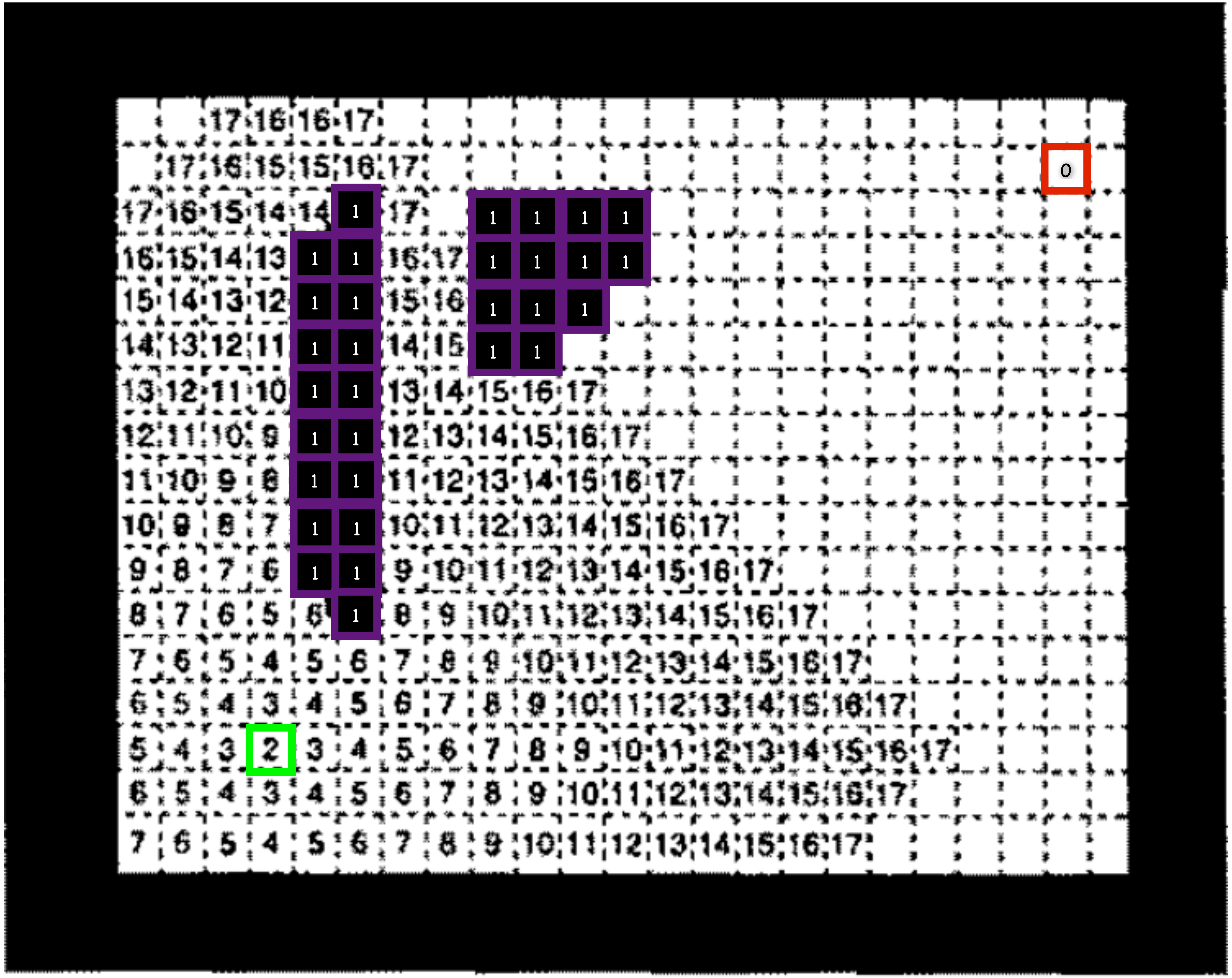
energy at empty cells on frontier =
current energy at frontier +
change in cell energy (1 in this case)

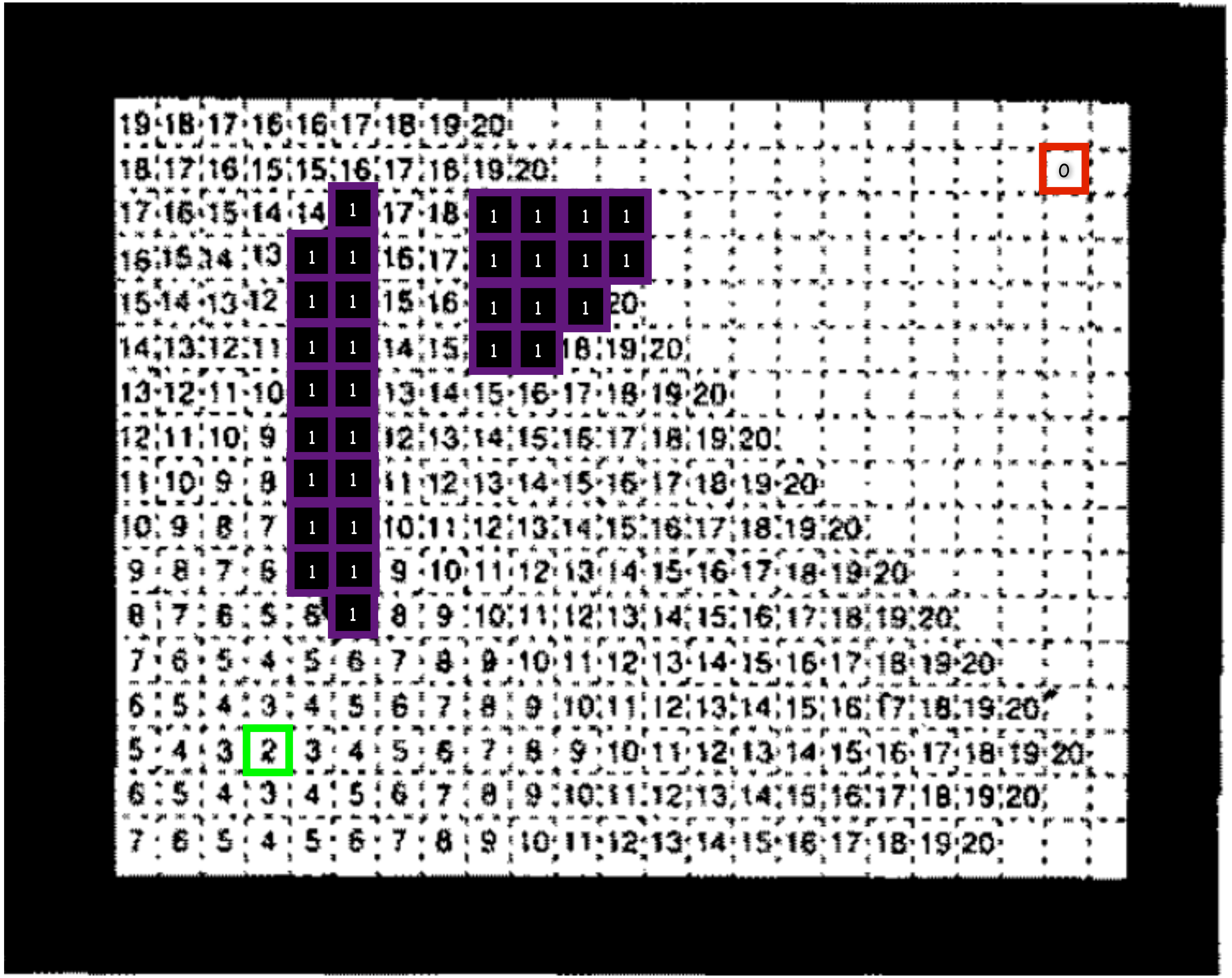


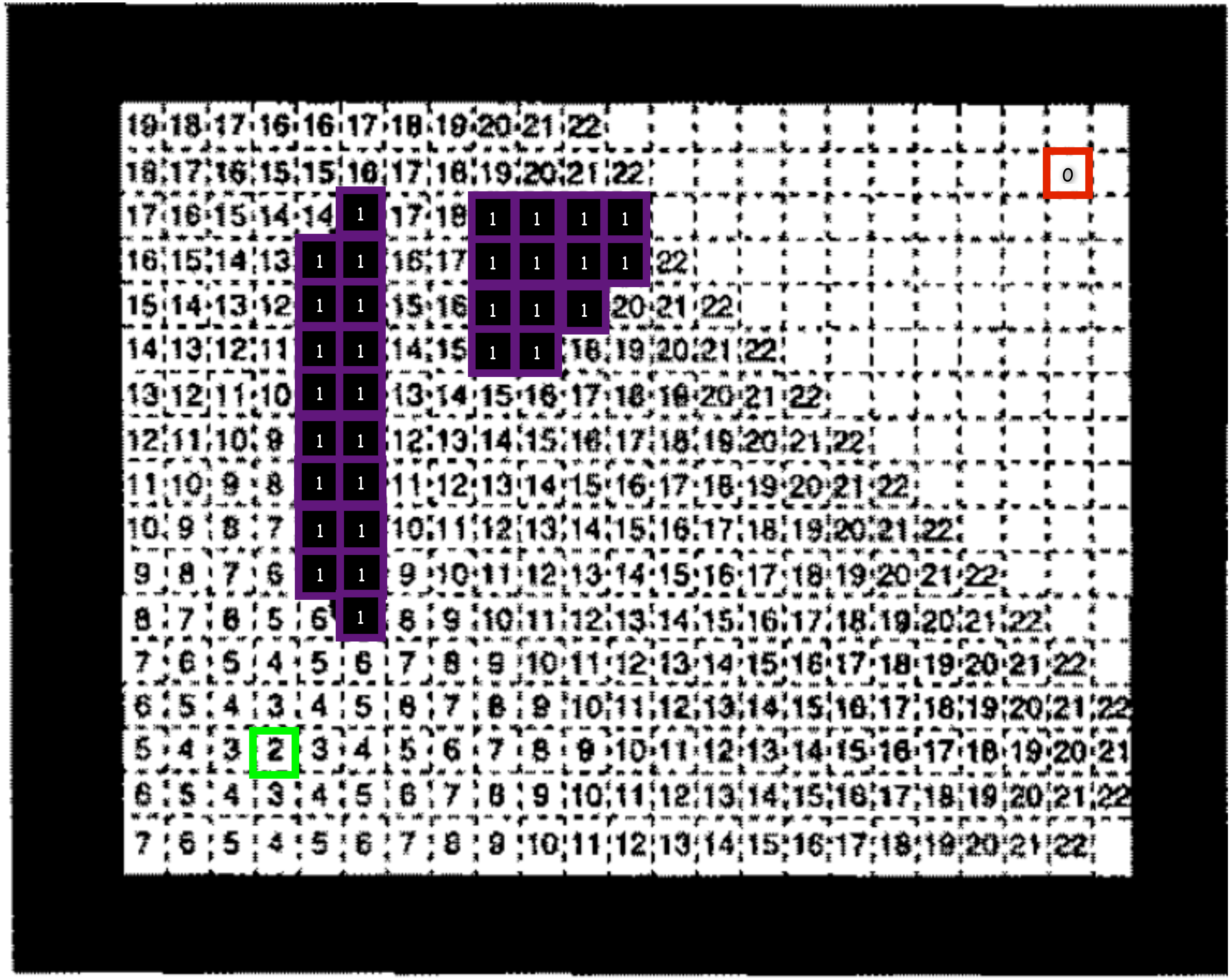
Brushfire away from goal:

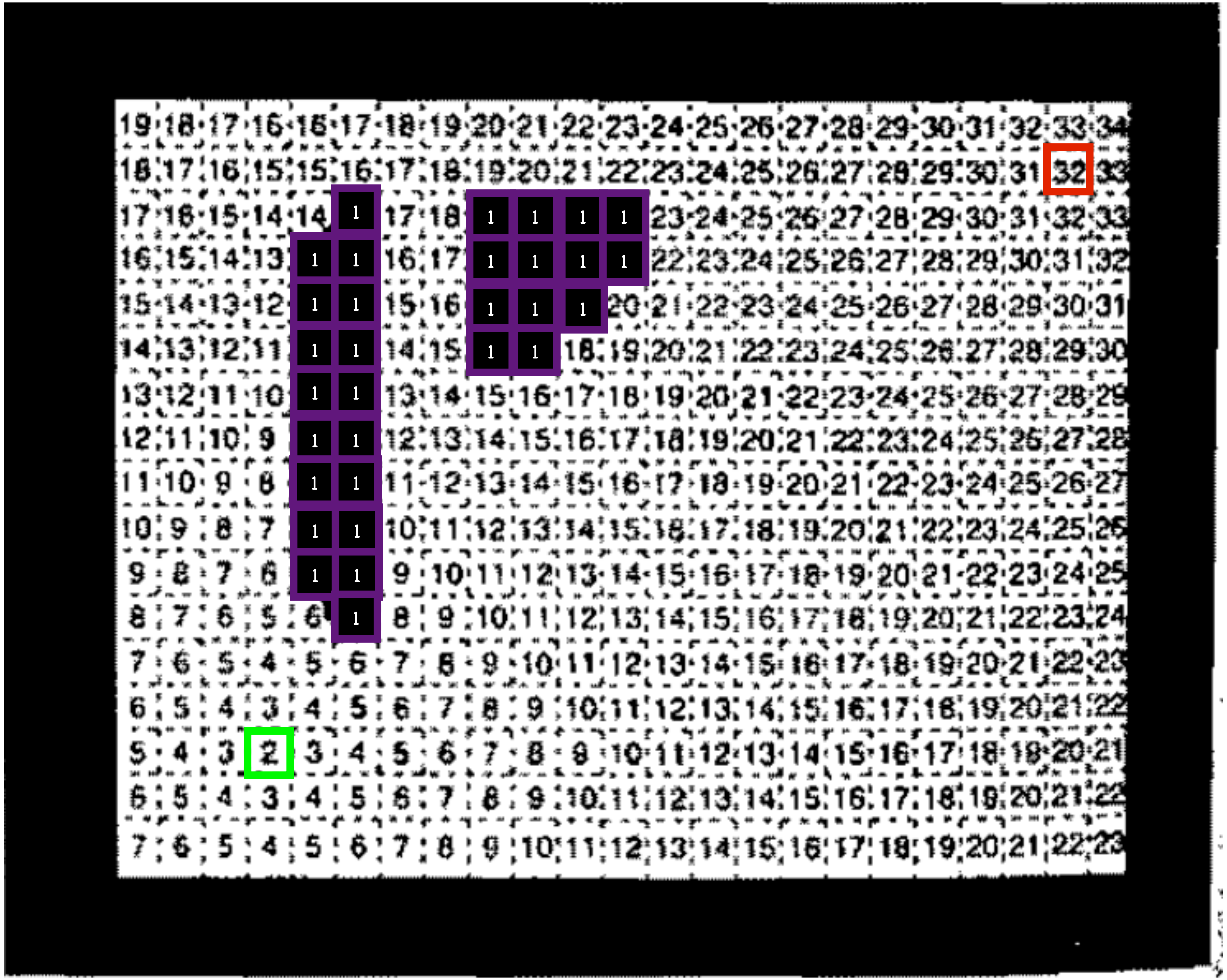
energy at empty cells on frontier = current energy at frontier + change in cell energy (1 in this case)



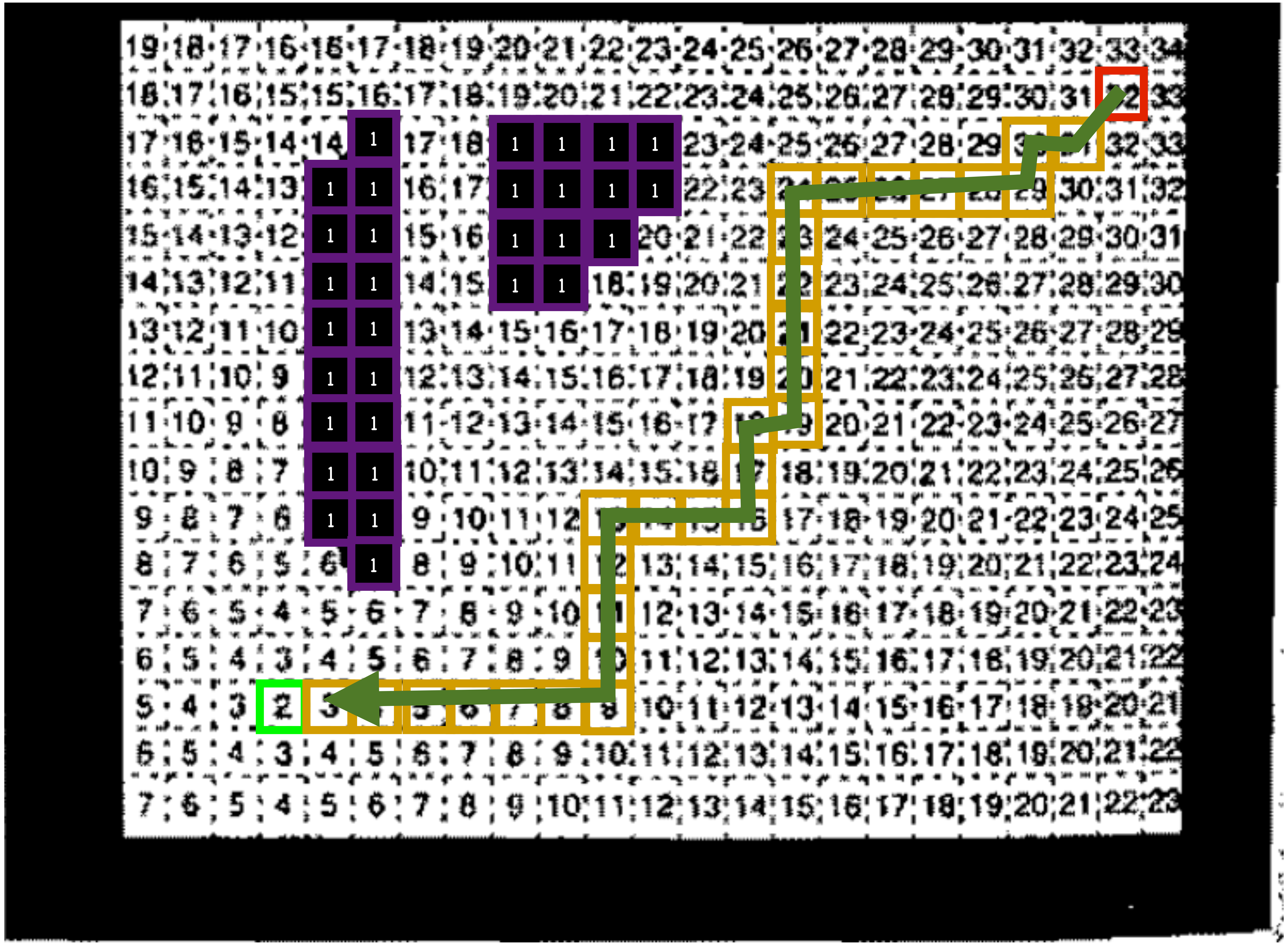






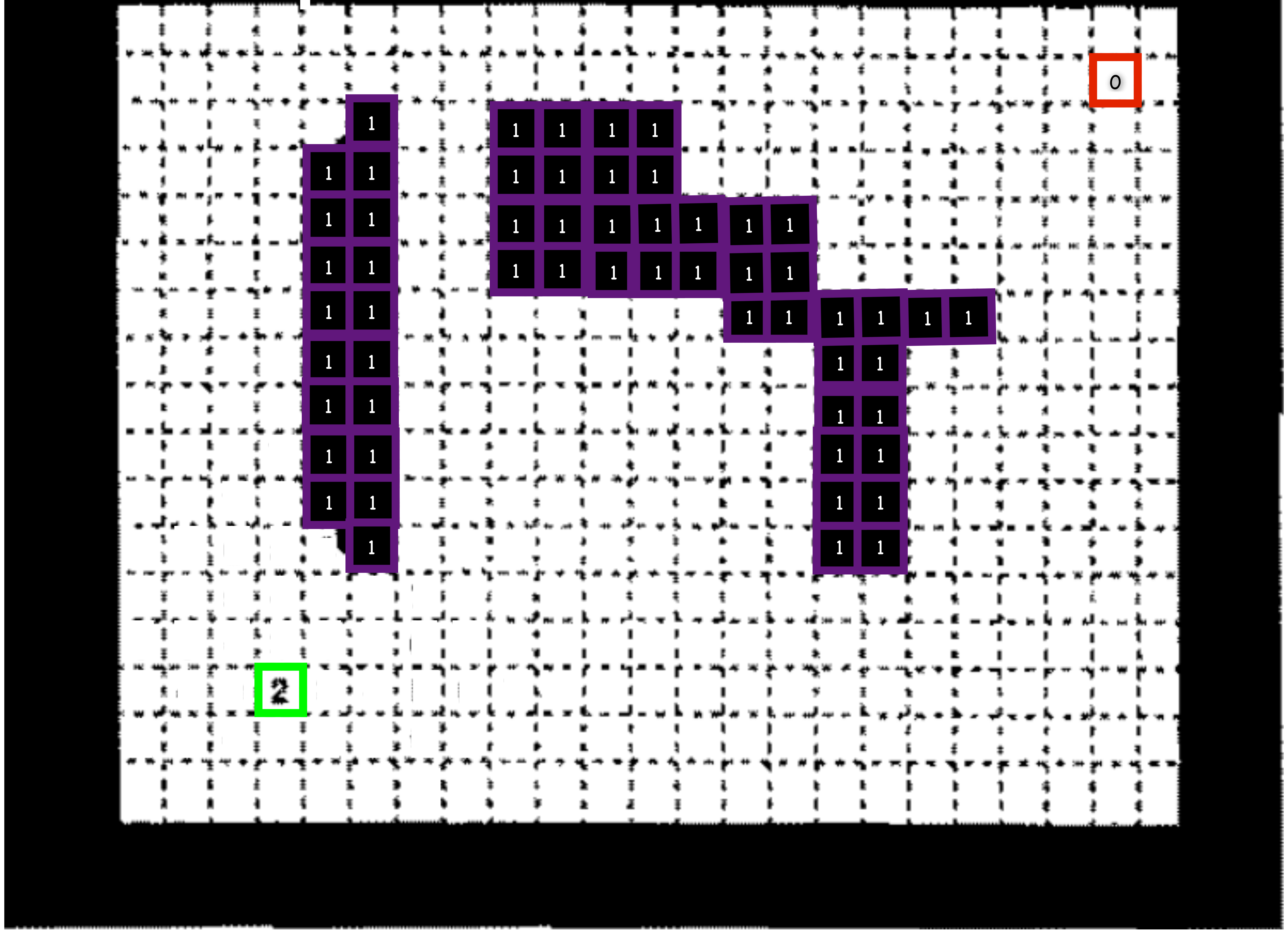


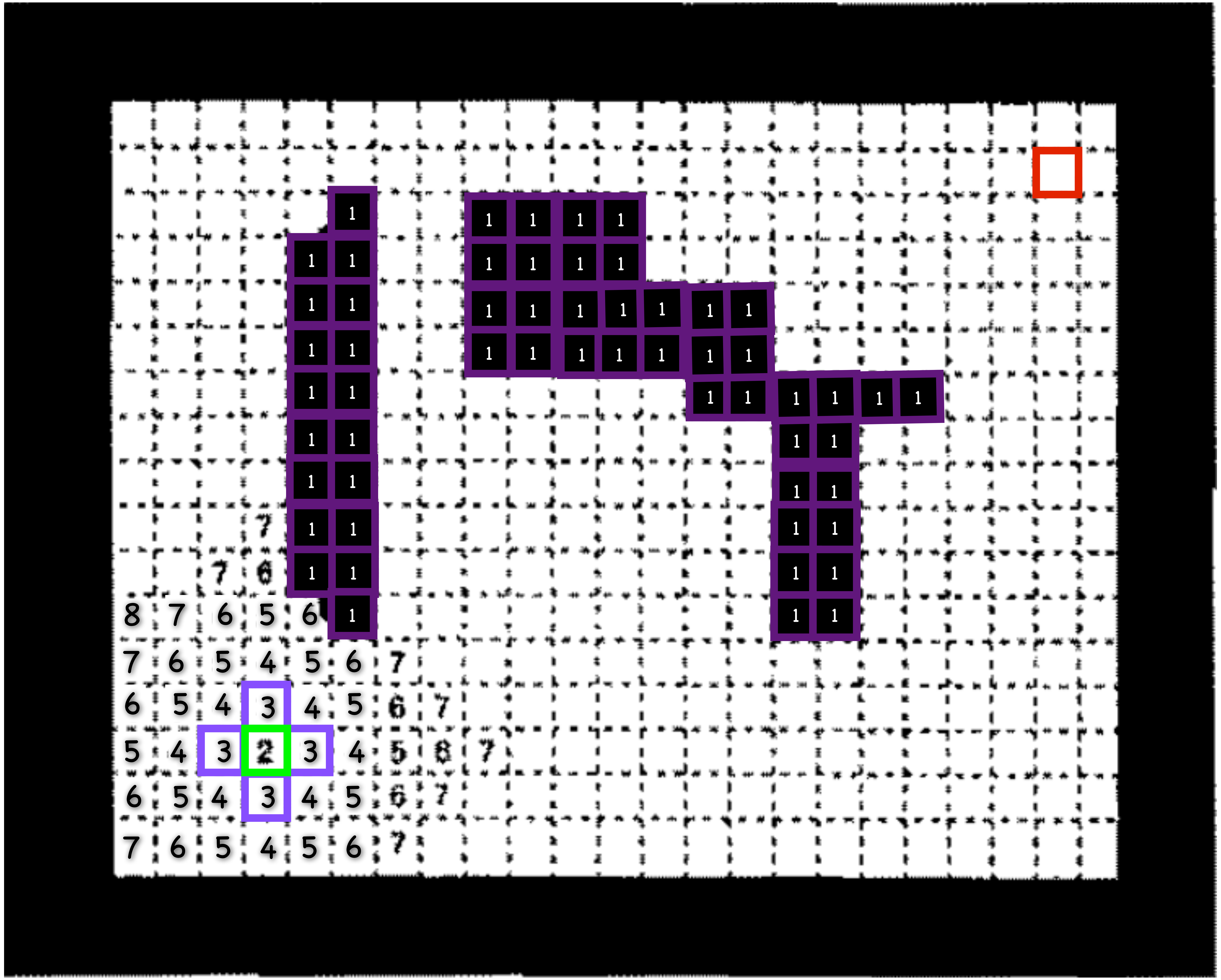
Once start reached,
follow brushfire potential to goal

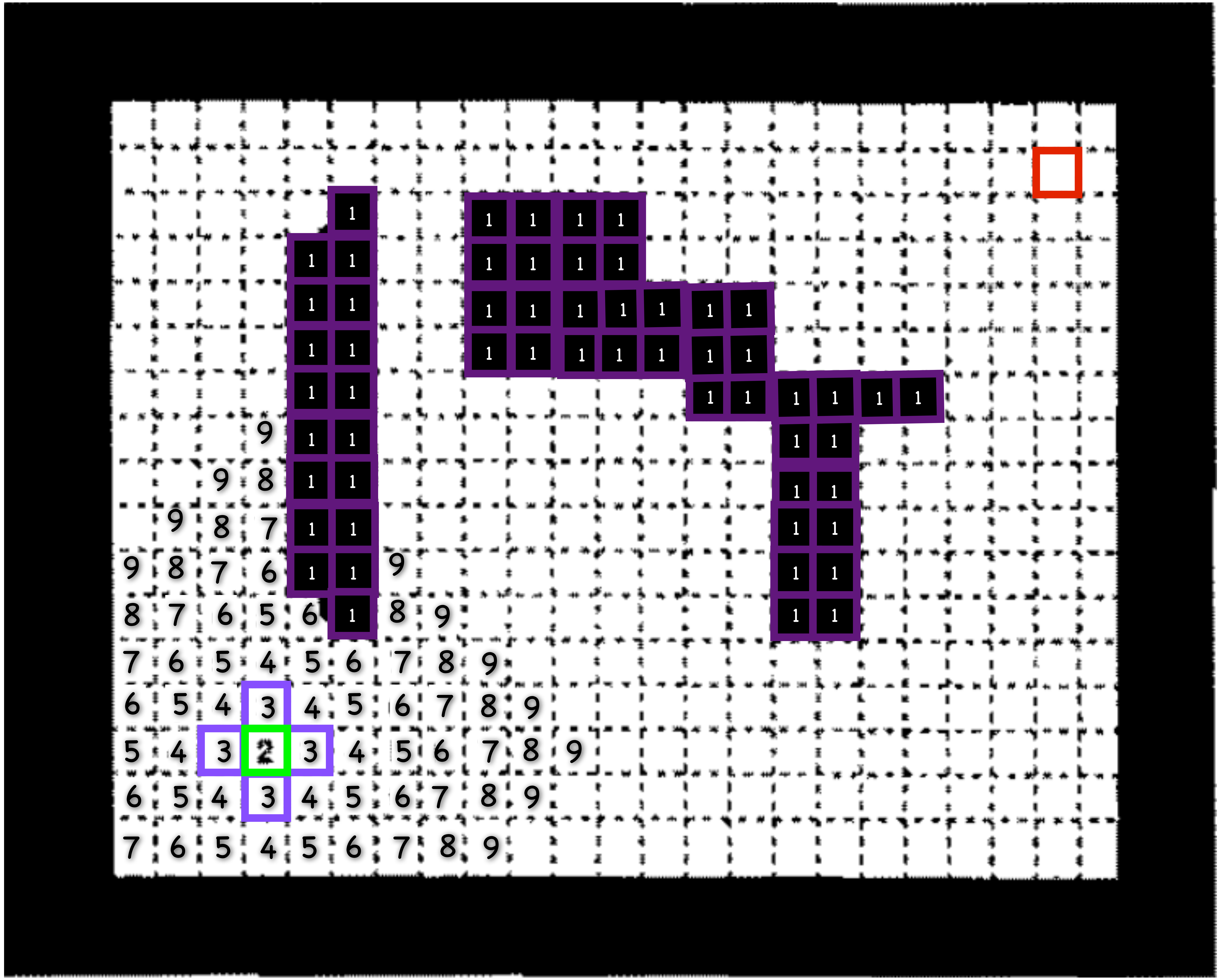


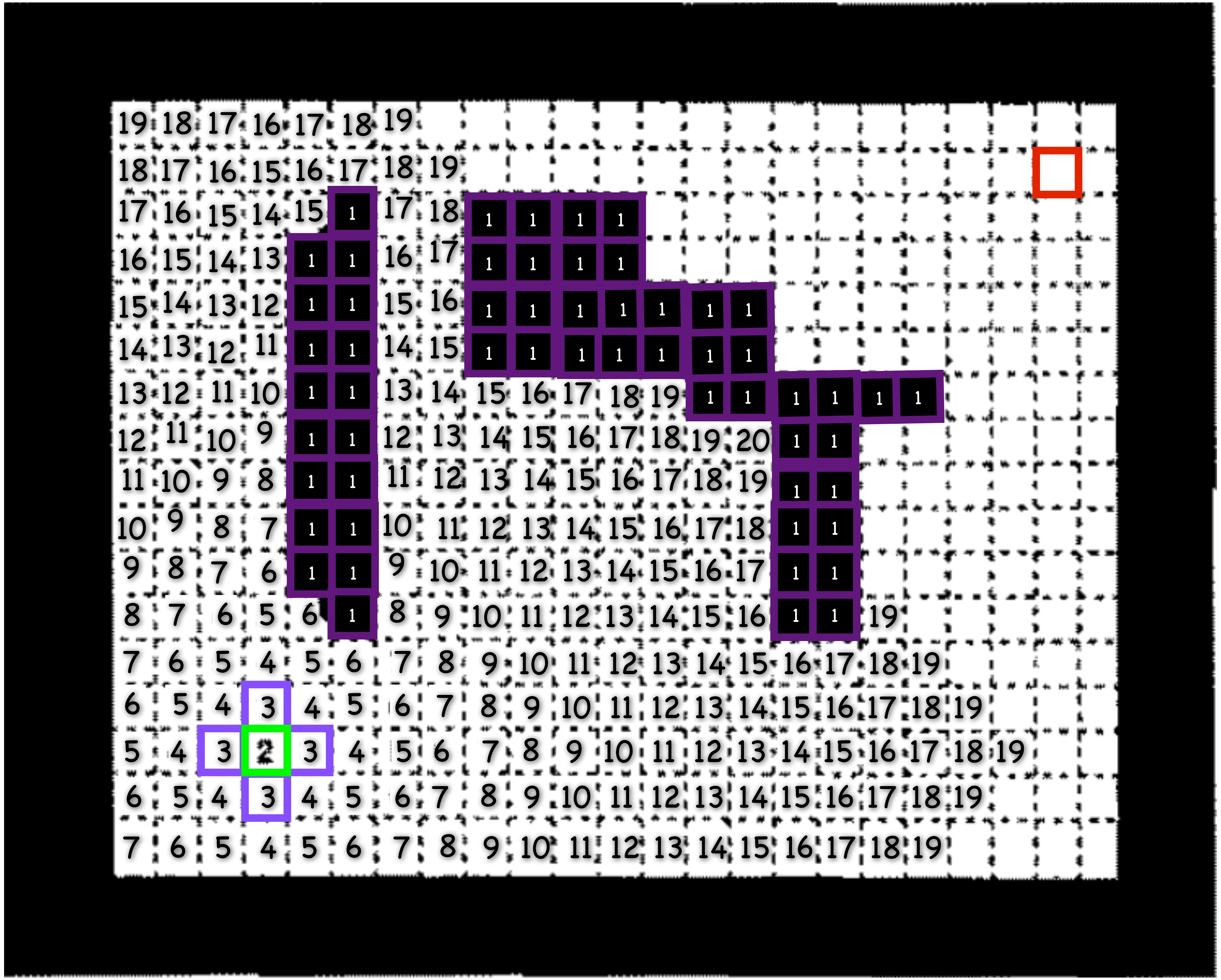
Example with Local Minima

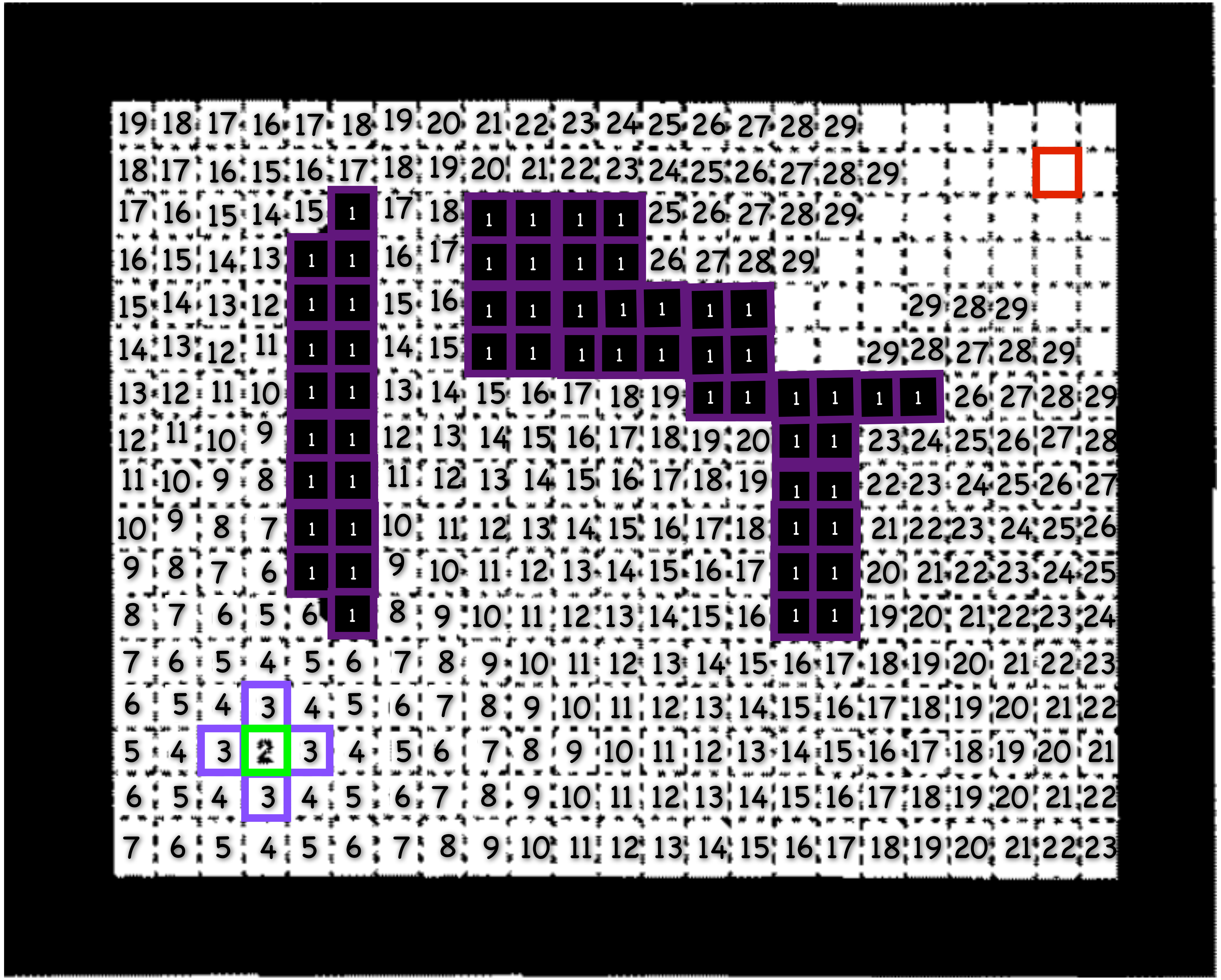
Example with Local Minima

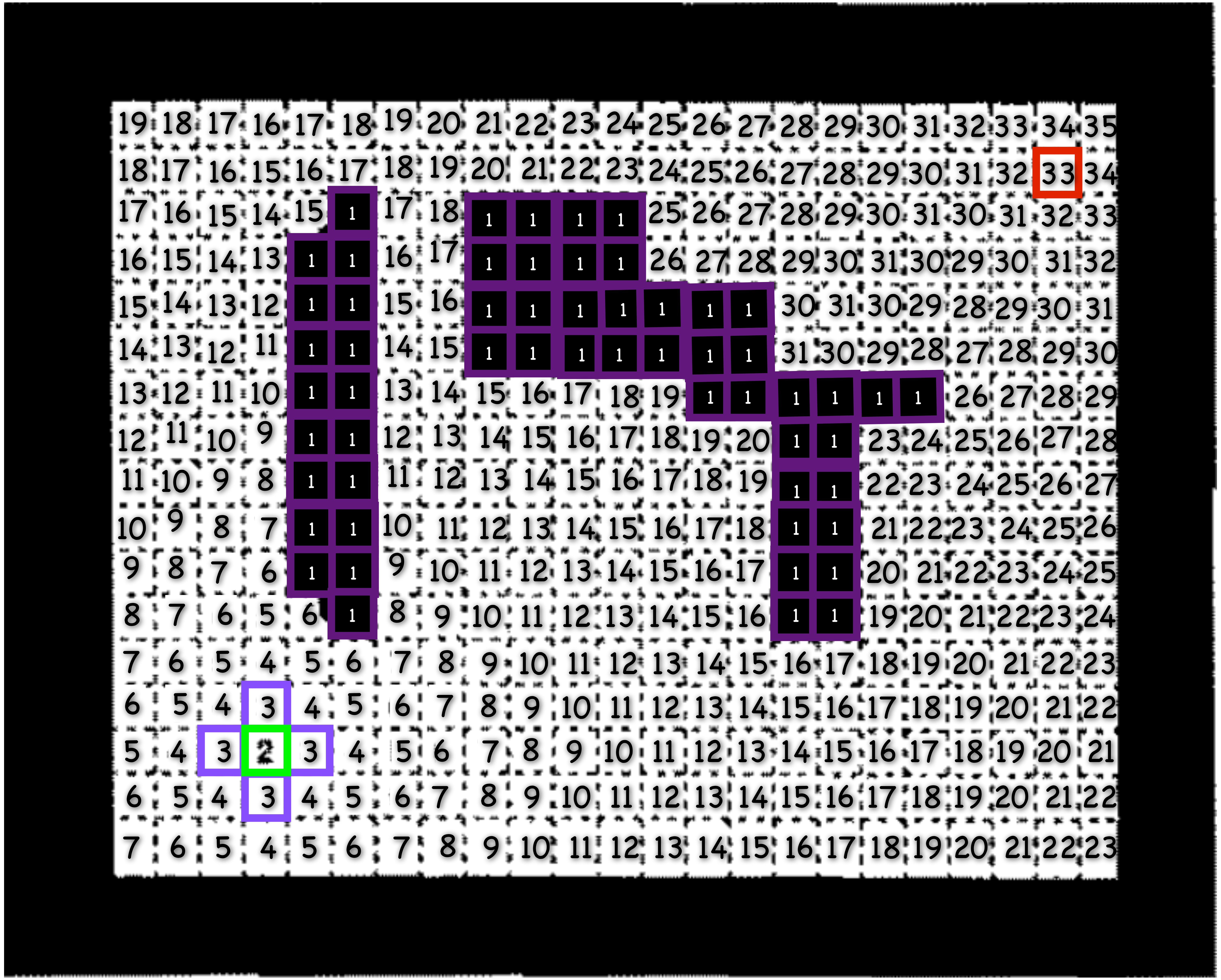


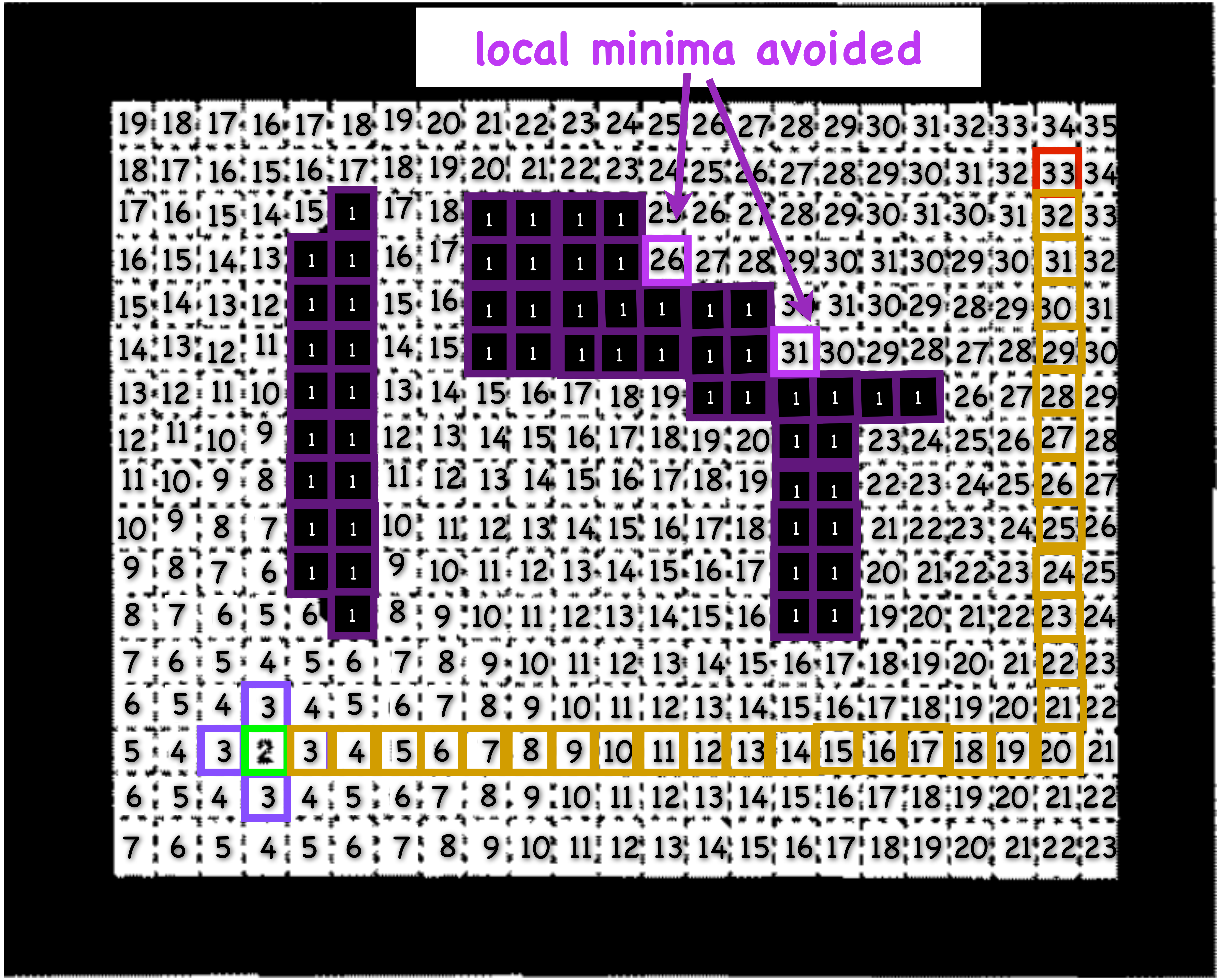




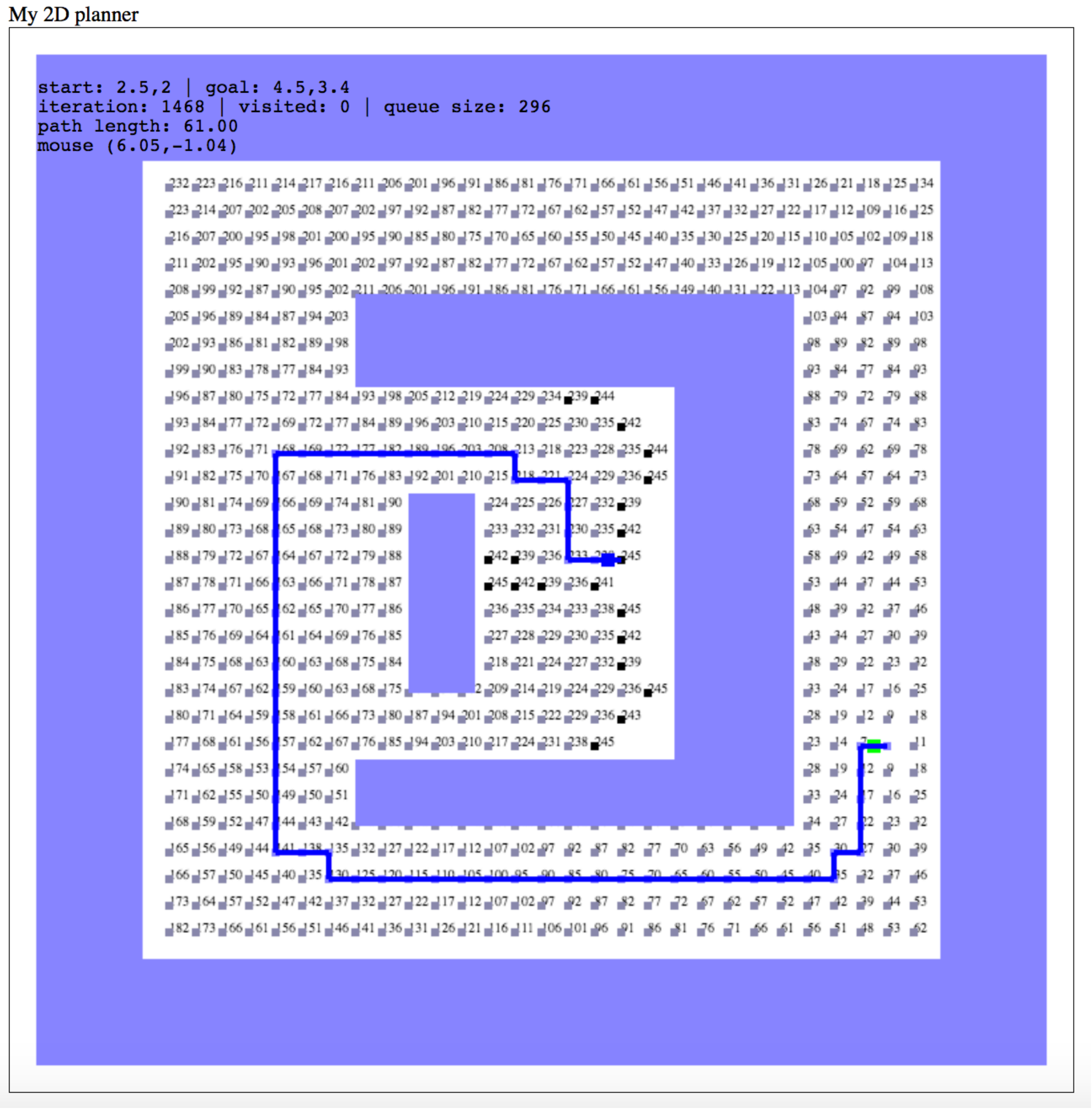







local minima avoided

Maithili's wavefront planner



Can we extend potential fields for arm navigation?

Potential Fields for Robot Arm

- Define endeffector goal as the attractive potential with cone potential
- Define repulsive potentials wrt. collision objects
  - Select points on robot links with “bowl” potential from nearest object
- Use manipulator Jacobian to transform potential at each point into velocites at robot joints
- Weighted sum of transformed velocities to generate control

Navigation Recap

Navigation Recap

Bug X
- Complete
- Non-optimal
- Planar

Subsumption and FSMs
- Fast but not adaptive
- Emphasis on good design

Potential Fields
- Complete in special cases
- Non-optimal
- General C-spaces
- Scales w/dimensionality

Grid Search/Wavefront
- Complete
- General C-spaces
- Limited dimensionality

Random walk
- Will find path eventually

Sampling roadmaps/RRT
- Probabilistically complete
- General
- Tractable (with good sampling)
- Scales w/dimensionality
- Not necessarily optimal


Next time ...
Collision Detection
U.S. Department of Transportation
https://www.youtube.com/watch?v=B8Ct5hjs0Jw