Planning with Sampling Roadmaps

Building a path to goal

EECS 367
Intro. to Autonomous Robotics

ROB 320
Robot Operating Systems

Winter 2022

autorob.org

Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- Graph Search (fixed graph)
    - Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first
- **Sampling-based Search (build graph):**
    - **Probabilistic Road Maps, Rapidly-exploring Random Trees**
- Optimization and local search:
    - Gradient descent, Potential fields, Simulated annealing, Wavefront

Where is this?

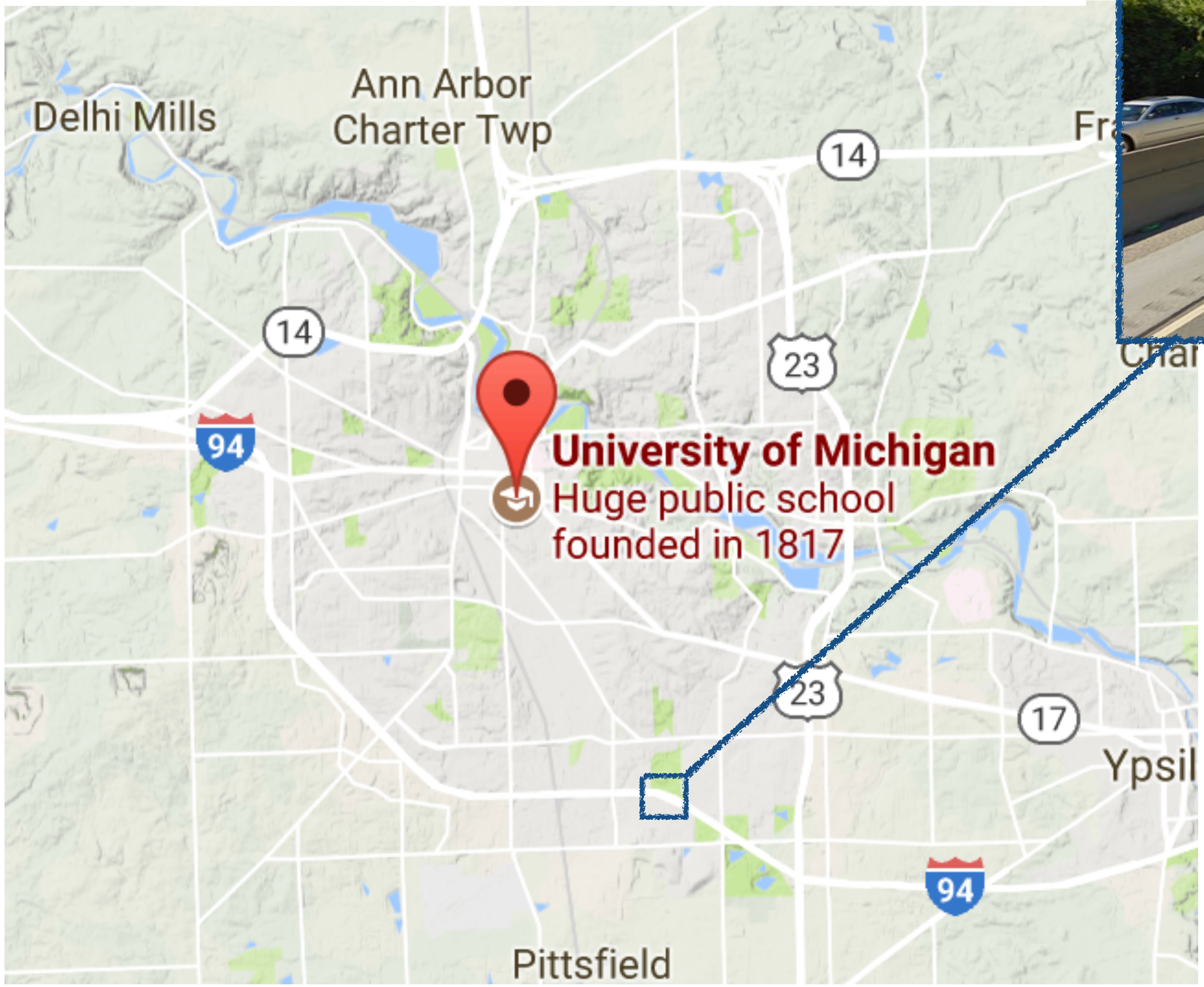

EXIT 177
State St
1 MILE
I-94

What is the configuration of this location?

What is the configuration of this location?

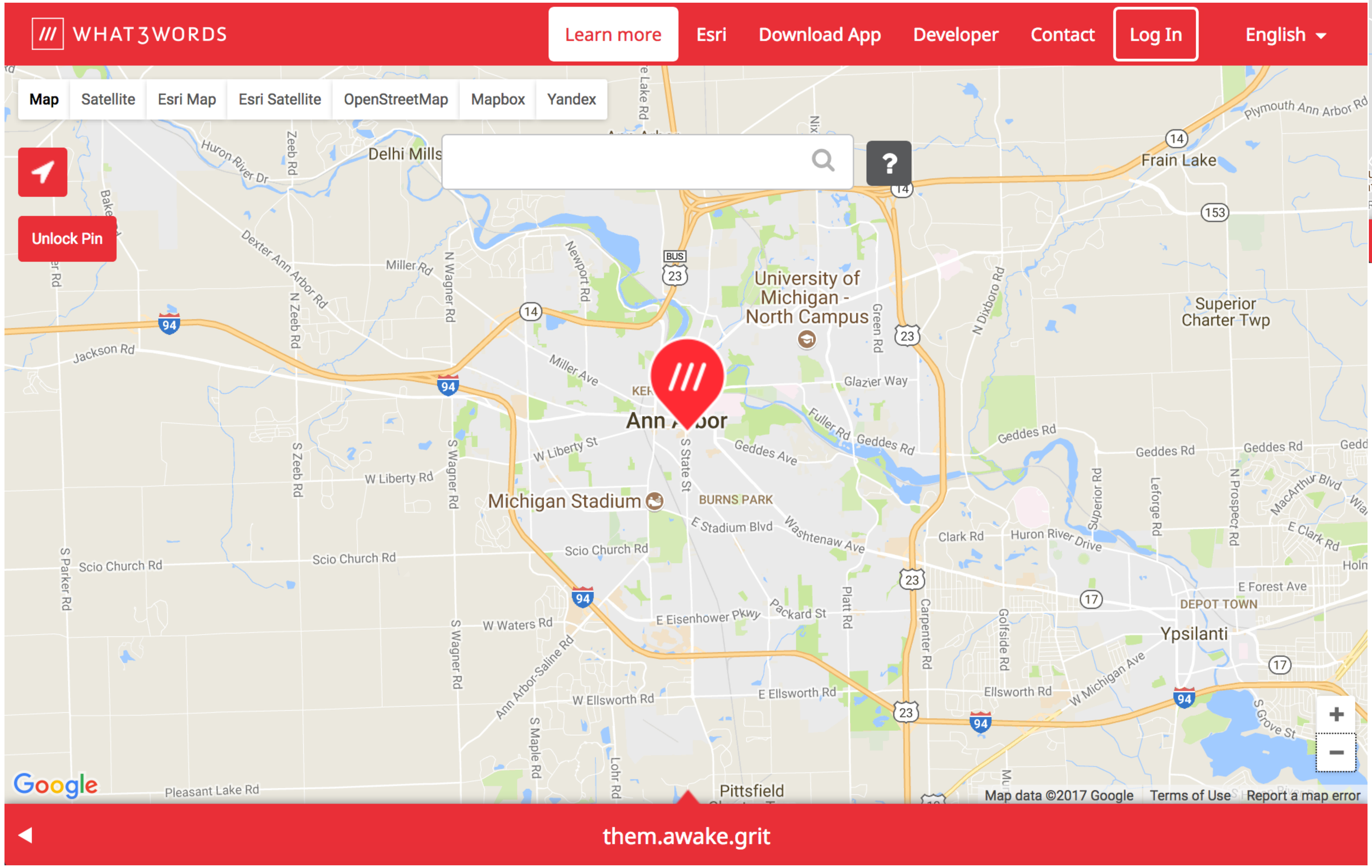
Configuration (lat,lon): [42.237631, -83.7147289]

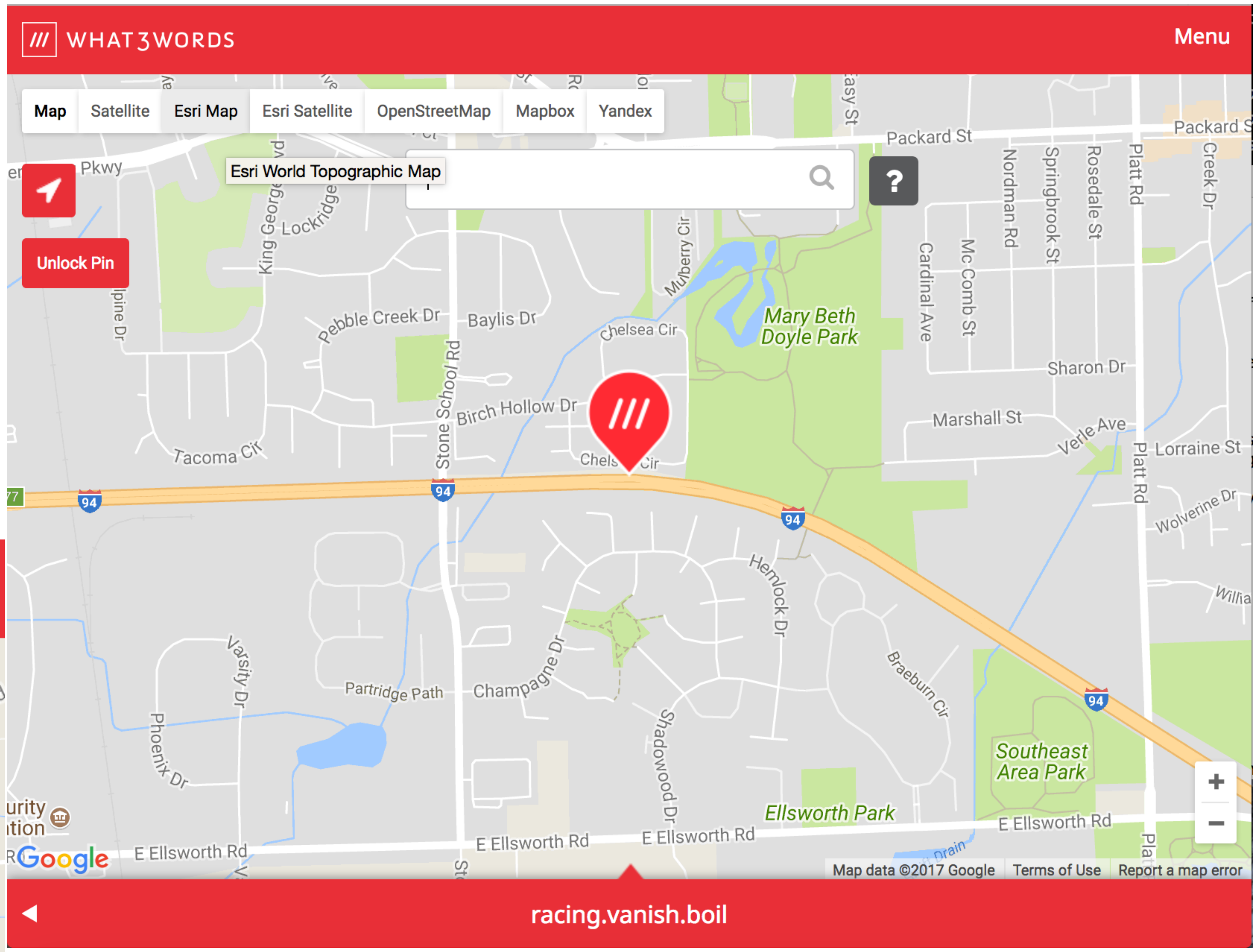
Configuration: [42.2780436,-83.7404128]



Alternatively...

Configuration: [them.awake.grit]

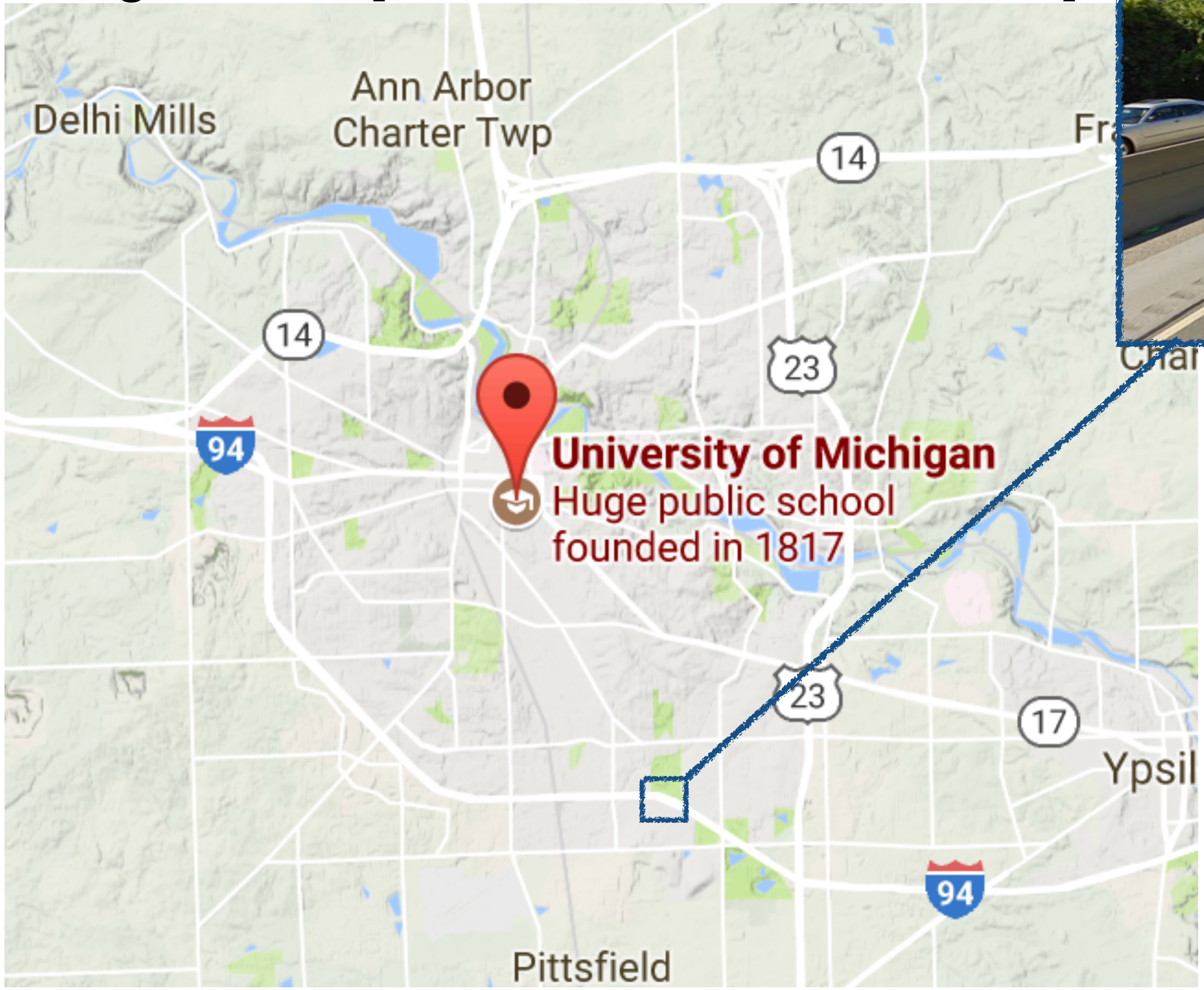




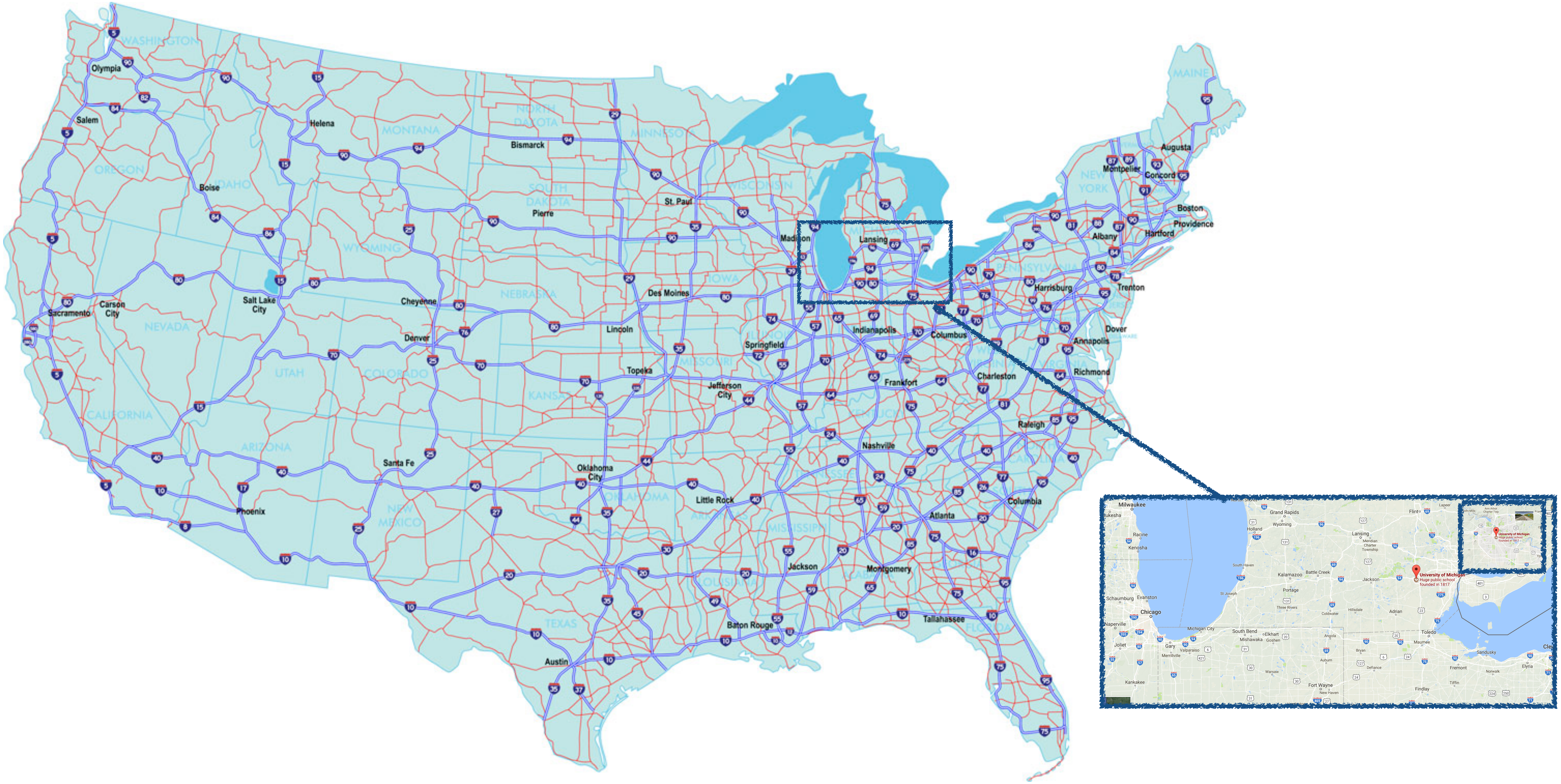
Configuration: [racing.vanish.boil]

Is this a usable representation for configuration?

Configuration: [42.2780436,-83.7404128]




Ann Arbor Charter Twp
Delhi Mills
University of Michigan
Huge public school founded in 1817
Ypsi
Pittsfield
Milwaukee
Waukesha
Racine
Kenosha
Norton Shores
Grand Rapids
Wyoming
Holland
Flint
Lansing
Meridian Charter Township
South Haven
Kalamazoo
Battle Creek
Jackson
Portage
St Joseph
Three Rivers
Coldwater
Hillsdale
Adrian
Chatham-Kent
Schaumburg
Evanston
Chicago
Naperville
Joliet
Gary
Michigan City
South Bend
Elkhart
Mishawaka
Goshen
Angola
Valparaiso
Merrillville
Toledo
Maumee
Bryan
Sandusky
Auburn
Defiance
Fremont
Elyria
Norwalk
Warsaw
Kankakee
Fort Wayne
New Haven
Tiffin
Findlay
Huntington
Van Wert
Ashland



Olympia
Salem
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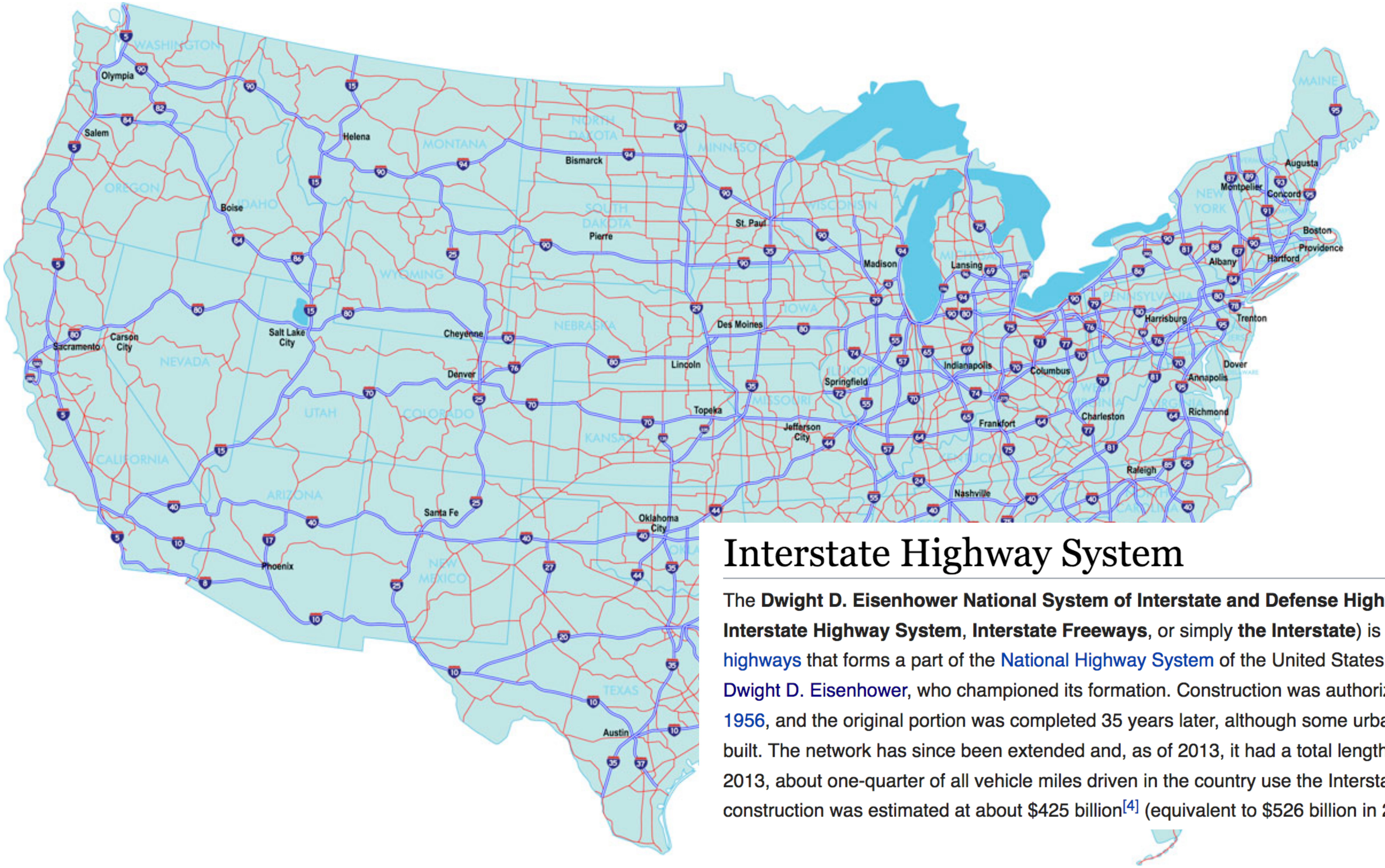
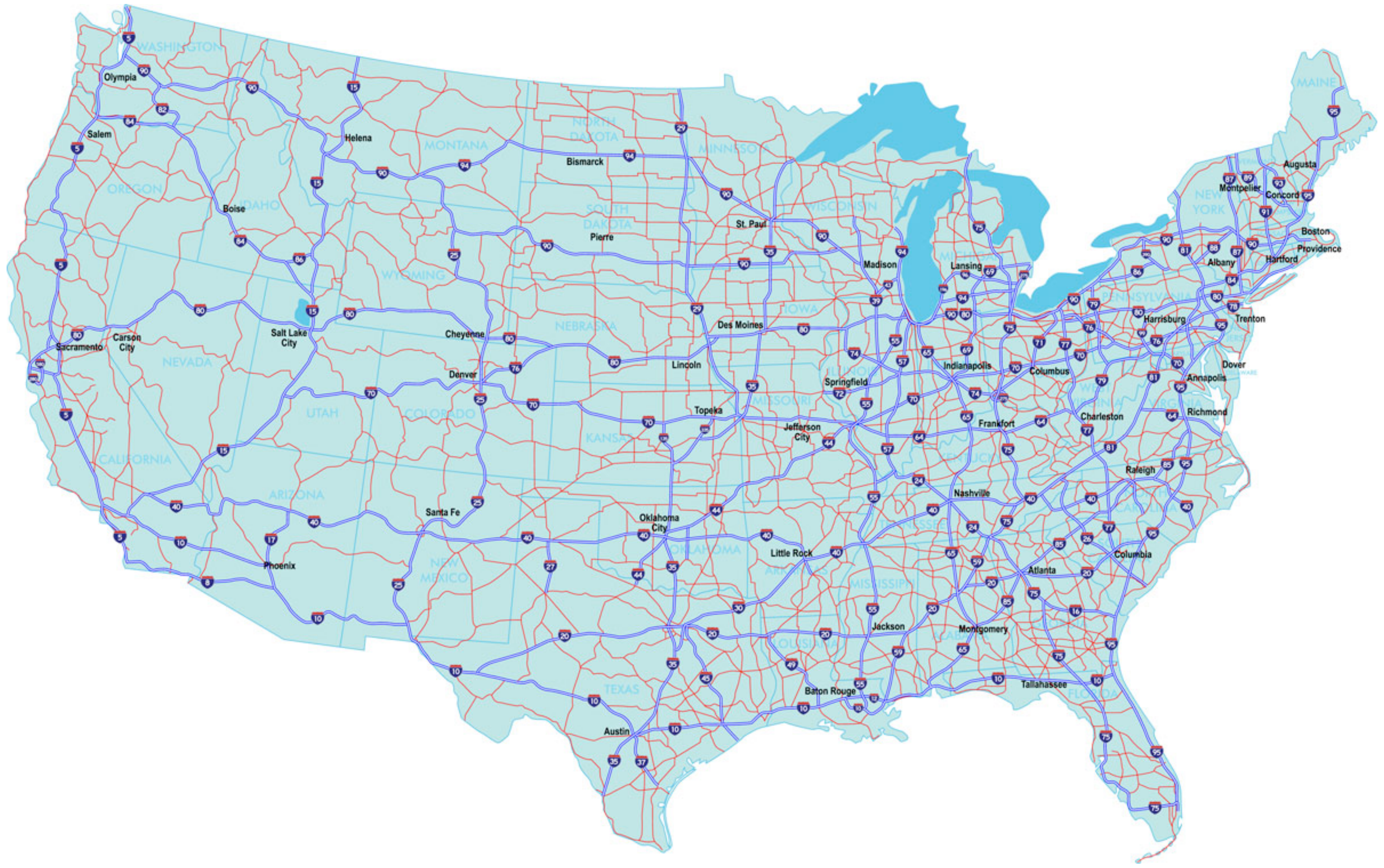
Boise
Bismarck
Pierre
St. Paul
Madison
Lansing
Salt Lake City
Carson City
Sacramento
Cheyenne
Des Moines
Denver
Lincoln
Springfield
Indianapolis
Columbus
Topeka
Jefferson City
Frankfort
Charleston
Richmond
Annapolis
Dover
Trenton
Harrisburg
Albany
Hartford
Providence
Boston
Concord
Montpelier
Augusta
Raleigh
Nashville
Columbia
Atlanta
Santa Fe
Oklahoma City
Little Rock
Phoenix
Jackson
Montgomery
Baton Rouge
Tallahassee
Austin
Milwaukee
Racine
Kenosha
Chicago
Grand Rapids
Kalamazoo
Battle Creek
Jackson
Toledo
University of Michigan



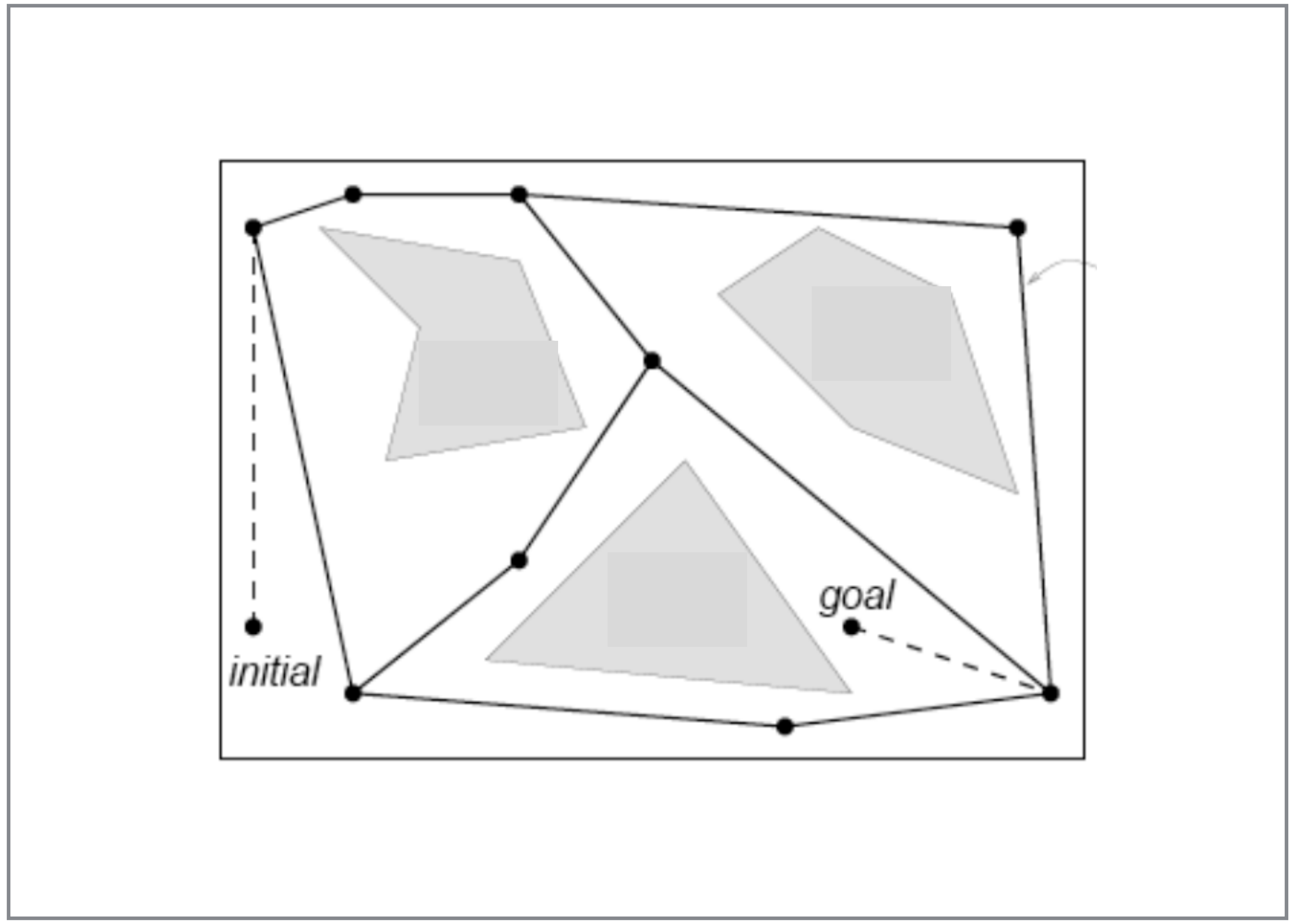
Interstate Highway System

The **Dwight D. Eisenhower National System of Interstate and Defense Highways** (commonly known as the **Interstate Highway System**, **Interstate Freeways**, or simply **the Interstate**) is a network of controlled-access highways that forms a part of the National Highway System of the United States. The system is named for President Dwight D. Eisenhower, who championed its formation. Construction was authorized by the Federal Aid Highway Act of 1956, and the original portion was completed 35 years later, although some urban routes were cancelled and never built. The network has since been extended and, as of 2013, it had a total length of 47,856 miles (77,017 km).[2] As of 2013, about one-quarter of all vehicle miles driven in the country use the Interstate system.[3] In 2006, the cost of construction was estimated at about \$425 billion[4] (equivalent to \$526 billion in 2016[5]).

Roadmaps

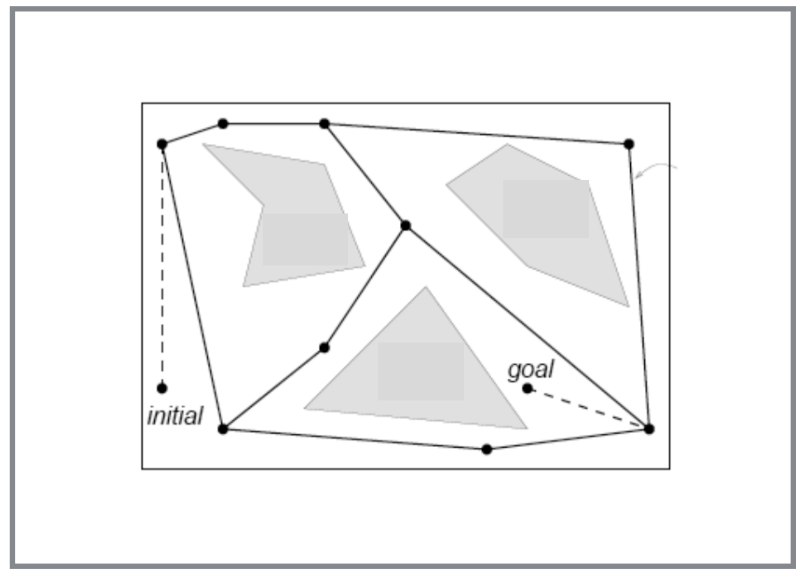


Roadmap over geolocations

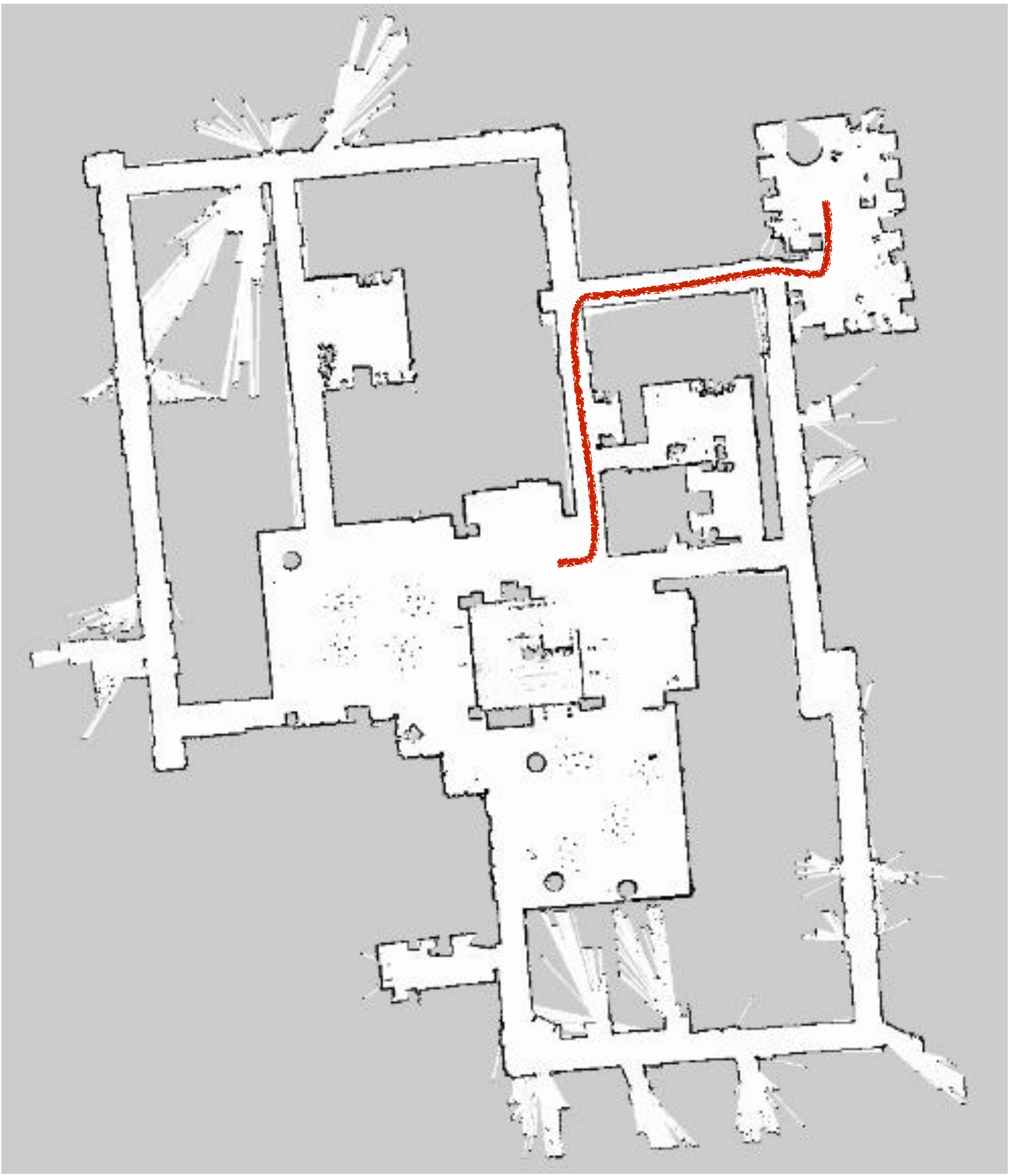


Roadmap over robot configurations

Roadmaps

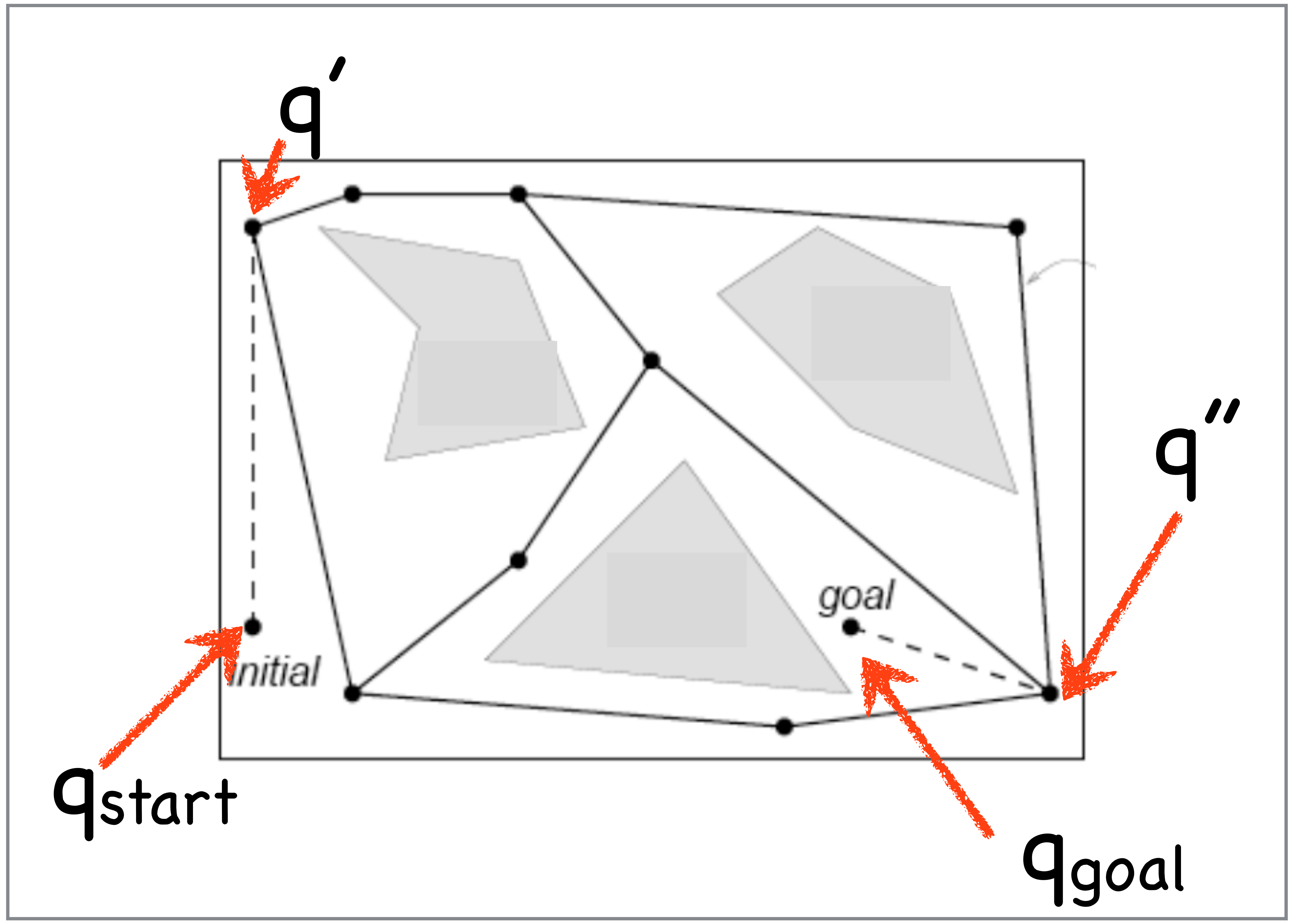


- Graph search assumed C-space as a fixed uniform grid
    - finite set of discretized cells
- How does this scale beyond planar navigation?
    - curse of dimensionality
- Roadmaps are a more general notion of graphs in C-space



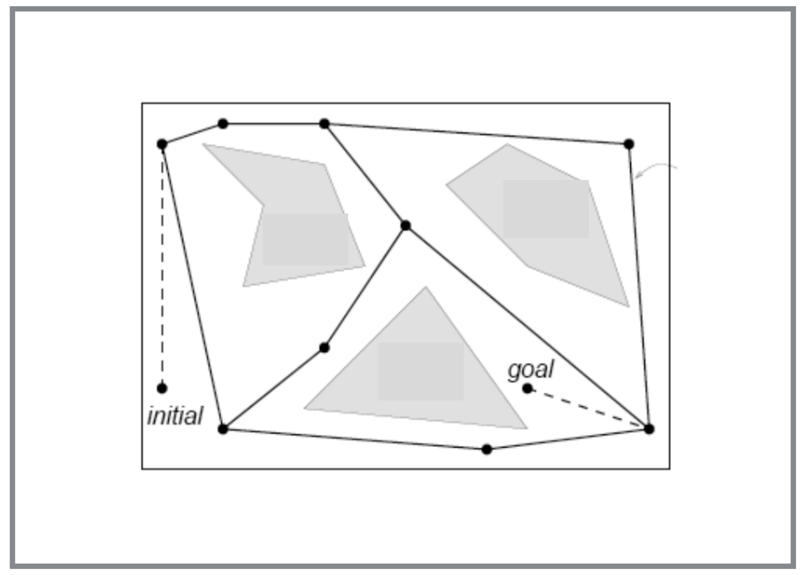
Roadmap Definition

- A roadmap *RM* is a union of curves s.t. all start and goal points in C-space (Q_{free}) can be connected by a path



- Roadmap properties:
  - **Accessibility**: There is a path from $q_{start} \in Q_{free}$ to some $q' \in RM$
  - **Departability**: There is a path from $q'' \in RM$ to $q_{goal} \in Q_{free}$
  - **Connectivity**: there exists a path in *RM* between *q'* and *q''*

Basic Roadmap Planner



1) Build the roadmap *RM* as graph *G(V,E)*

   - *V*: nodes are "valid" in C-space in Q_{free}
     - a configuration *q* is valid if it is not in collision and within joint limits
   - *E*: an edge $e(q_1,q_2)$ connects two nodes if a free path connects q_1 and q_2
     - all configurations along edge assumed to be valid

2) Connect start and goal configurations to *RM* at *q'* and *q''*, respectively

3) Find path in *RM* between *q'* and *q''*

How to build a roadmap?

How to build a roadmap?

2 Approaches

2 Approaches to Roadmaps

Deterministic:

complete algorithms

- Visibility Graph
    - trace lines connecting obstacle polygon vertices
- Voronoi Planning
    - trace edges equidistant from obstacles

Probabilistic:

C-space sampling

- Probabilistic Roadmap (PRM)
    - sample and connect vertices in graph for multiple planning queries
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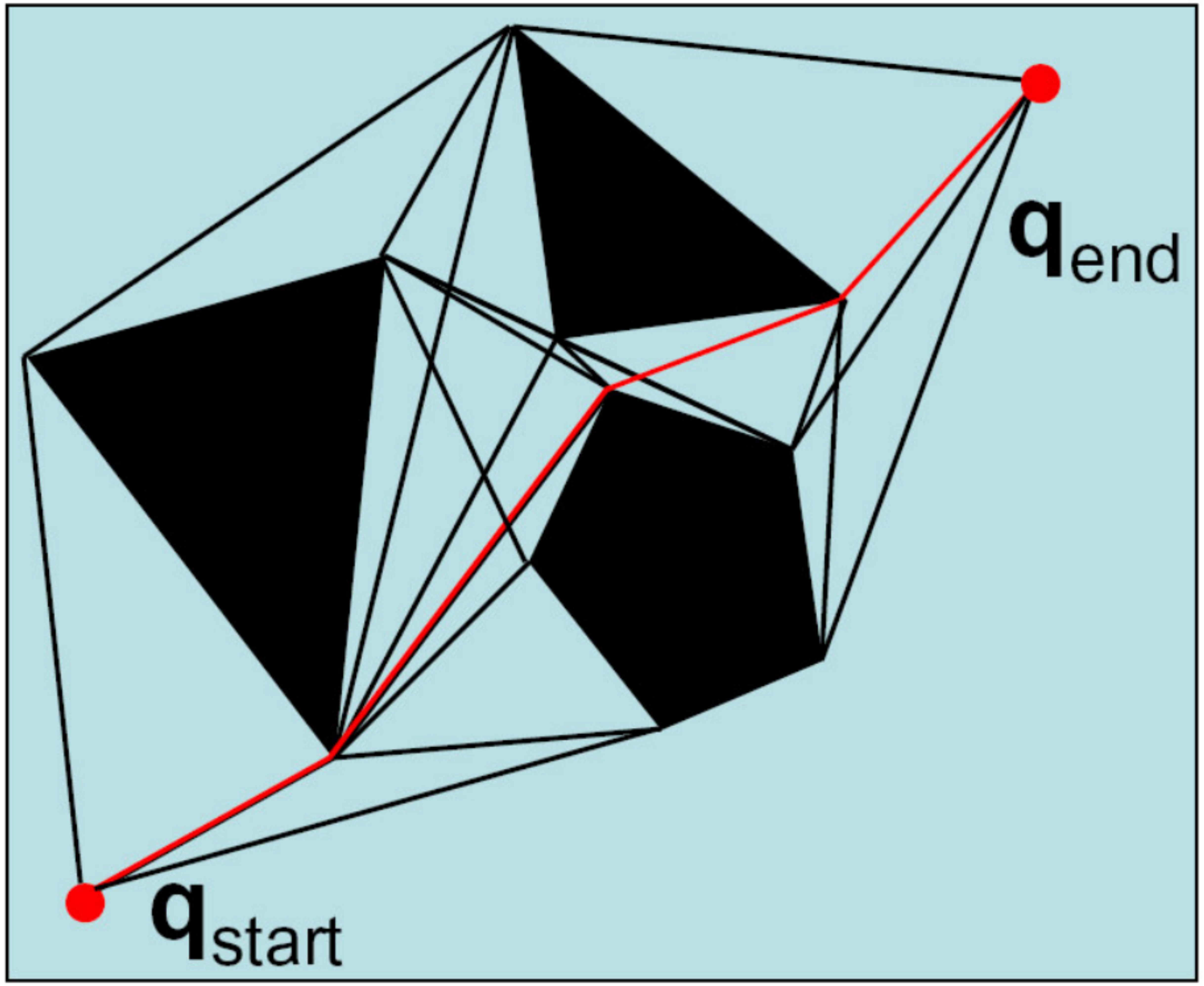

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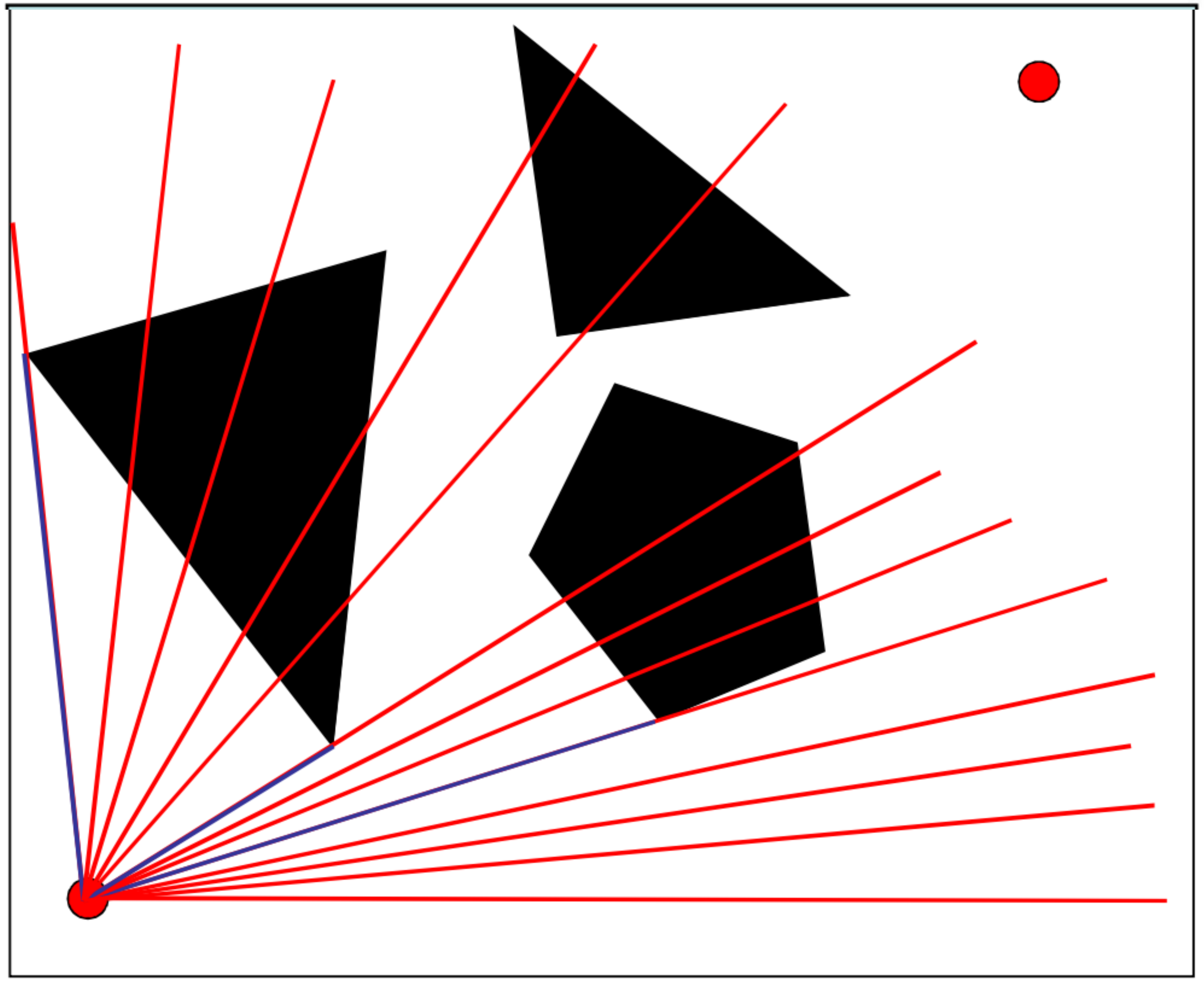

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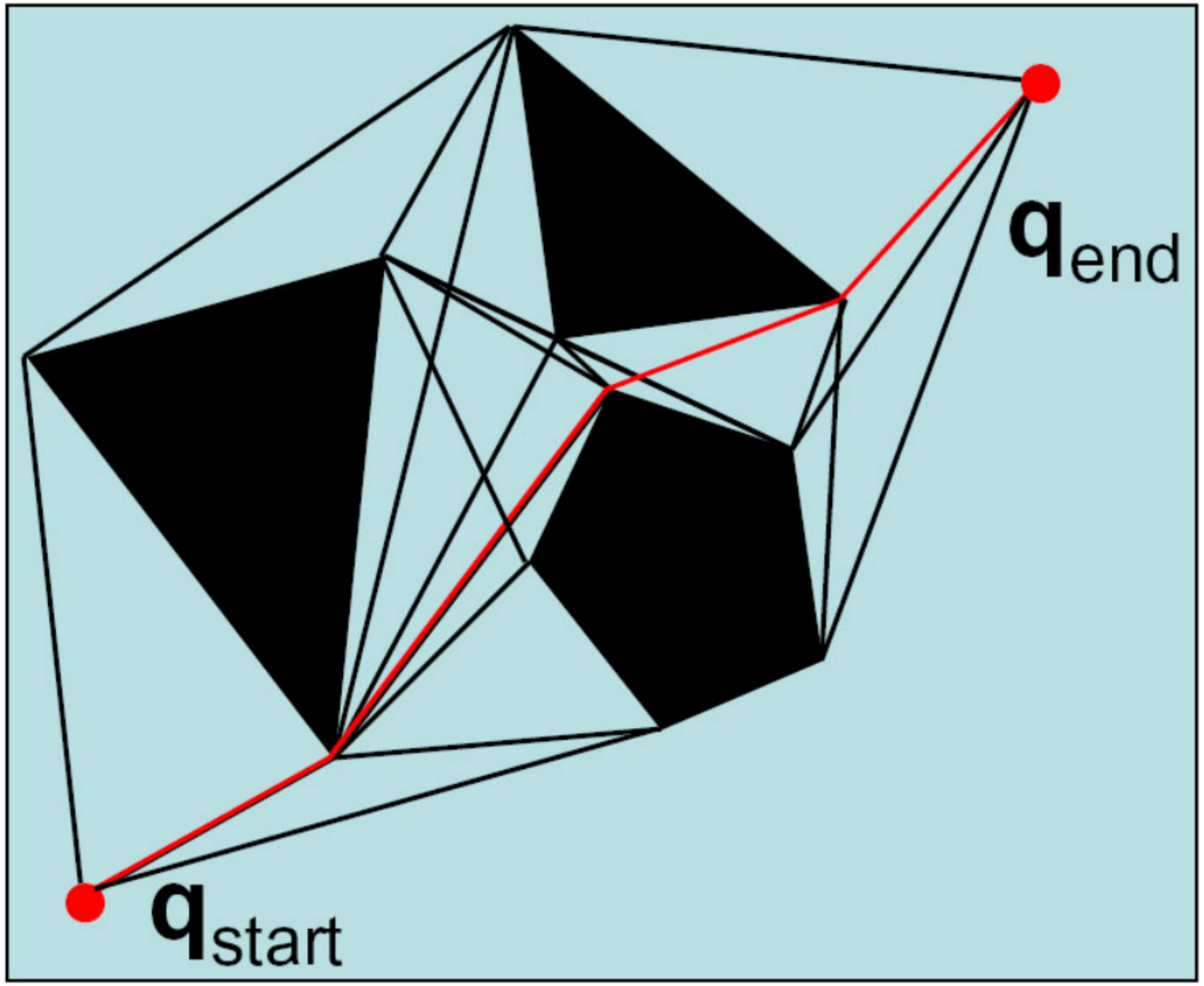

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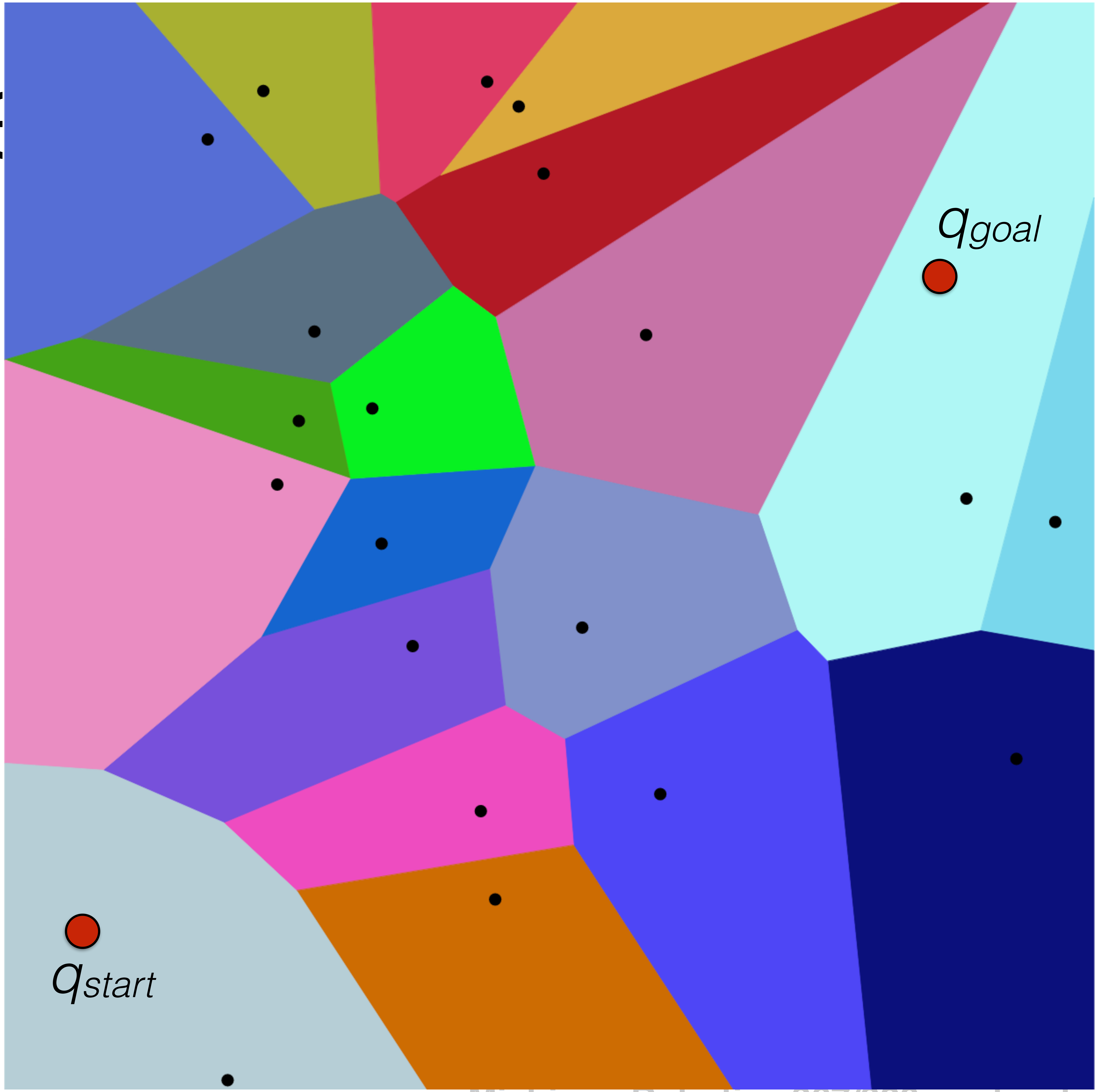
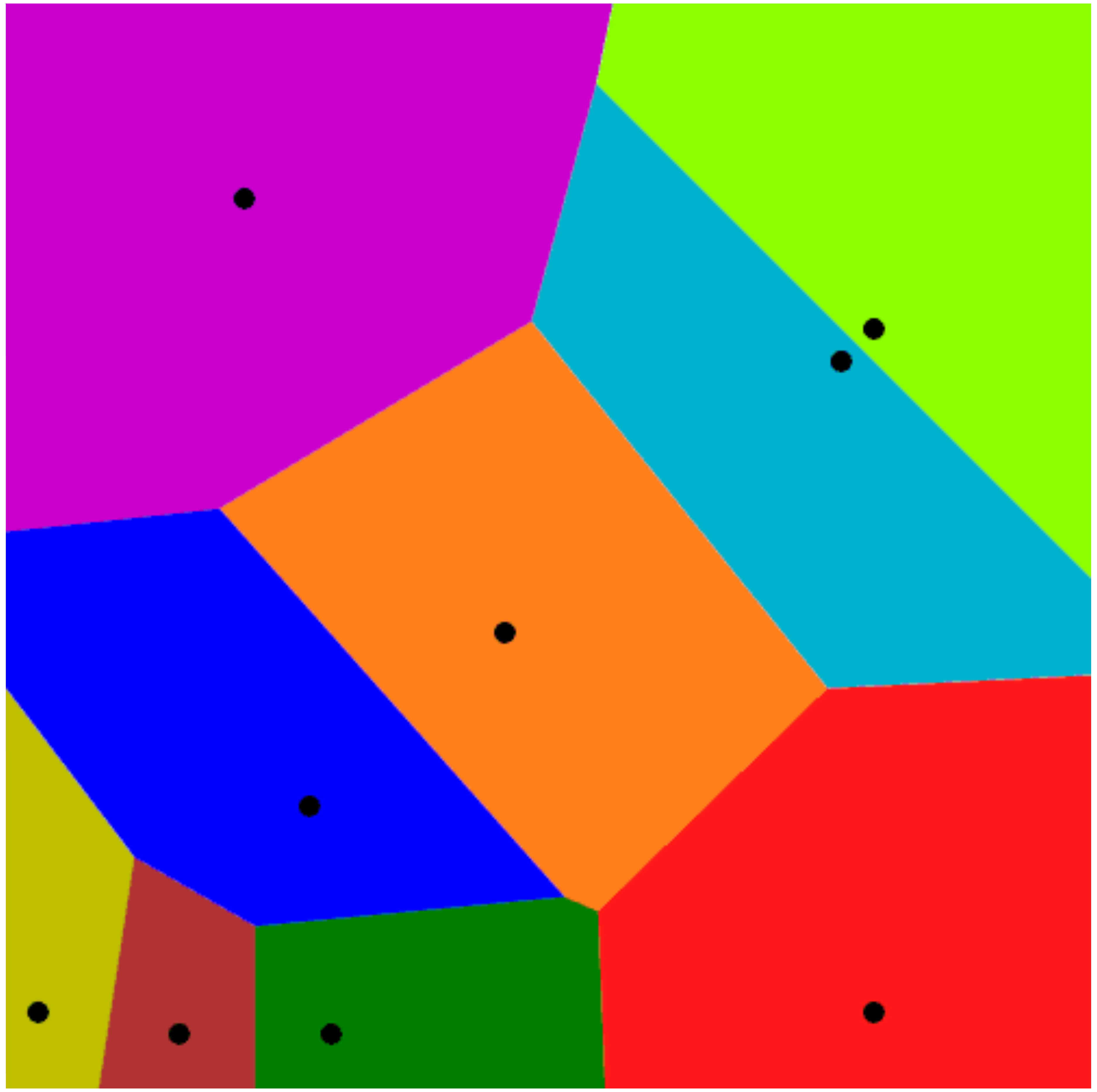


Voronoi Diagram

- Given N input points in a d dimensional space
- Find region boundaries such that each point on a boundary are equidistant to two or more input points
- Delaunay triangulation is a dual to the Voronoi diagram



https://en.wikipedia.org/wiki/Voronoi_diagram#/media/File:Voronoi_growth_euclidean.gif

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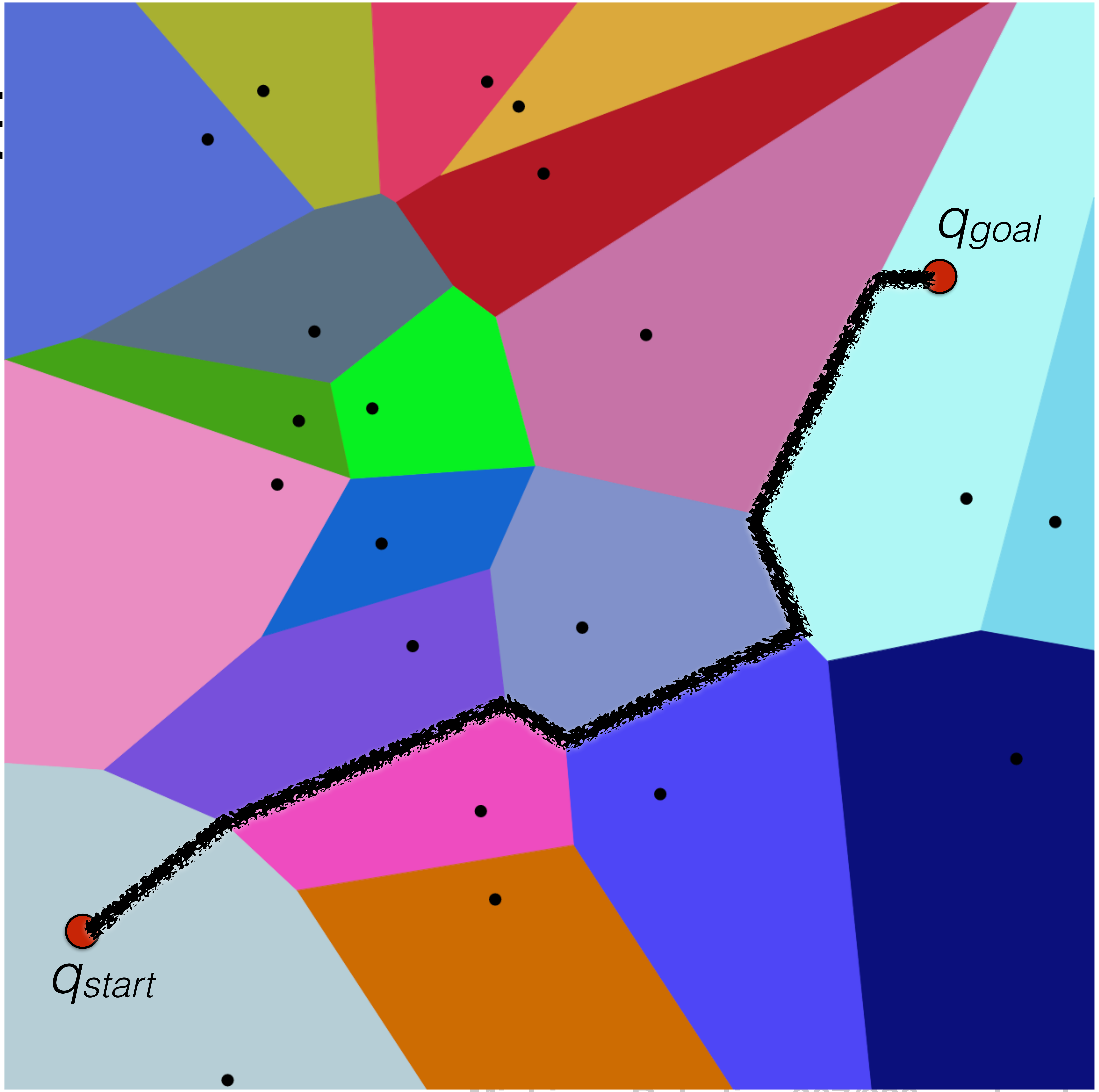

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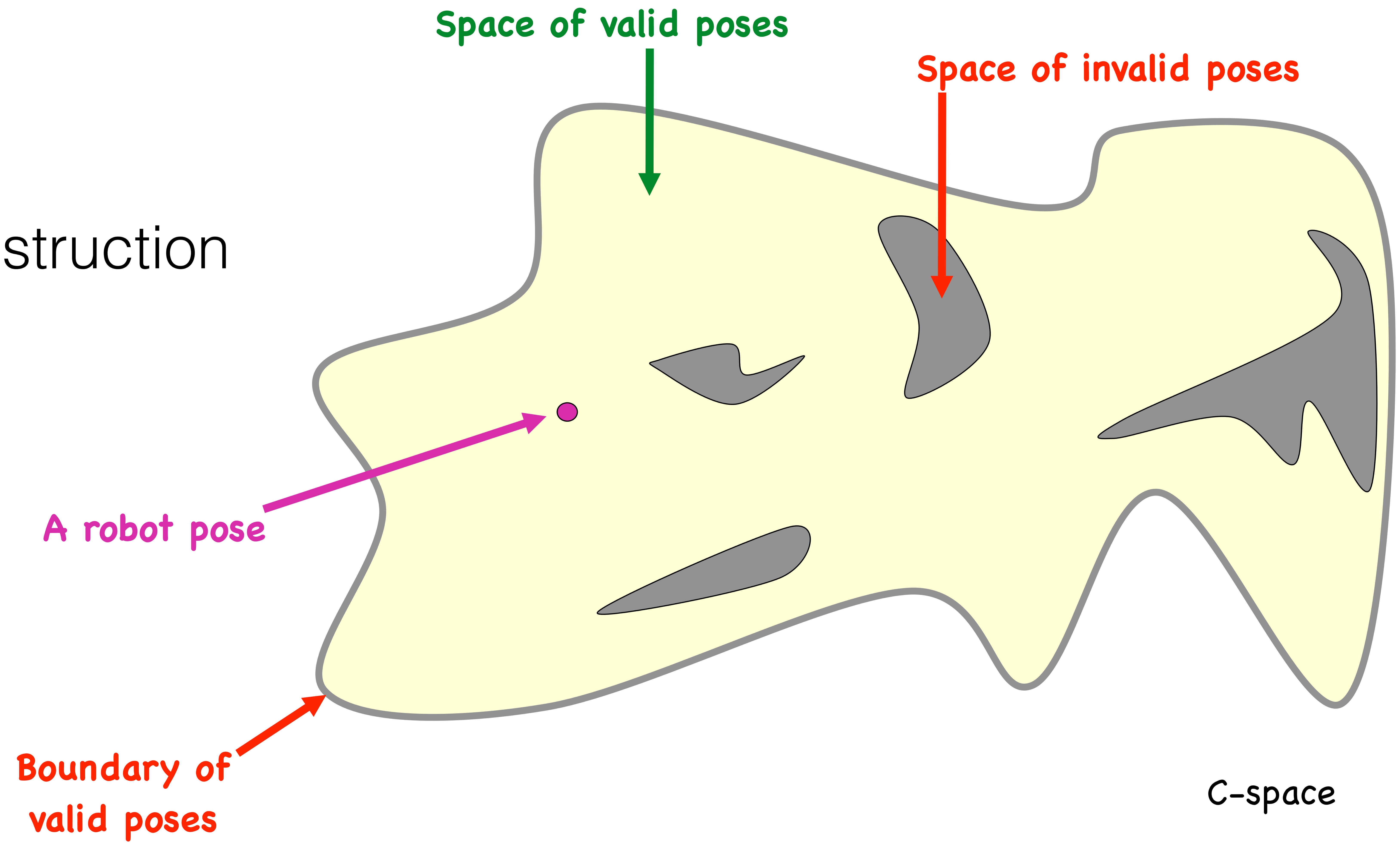
Probabilistic road maps

- Two phases
  - Roadmap construction
  - Path Query

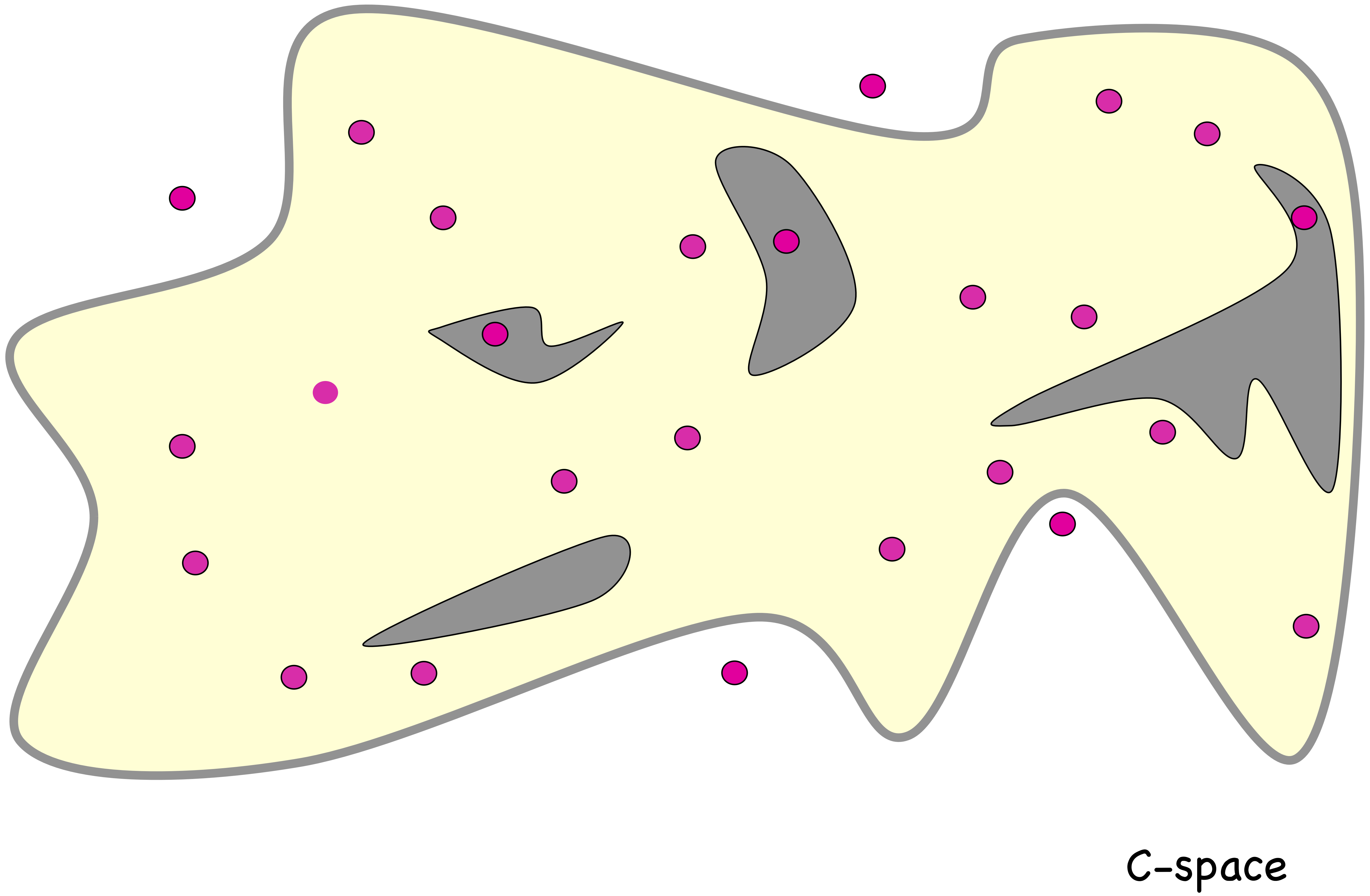


PRM: construction phase

1) **Select N sample poses at random**

2) Eliminate invalid poses

3) Connect neighboring poses



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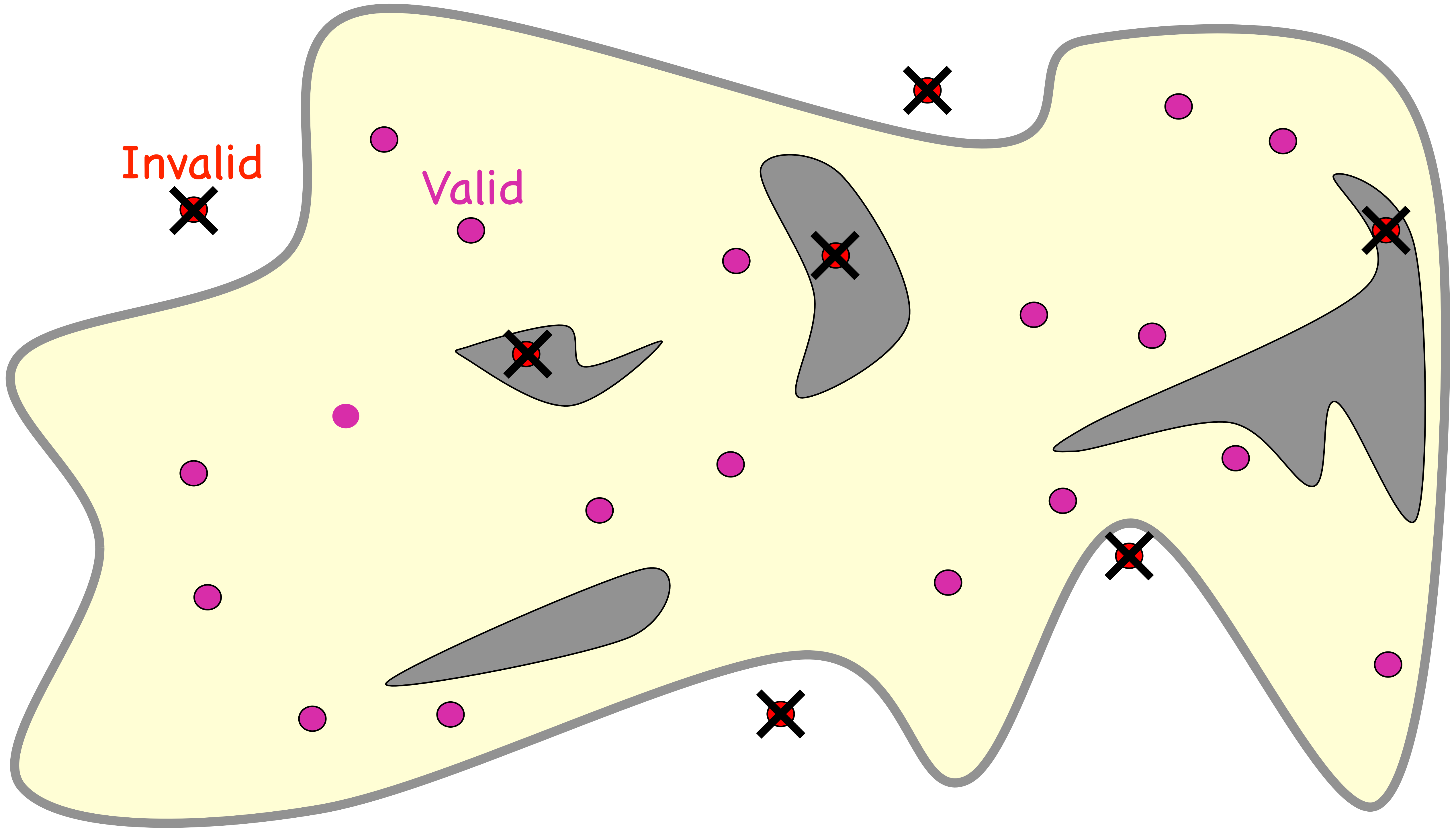

Collision detection will be covered later

PRM: construction phase

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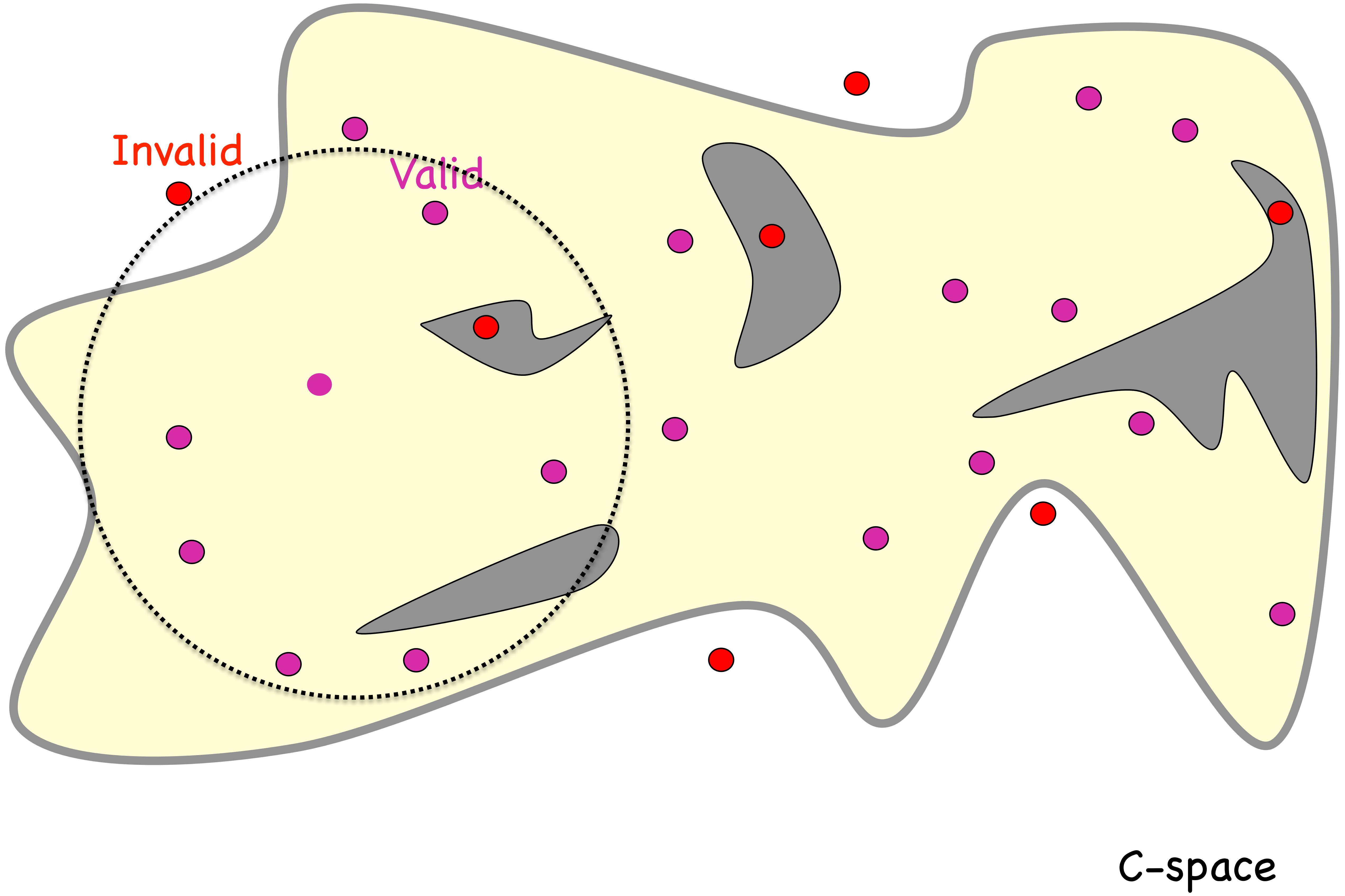
3) **Connect neighboring poses**



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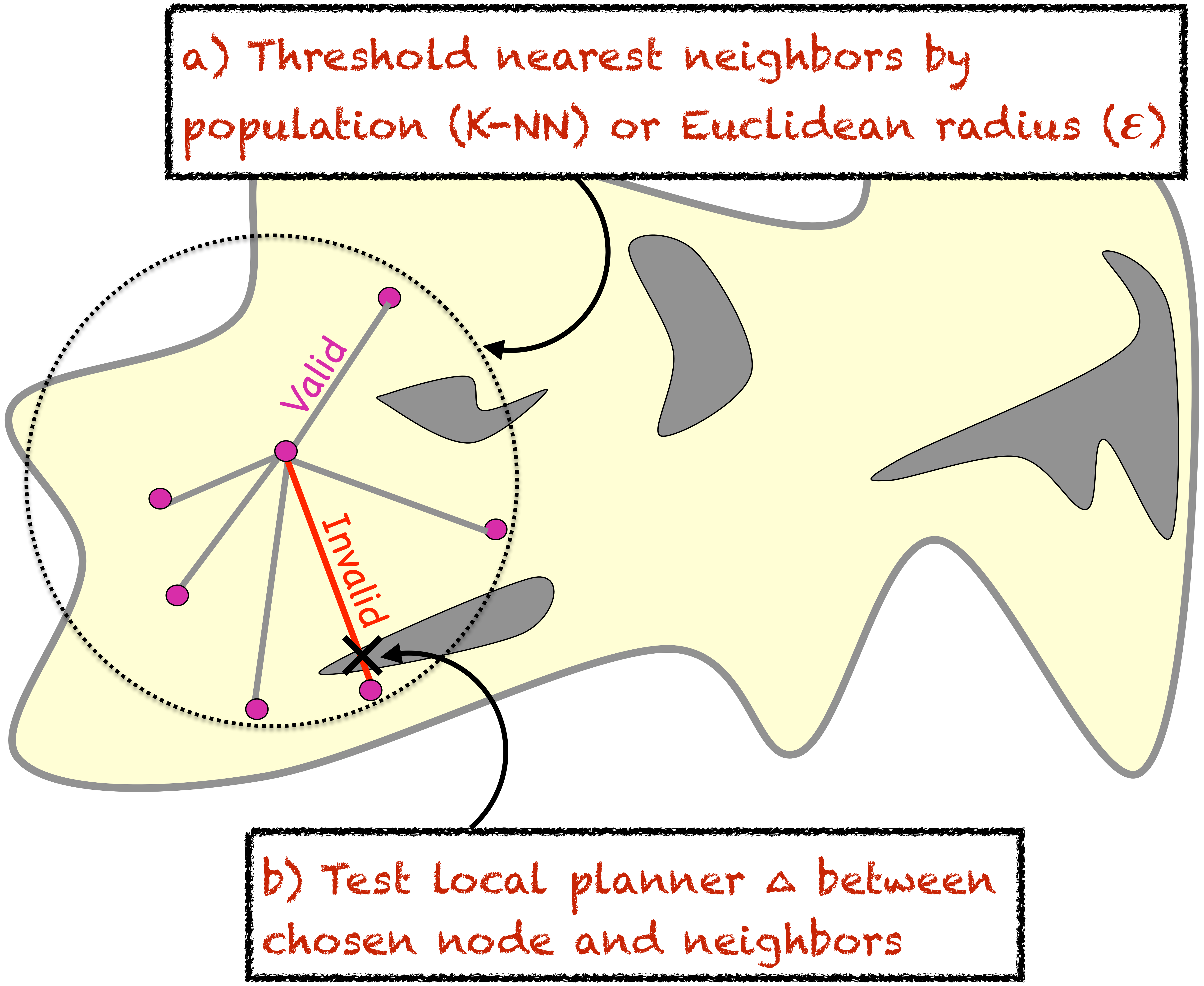


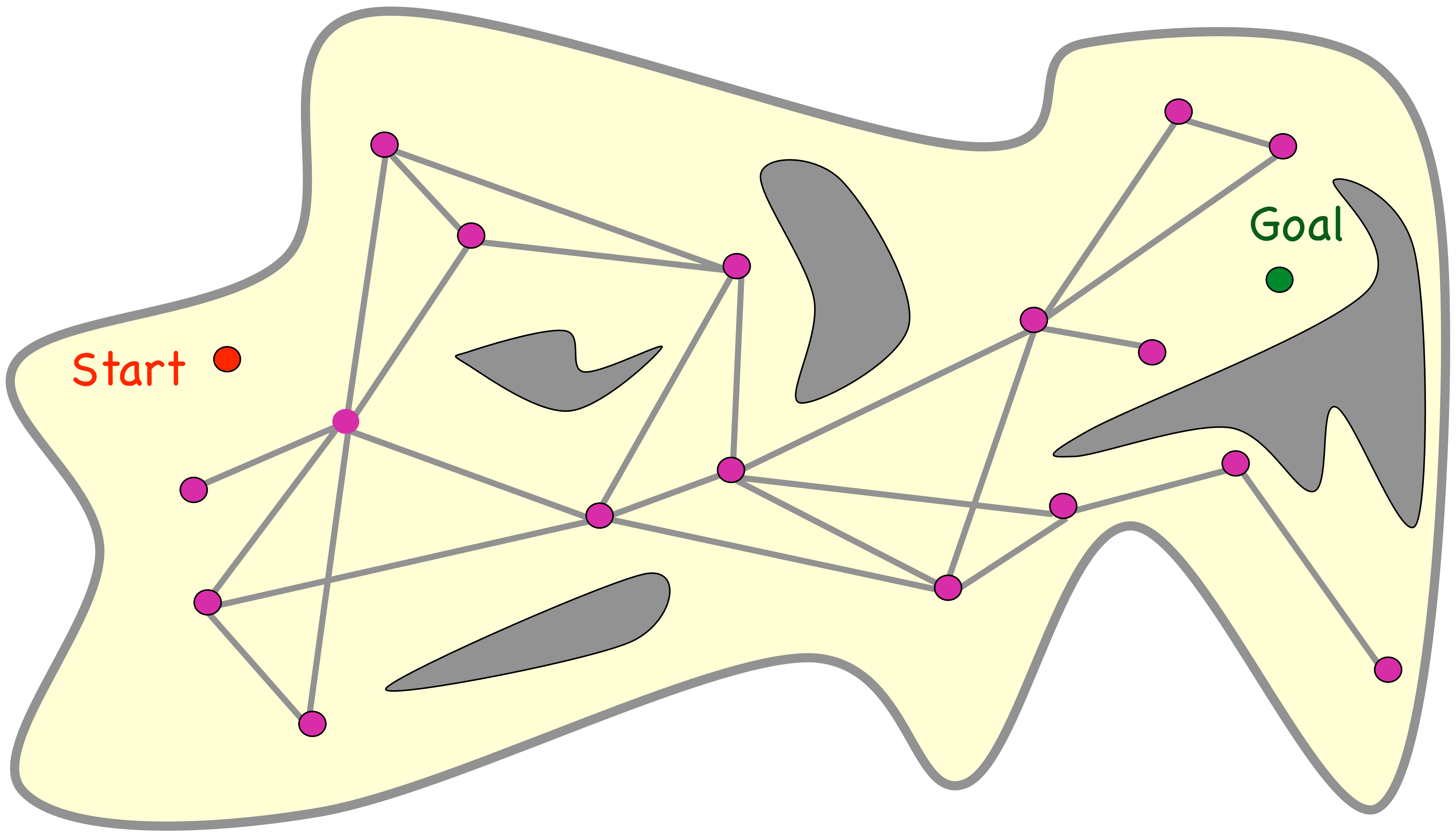
PRM: query phase

1) **Given constructed roadmap, start pose, and goal pose**

2) Attach goal and start to nearest roadmap entry nodes

3) Search for path between roadmap entry nodes

4) Return path with entry and departure edges



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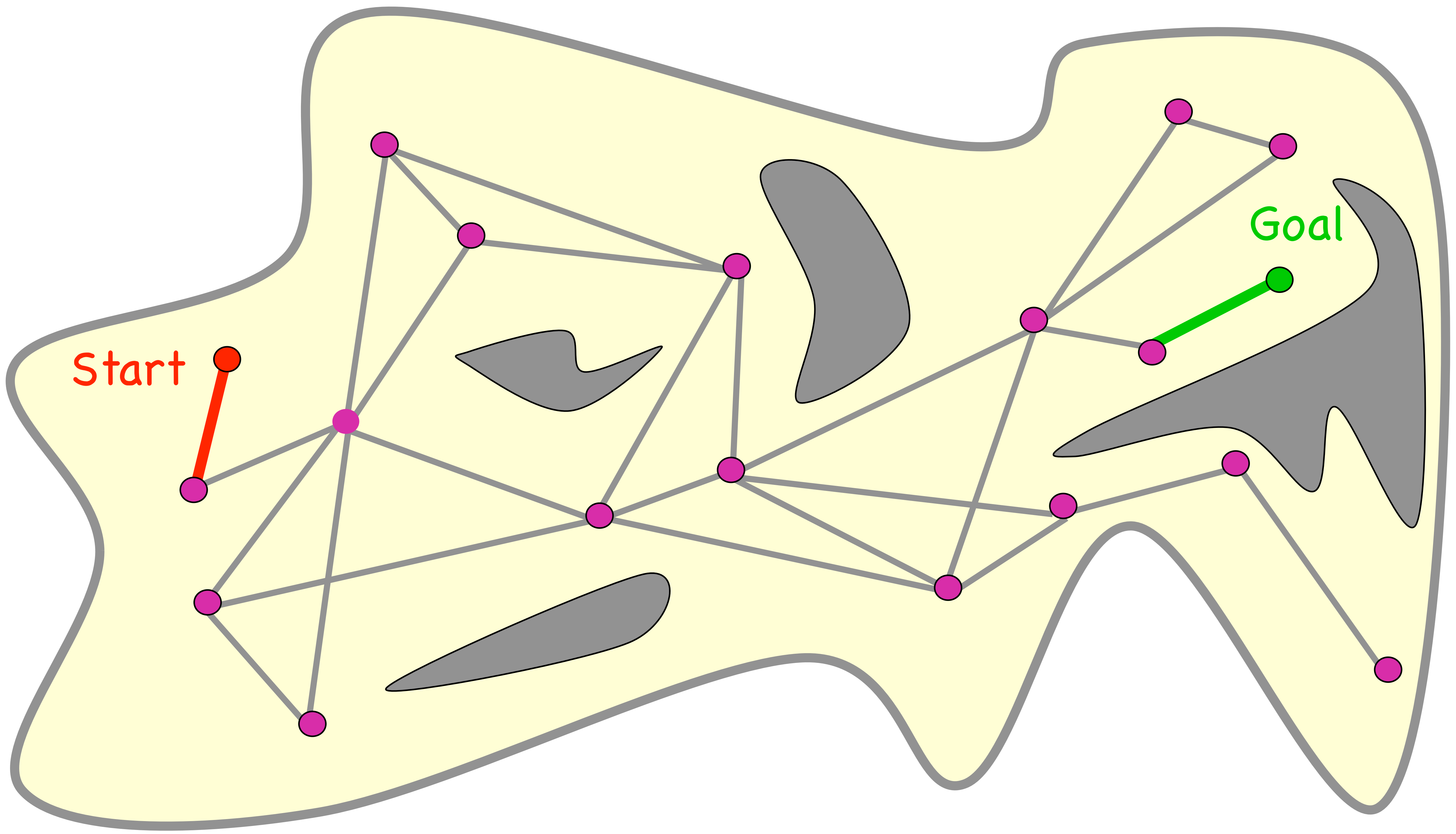

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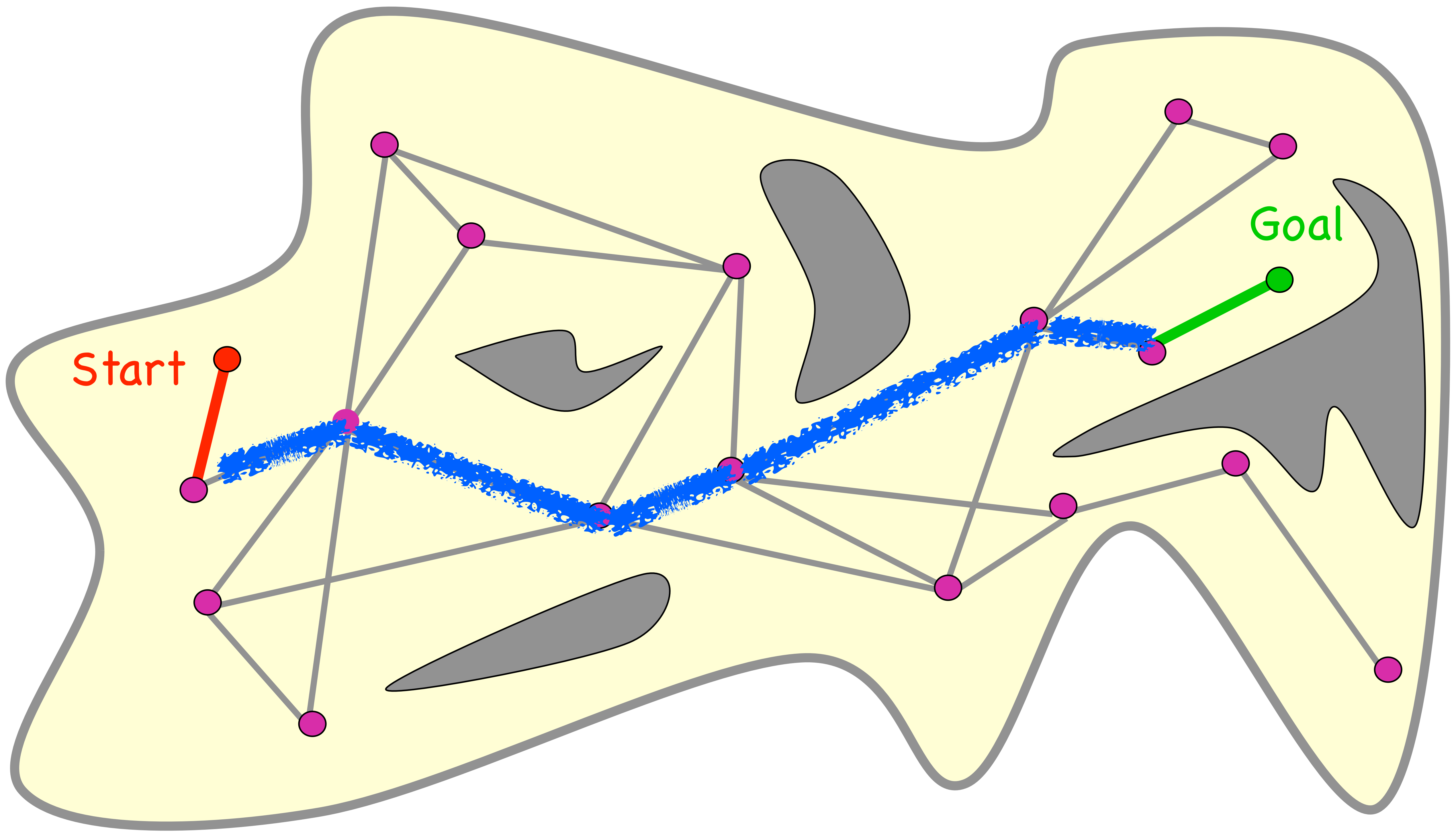

Remember: graph search algorithms
A*, Dijkstra, BFS, DFS

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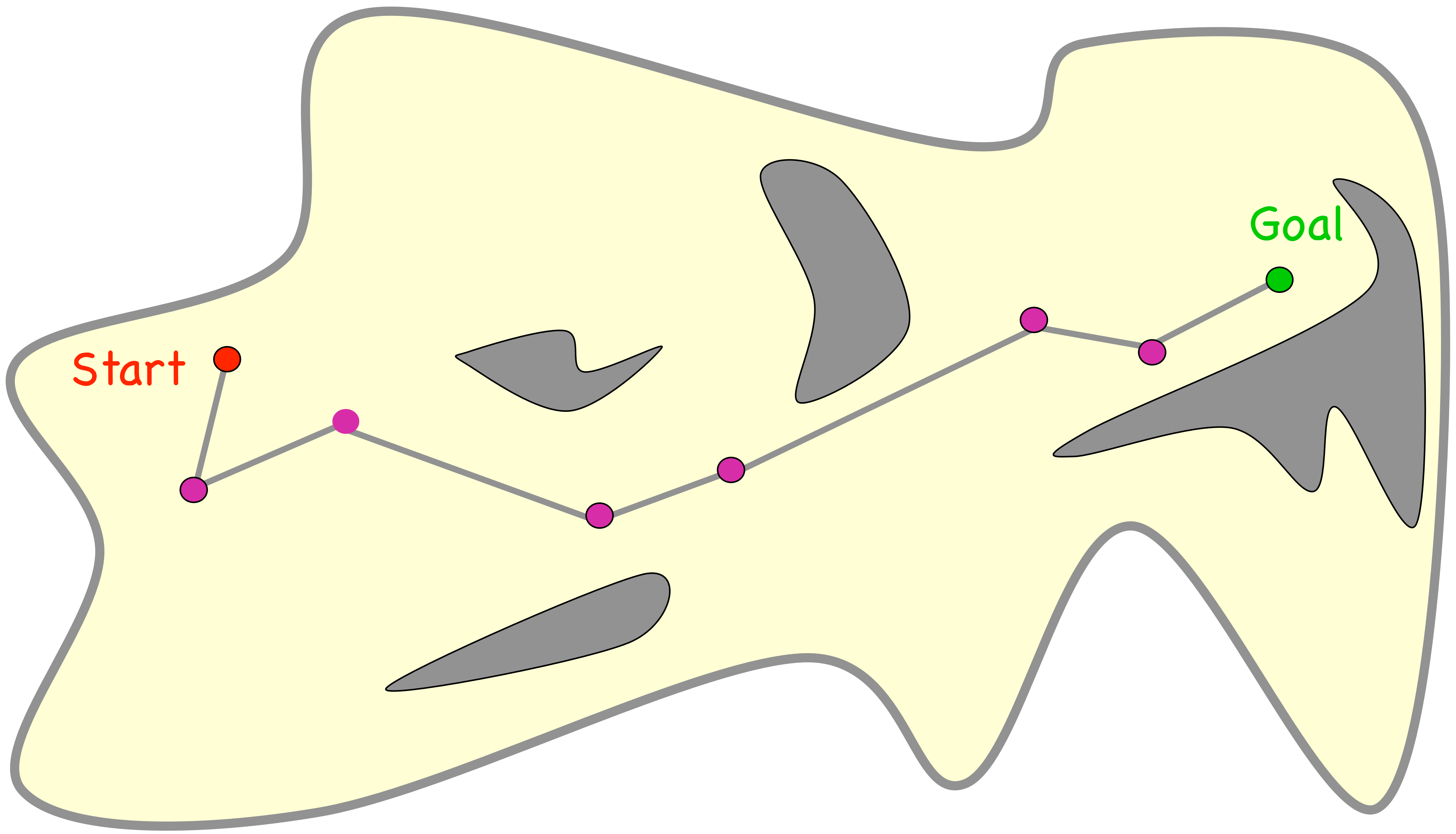

Multi-query planning: Considerations

- Number of samples wrt. C-space dimensionality
- Balanced sampling over C-space
- Choice of distance (e.g., Euclidean)
- Choice of local planner (e.g., line subdivision)
- Selecting neighbors: (e.g., K-NN, kd-tree, cell hashing)

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Single Query Planning

- Given specific start and goal configurations
- Grow trees from start and goal towards each other
- Path is found once trees connect
- Focus sampling in unexplored areas of C-space and moving towards start/goal
- Common algorithms:
  - ESTs (expansive space trees)
  - **RRTs (rapidly exploring random trees)**

RRT Algorithm

[Kuffner, LaValle 2000]

RRT Algorithm

Extend graph towards a random configuration and repeat

BUILD_RRT(q_{init})

$\mathcal{T}$.init(q_{init});
for $k = 1$ **to** K **do**
    $q_{rand} \leftarrow$ RANDOM_CONFIG();
    EXTEND($\mathcal{T}, q_{rand}$);
Return $\mathcal{T}$

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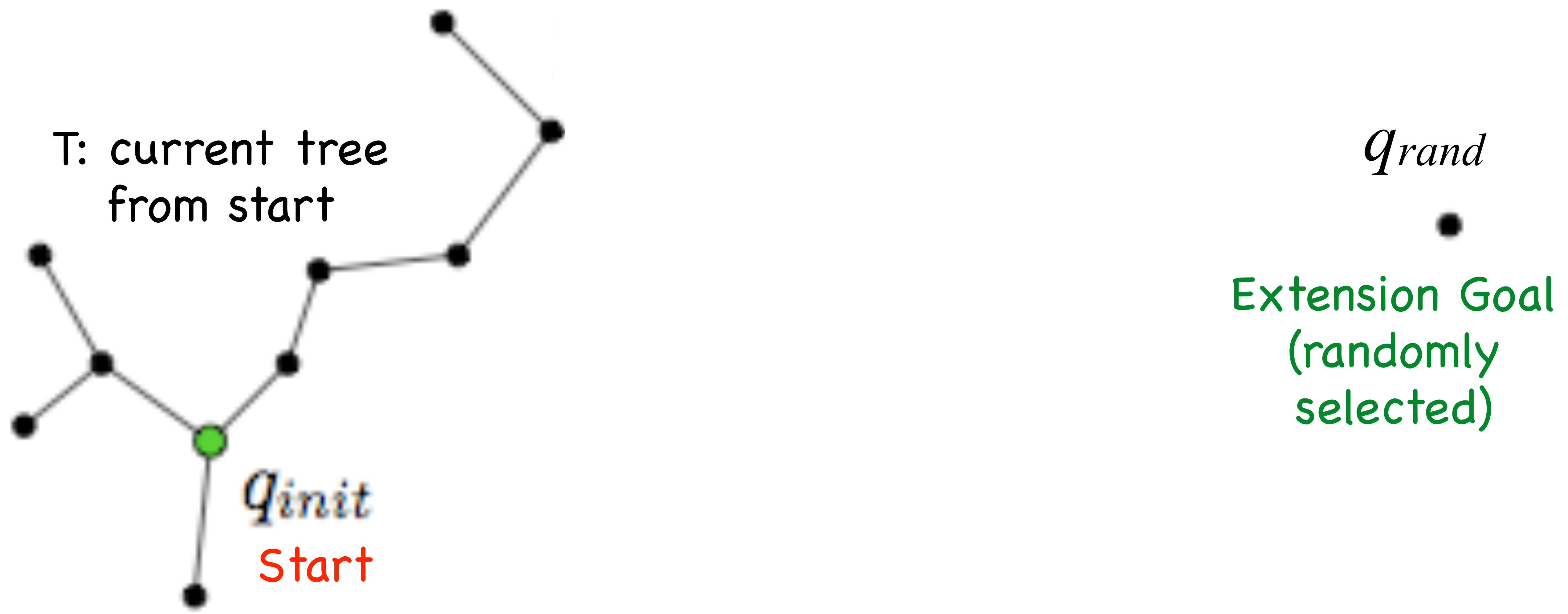

Figure 3: The EXTEND operation.

[Kuffner, LaValle 2000]

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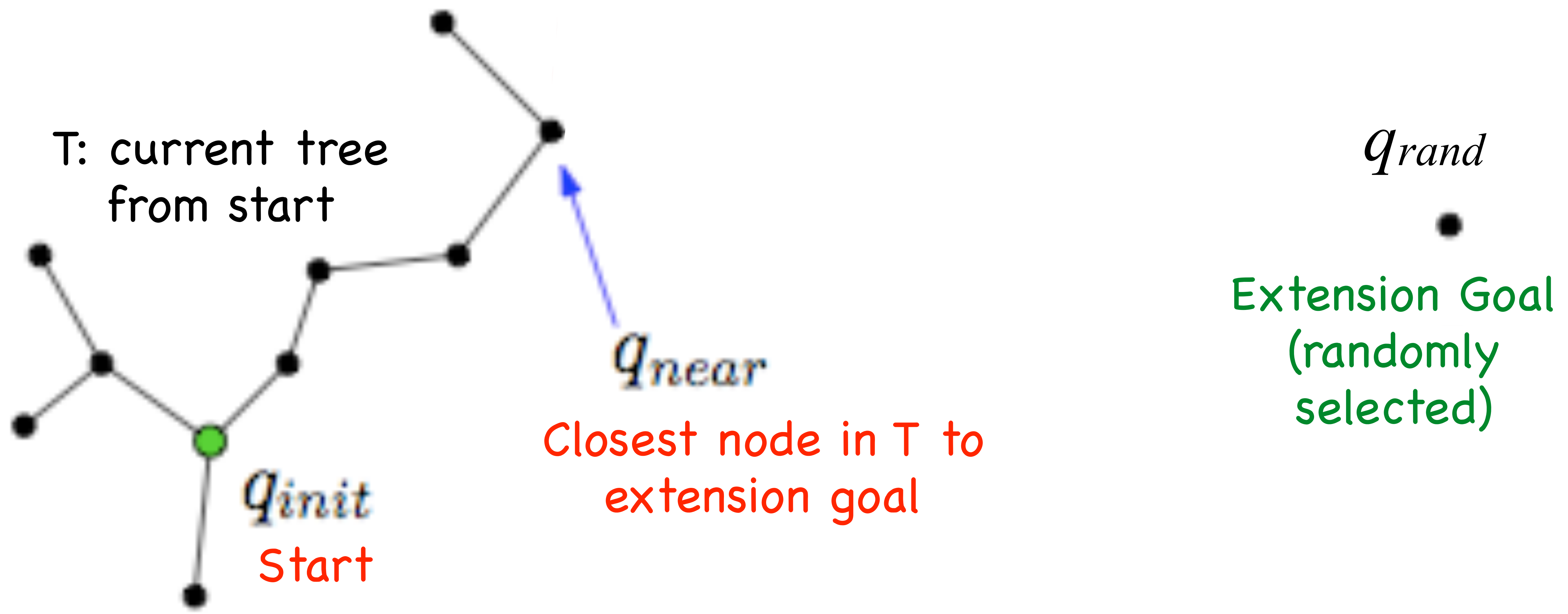
```
EXTEND(T, q)
  q_near ← NEAREST_NEIGHBOR(q, T);
  if NEW_CONFIG(q, q_near, q_new) then
      T.add_vertex(q_new);
      T.add_edge(q_near, q_new);
      if q_new = q then
          Return Reached;
      else
          Return Advanced;
  Return Trapped;
```

Extend graph towards a random configuration



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EXTEND($\mathcal{T}, q$)

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        Return *Reached*;
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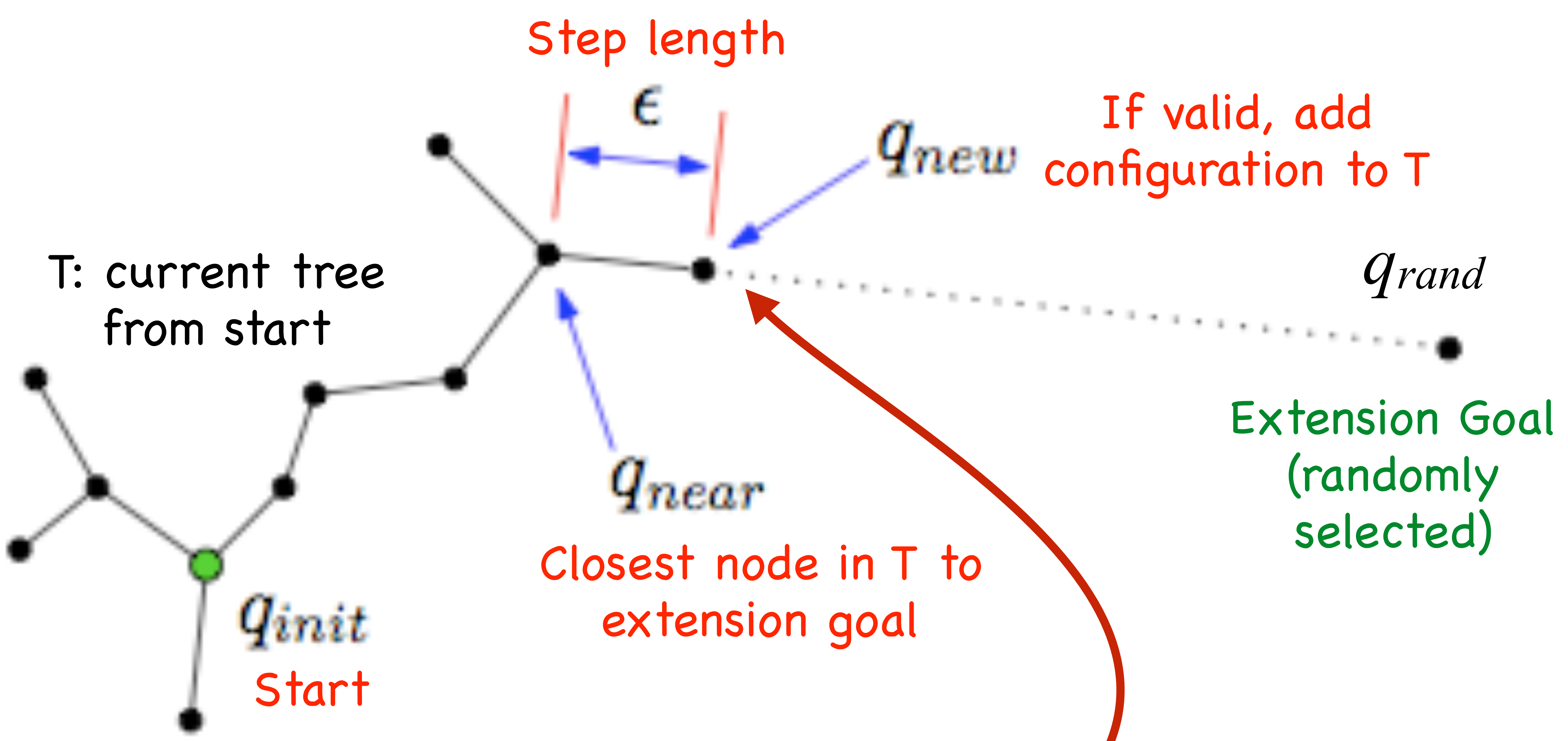
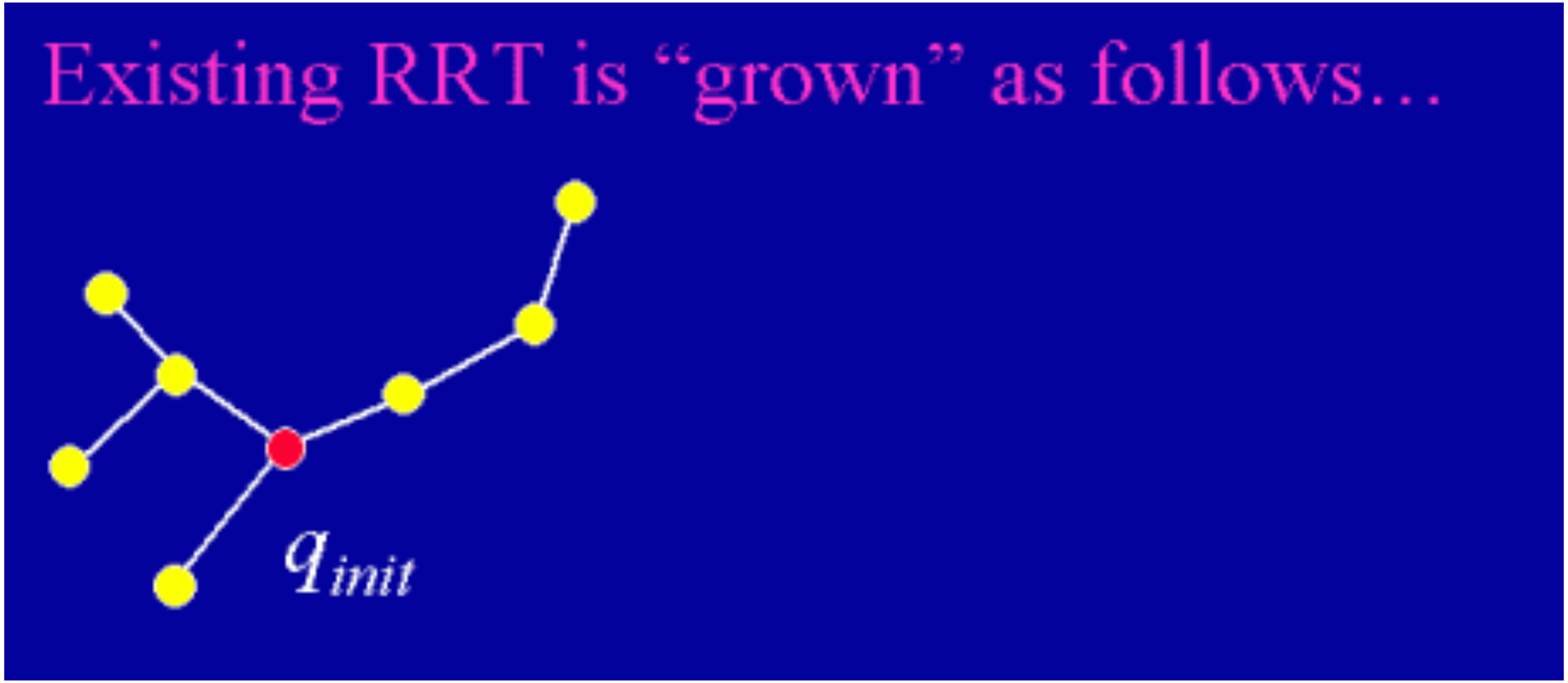
Extend graph towards a random configuration



Generate and test new configuration along vector in C-space from q_{near} to q_{rand}

RRT Extend animation



http://www.kuffner.org/james/plan/algorithm.php

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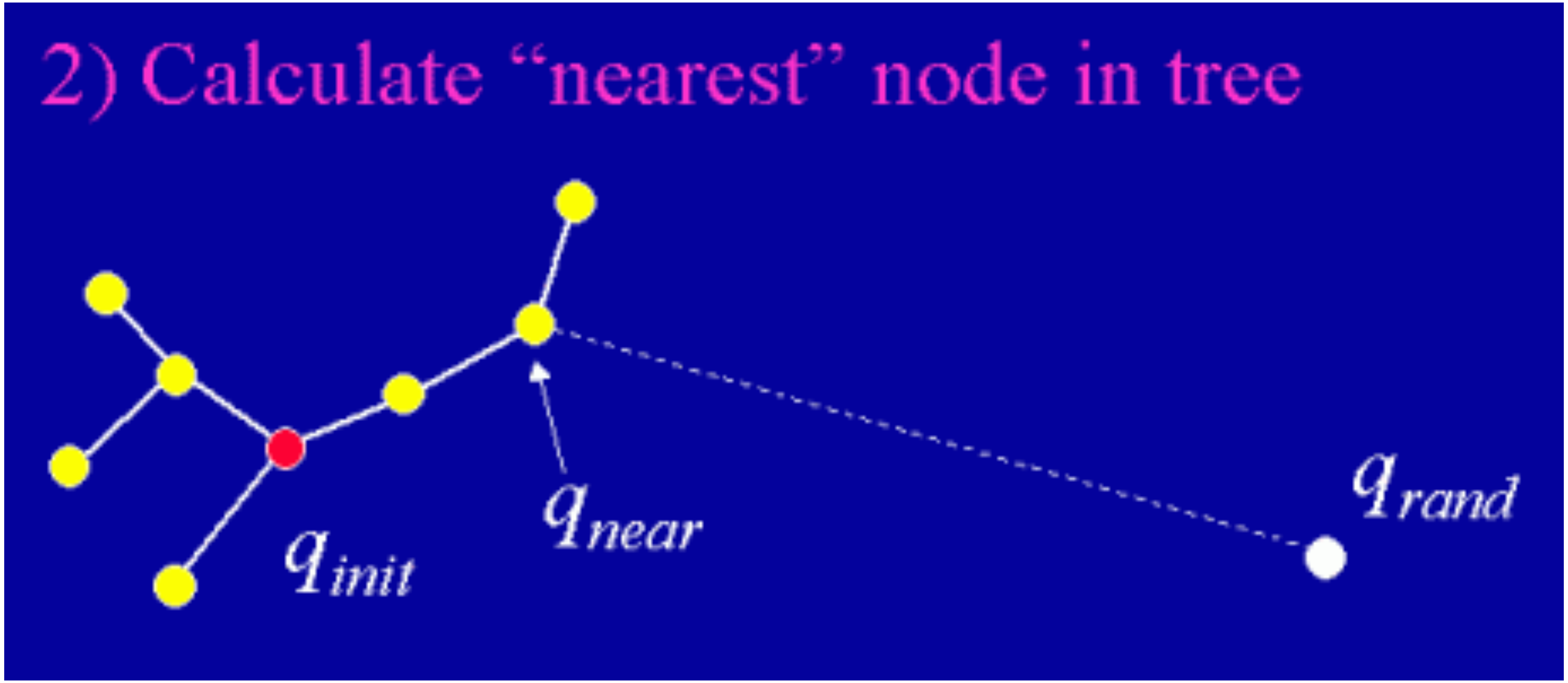

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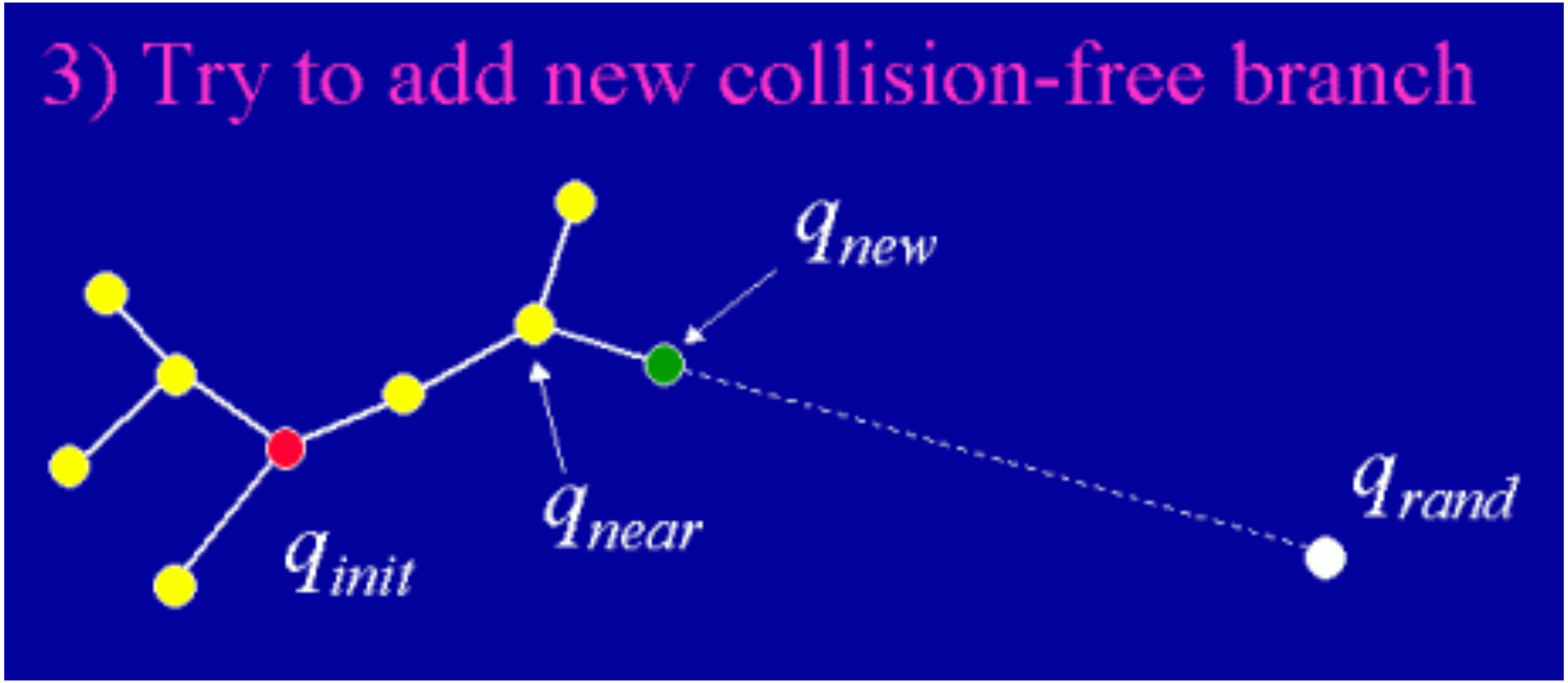

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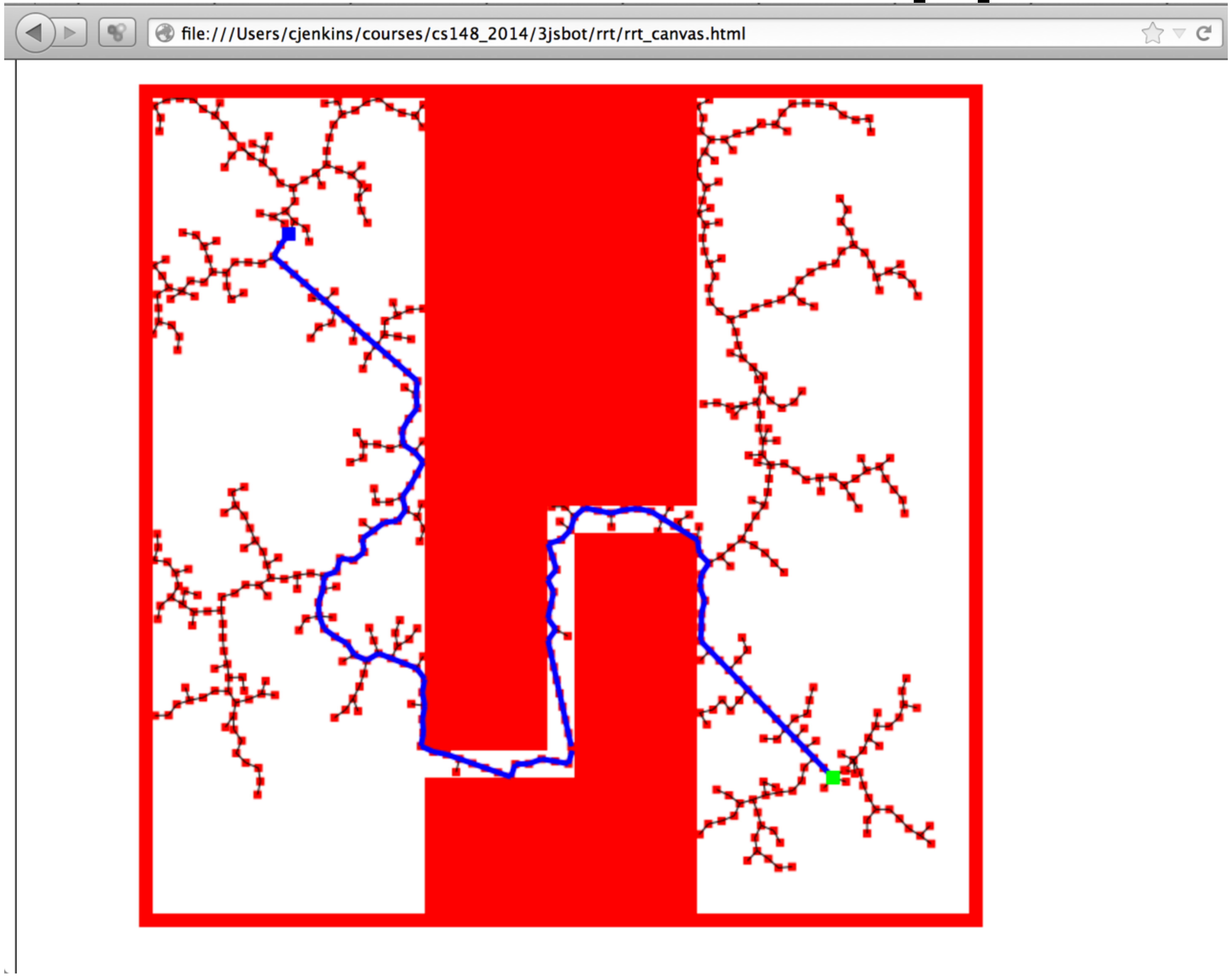
Let's see what happens

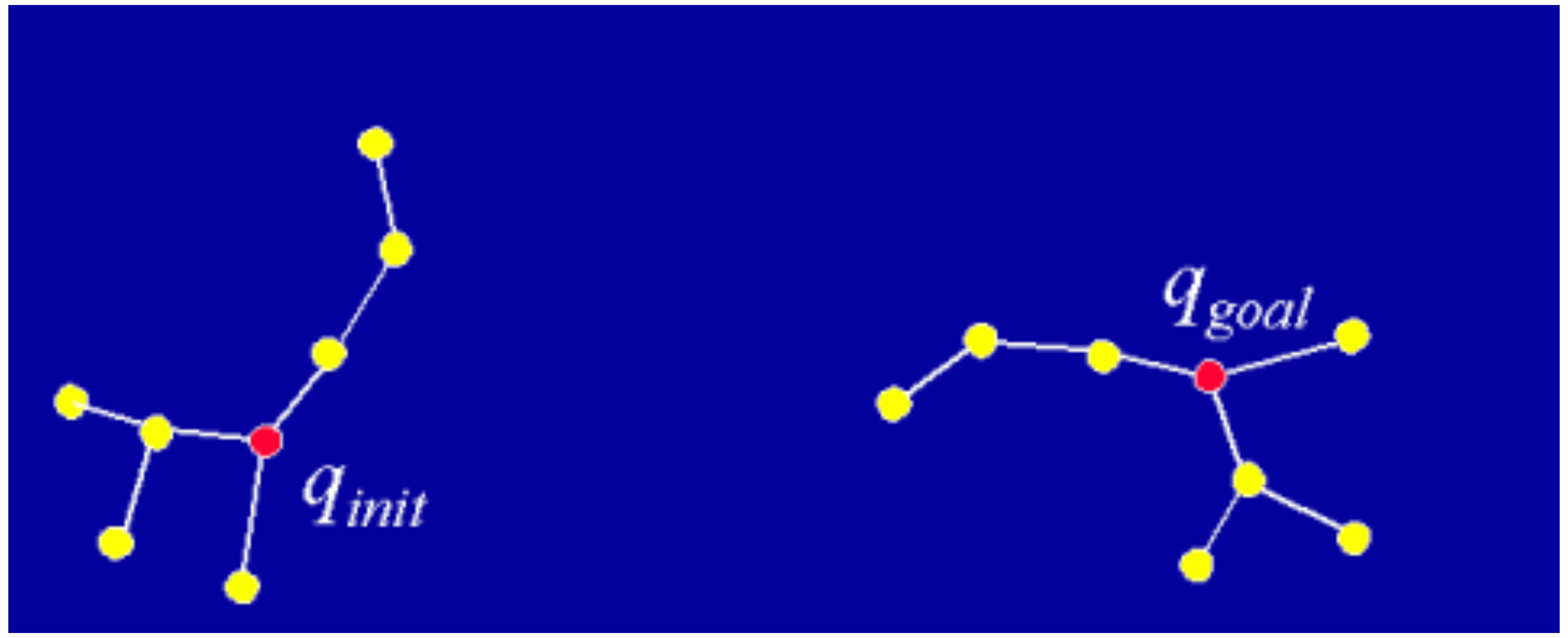


[Kuffner, LaValle 2000]

RRT Connect

0) Use 2 trees (A and B) rooted at start and goal configurations

```
RRT_CONNECT_PLANNER(q_init, q_goal)
  T_a.init(q_init); T_b.init(q_goal);
  for k = 1 to K do
      q_rand ← RANDOM_CONFIG();
      if not (EXTEND(T_a, q_rand) = Trapped) then
          if (CONNECT(T_b, q_new) = Reached) then
              Return PATH(T_a, T_b);
      SWAP(T_a, T_b);
  Return Failure
```



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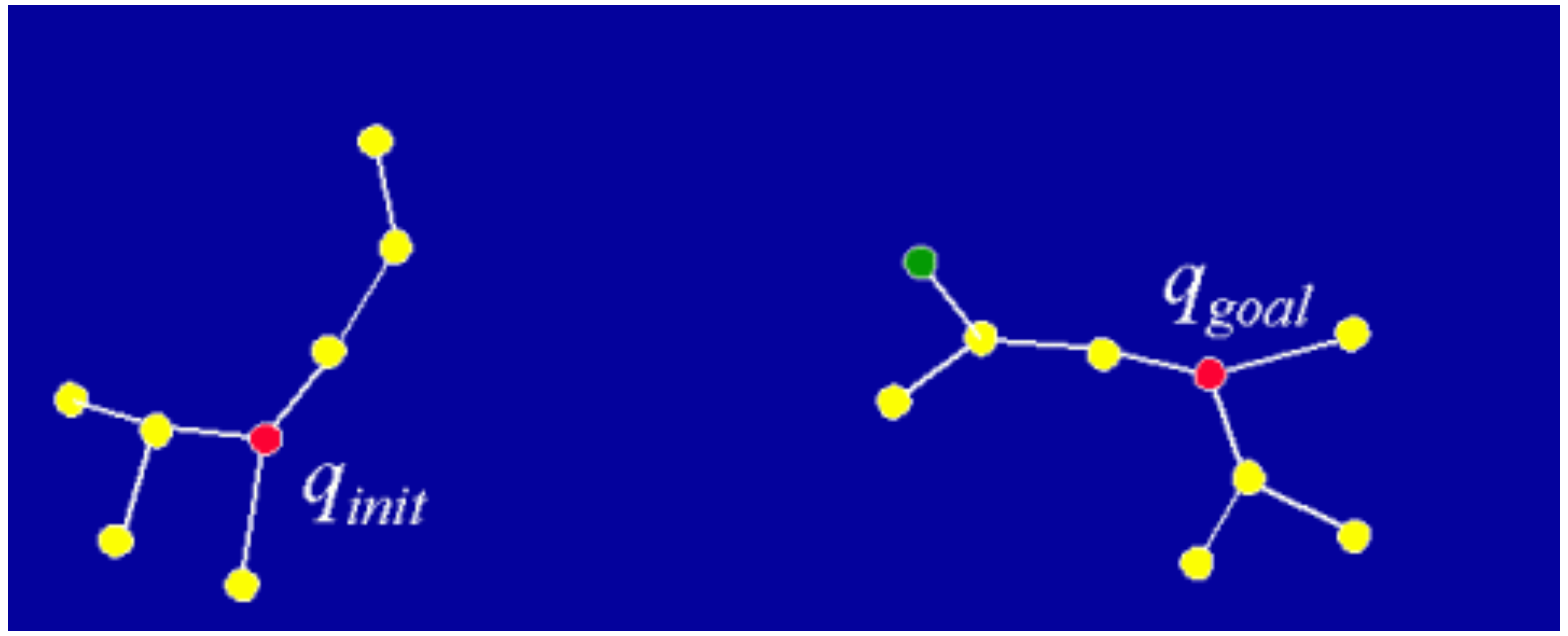
1) Extend tree A towards a random configuration



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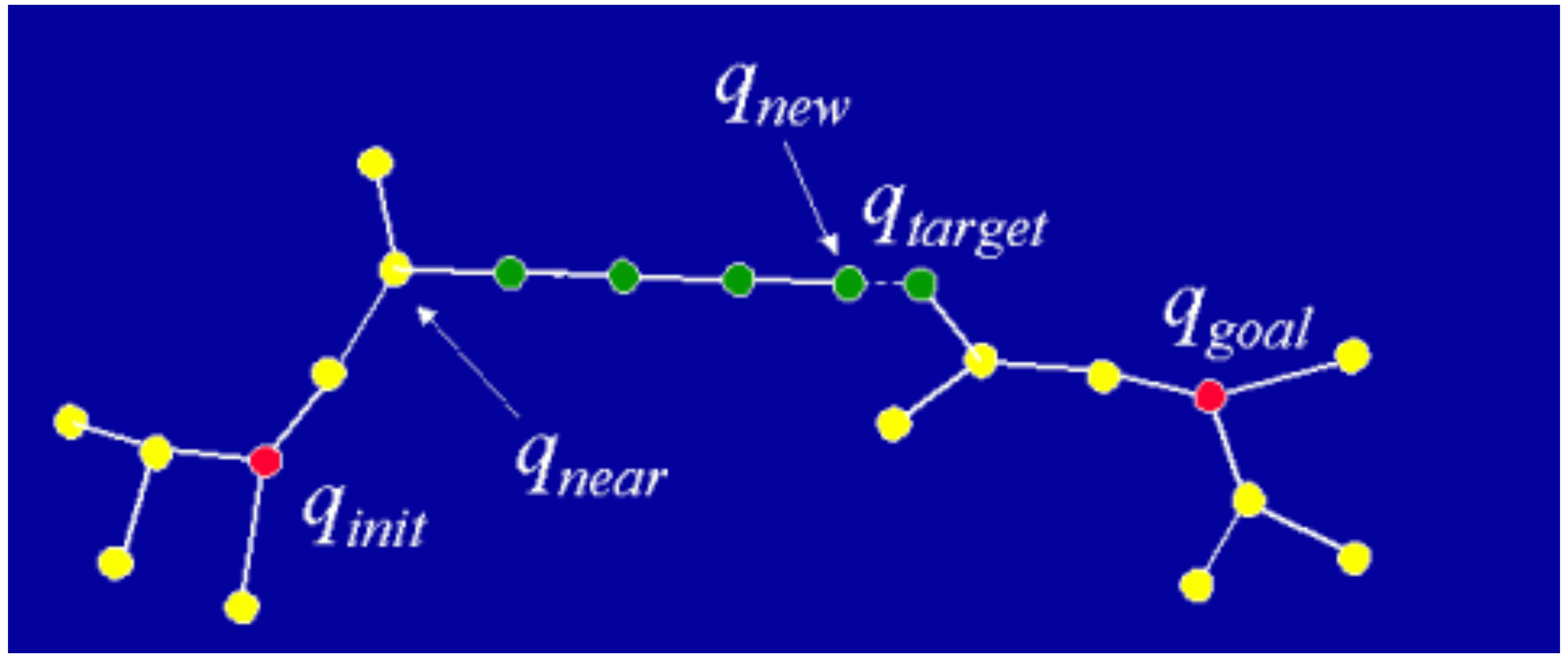
$q_{near} \leftarrow$ NEAREST_NEIGHBOR($q, \mathcal{T}$);
if NEW_CONFIG(q, q_{near}, q_{new}) **then**
    $\mathcal{T}$.add_vertex(q_{new});
    $\mathcal{T}$.add_edge(q_{near}, q_{new});
    if $q_{new} = q$ **then**
        Return *Reached*;
    else
        Return *Advanced*;
Return *Trapped*;

1) Extend tree A towards a random configuration

CONNECT($\mathcal{T}, q$)
repeat
    $S \leftarrow$ EXTEND($\mathcal{T}, q$);
until not ($S =$ *Advanced*)
Return S;



2) Try to connect tree B to tree A by extending repeatedly from its nearest neighbor

RRT Connect

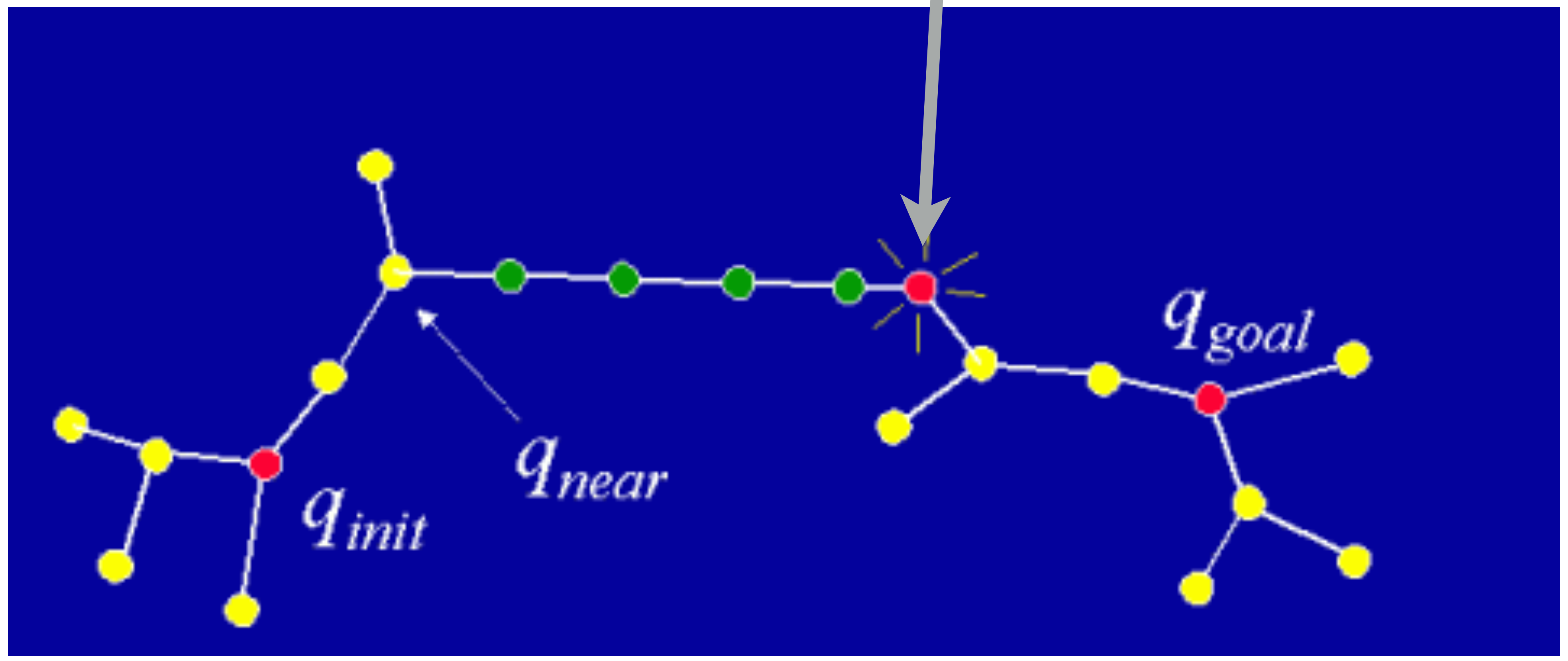
[Kuffner, LaValle 2000]

0) Use 2 trees (A and B) rooted at start and goal configurations

RRT_CONNECT_PLANNER(q_{init}, q_{goal})

$\mathcal{T}_a$.init(q_{init}); $\mathcal{T}_b$.init(q_{goal});
for $k = 1$ **to** K **do**
    $q_{rand} \leftarrow$ RANDOM_CONFIG();
    if not (EXTEND($\mathcal{T}_a, q_{rand}$) $=$ *Trapped*) **then**
        if (CONNECT($\mathcal{T}_b, q_{new}$) $=$ *Reached*) **then**
            Return PATH($\mathcal{T}_a, \mathcal{T}_b$);
    SWAP($\mathcal{T}_a, \mathcal{T}_b$);
Return *Failure*

search succeeds if trees connect



EXTEND($\mathcal{T}, q$)

$q_{near} \leftarrow$ NEAREST_NEIGHBOR($q, \mathcal{T}$);
if NEW_CONFIG(q, q_{near}, q_{new}) **then**
    $\mathcal{T}$.add_vertex(q_{new});
    $\mathcal{T}$.add_edge(q_{near}, q_{new});
    if $q_{new} = q$ **then**
        Return *Reached*;
    else
        Return *Advanced*;
Return *Trapped*;

1) Extend tree A towards a random configuration

CONNECT($\mathcal{T}, q$)

repeat
    $S \leftarrow$ EXTEND($\mathcal{T}, q$);
until not ($S =$ *Advanced*)
Return S;

2) Try to connect tree B to tree A by extending repeatedly from its nearest neighbor

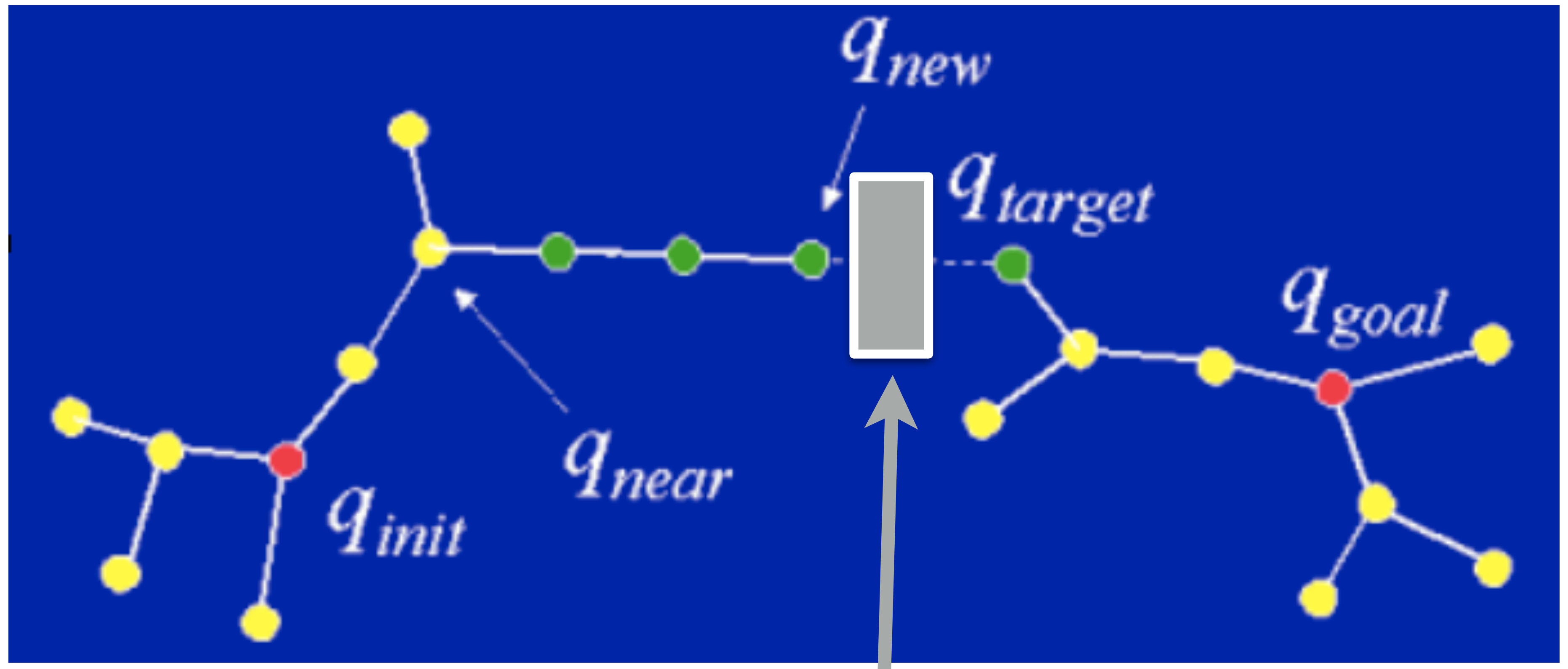
RRT Connect

[Kuffner, LaValle 2000]

0) Use 2 trees (A and B) rooted at start and goal configurations

```
RRT_CONNECT_PLANNER(q_init, q_goal)
  T_a.init(q_init); T_b.init(q_goal);
  for k = 1 to K do
      q_rand ← RANDOM_CONFIG();
      if not (EXTEND(T_a, q_rand) = Trapped) then
          if (CONNECT(T_b, q_new) = Reached) then
              Return PATH(T_a, T_b);
      SWAP(T_a, T_b);
  Return Failure
```

3) reverse roles for trees A and B and repeat



collision encountered

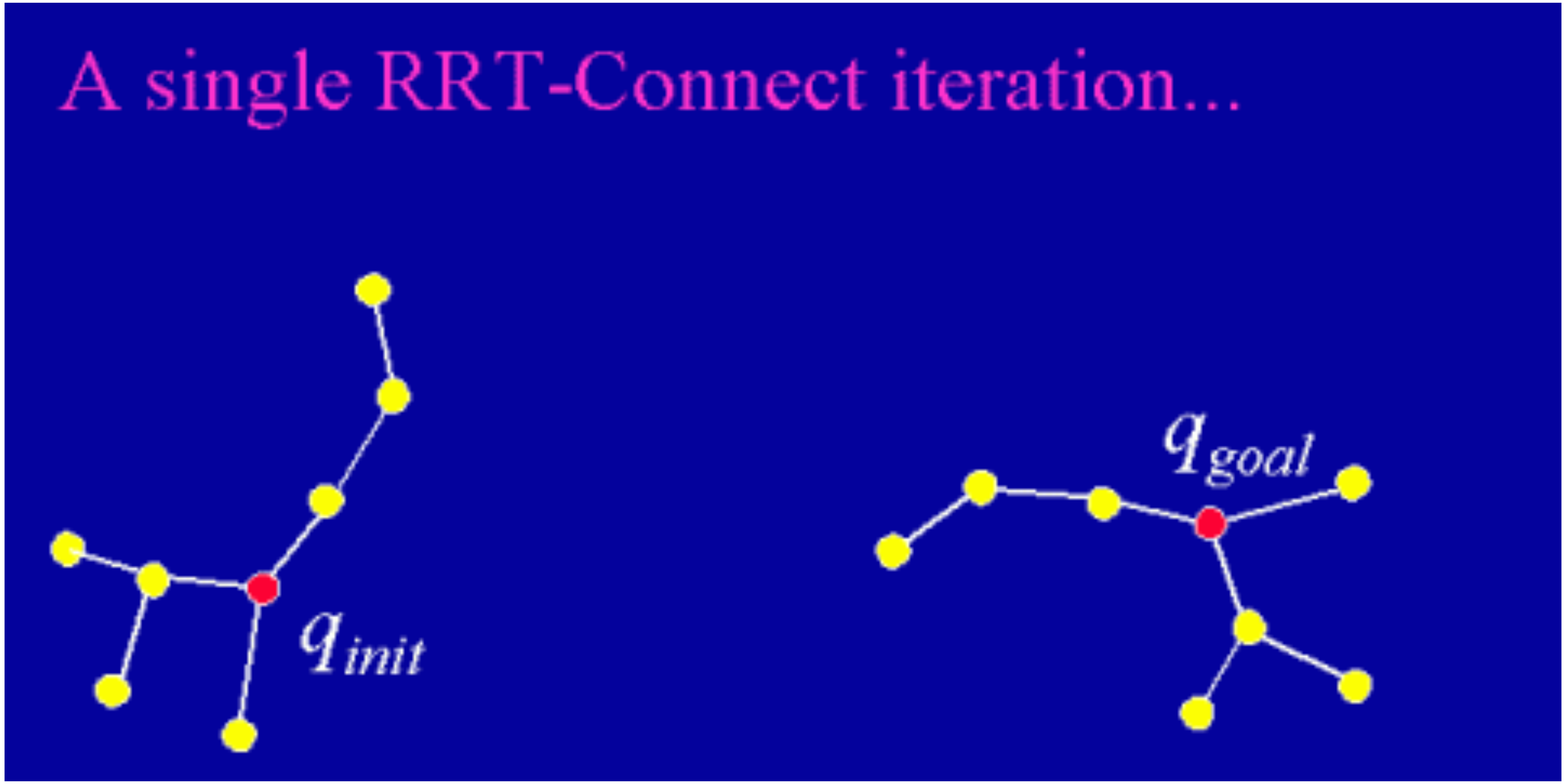
```
EXTEND(T, q)
  q_near ← NEAREST_NEIGHBOR(q, T);
  if NEW_CONFIG(q, q_near, q_new) then
      T.add_vertex(q_new);
      T.add_edge(q_near, q_new);
      if q_new = q then
          Return Reached;
      else
          Return Advanced;
  Return Trapped;
```

1) Extend tree A towards a random configuration

```
CONNECT(T, q)
  repeat
      S ← EXTEND(T, q);
  until not (S = Advanced)
  Return S;
```

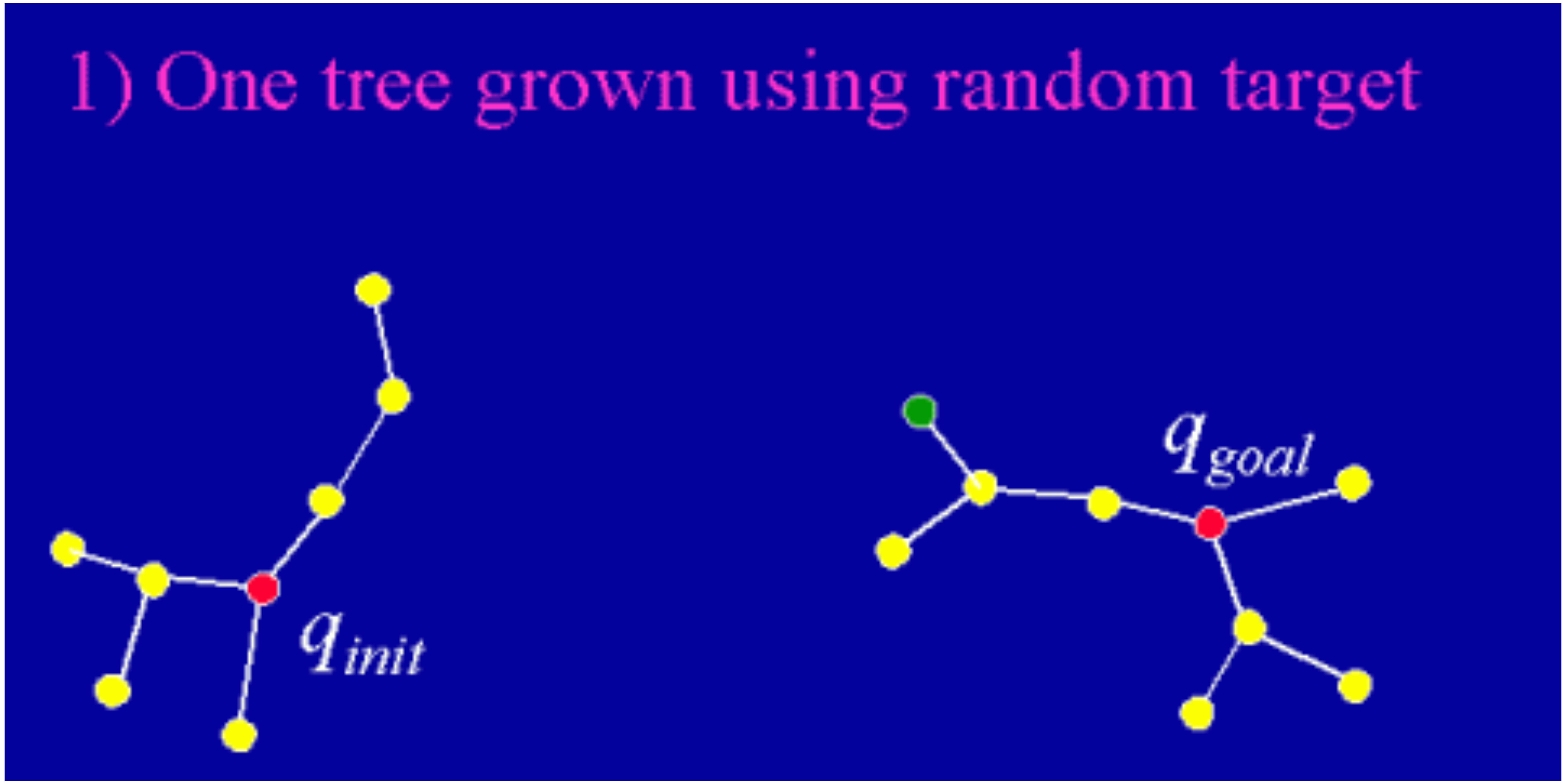
2) Try to connect tree B to tree A by extending repeatedly from its nearest neighbor

RRT-Connect animation



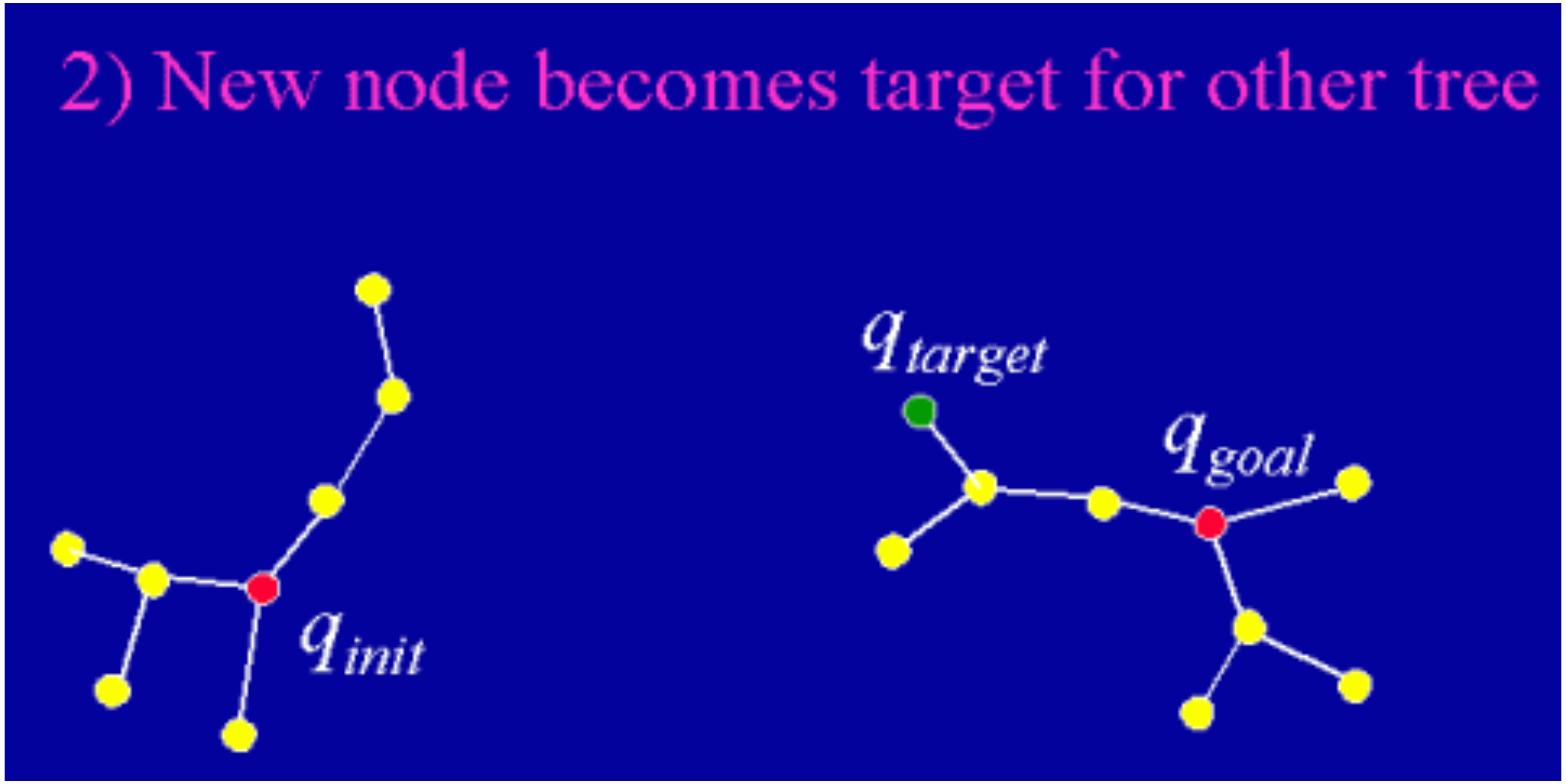
http://www.kuffner.org/james/plan/algorithm.php

RRT-Connect animation



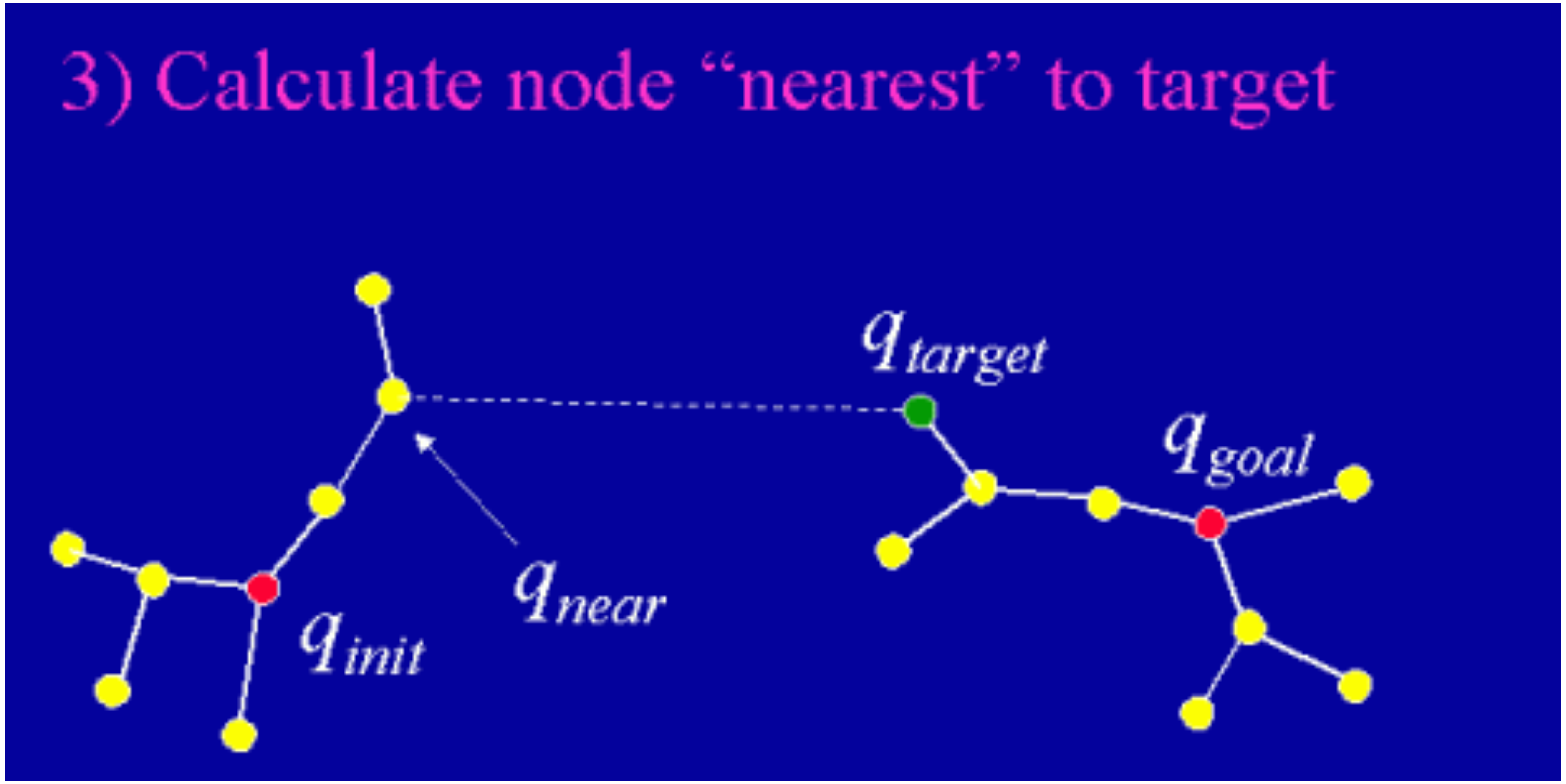
http://www.kuffner.org/james/plan/algorithm.php

RRT-Connect animation



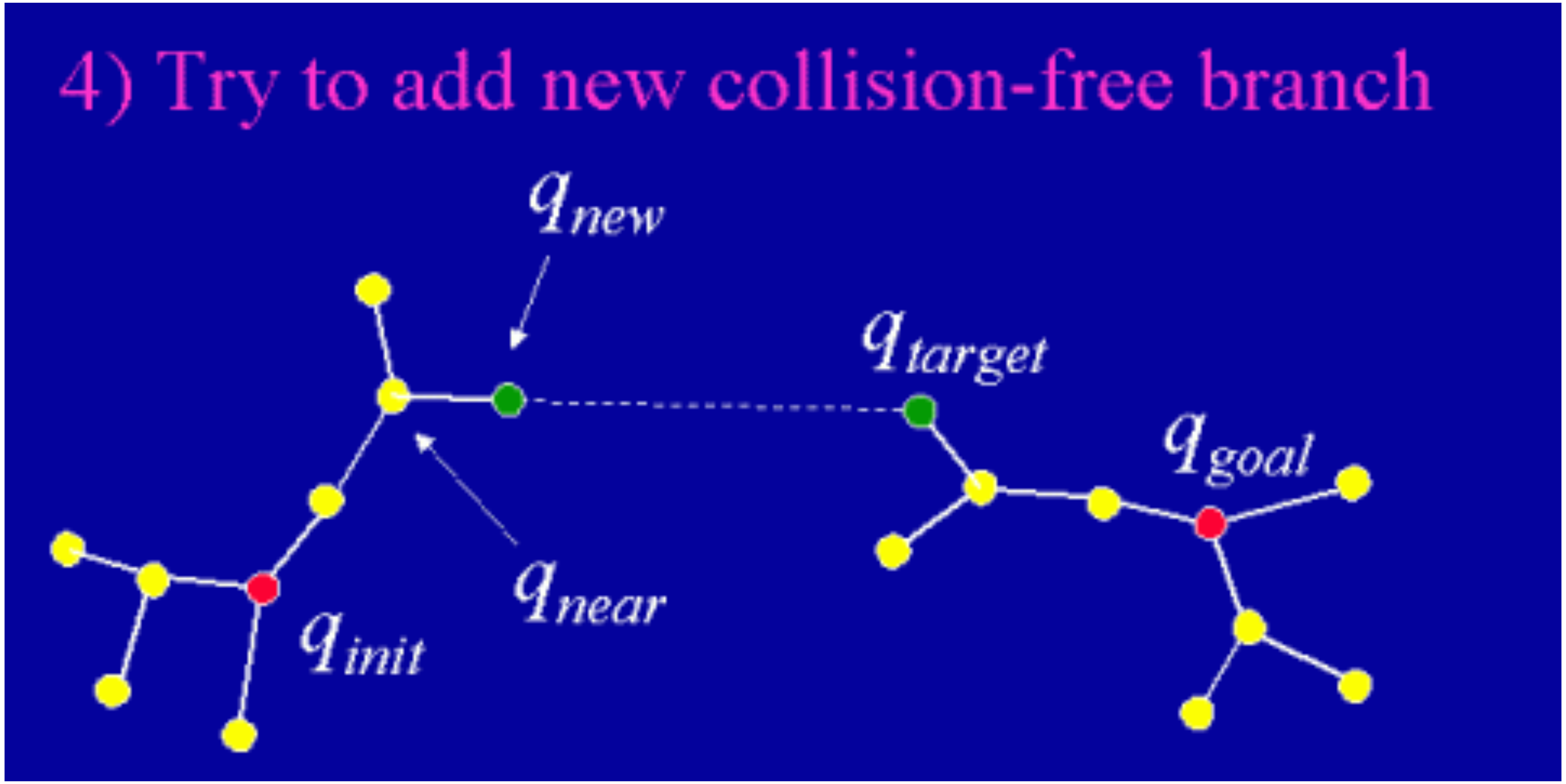
http://www.kuffner.org/james/plan/algorithm.php

RRT-Connect animation



http://www.kuffner.org/james/plan/algorithm.php

RRT-Connect animation



http://www.kuffner.org/james/plan/algorithm.php

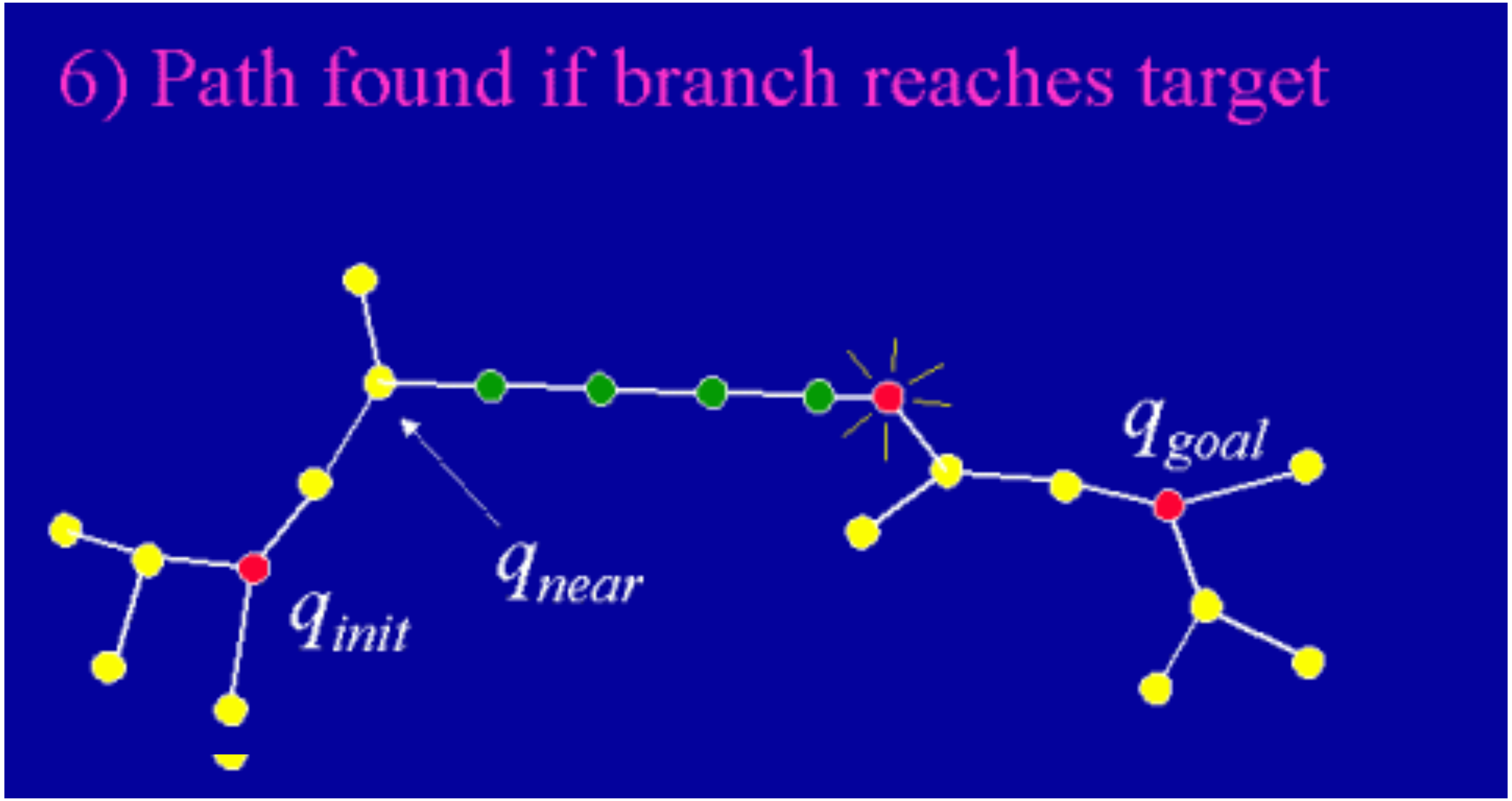
RRT-Connect animation

http://www.kuffner.org/james/plan/algorithm.php

RRT-Connect animation

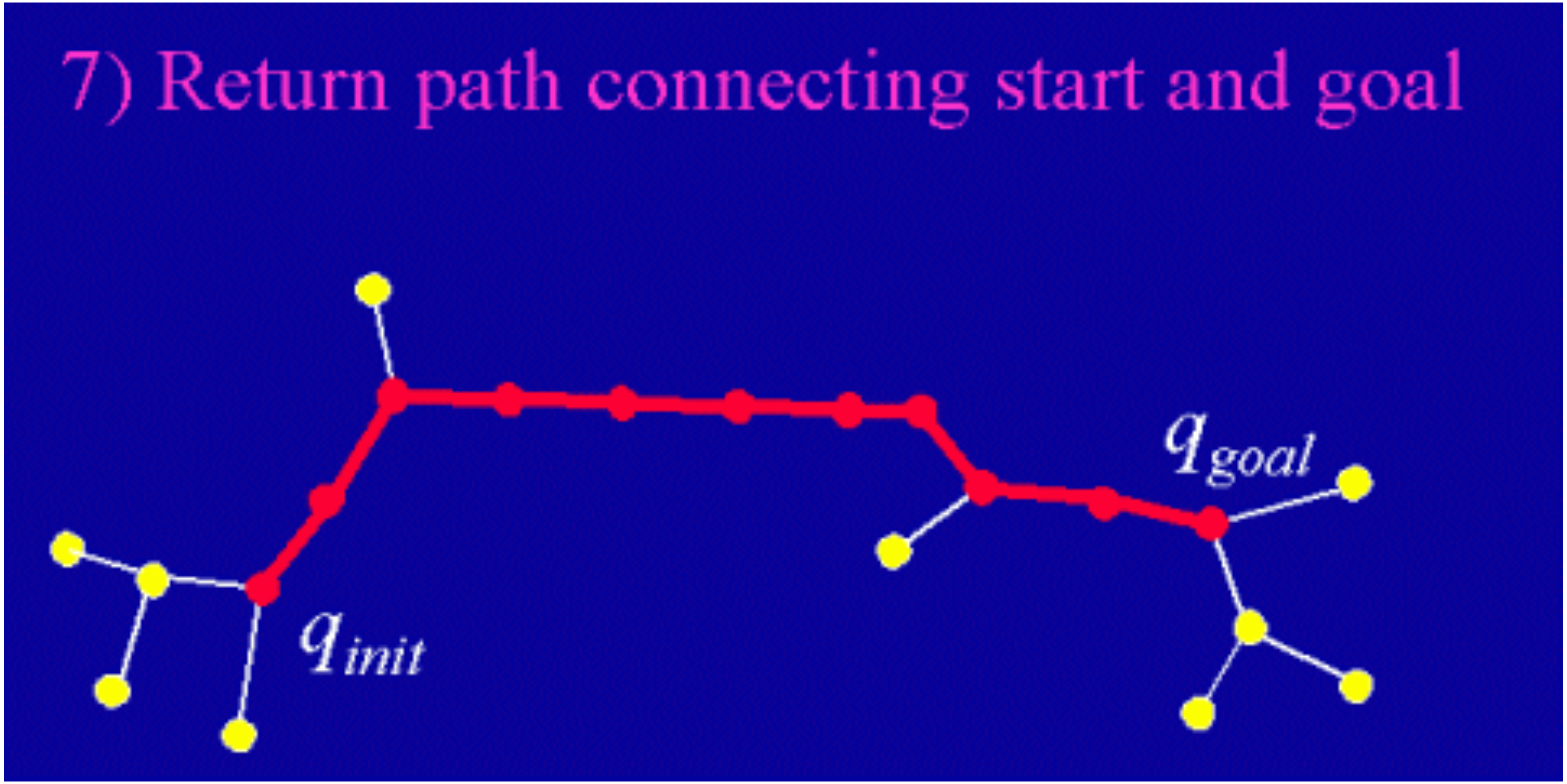
http://www.kuffner.org/james/plan/algorithm.php

RRT-Connect animation



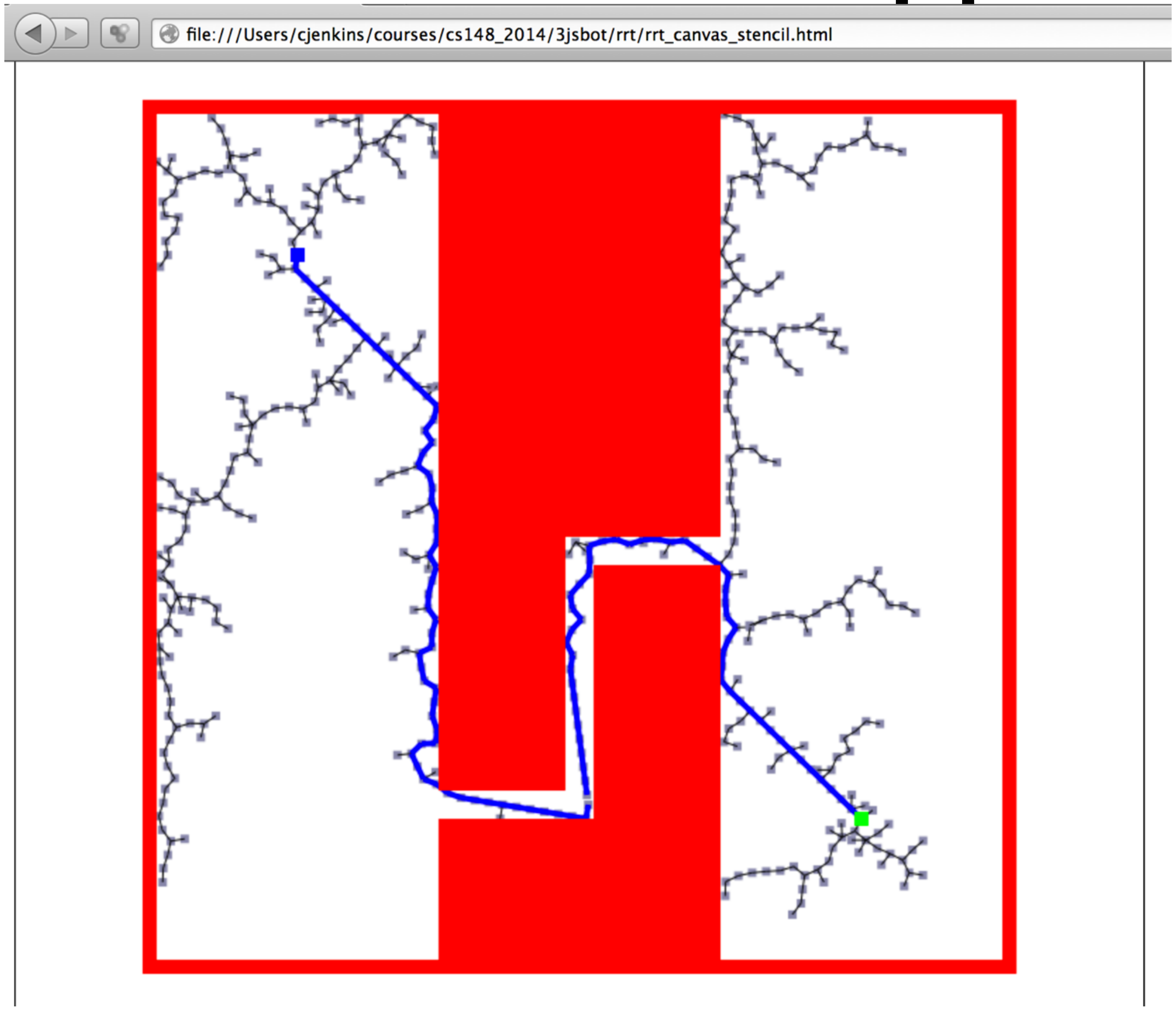
http://www.kuffner.org/james/plan/algorithm.php

RRT-Connect animation



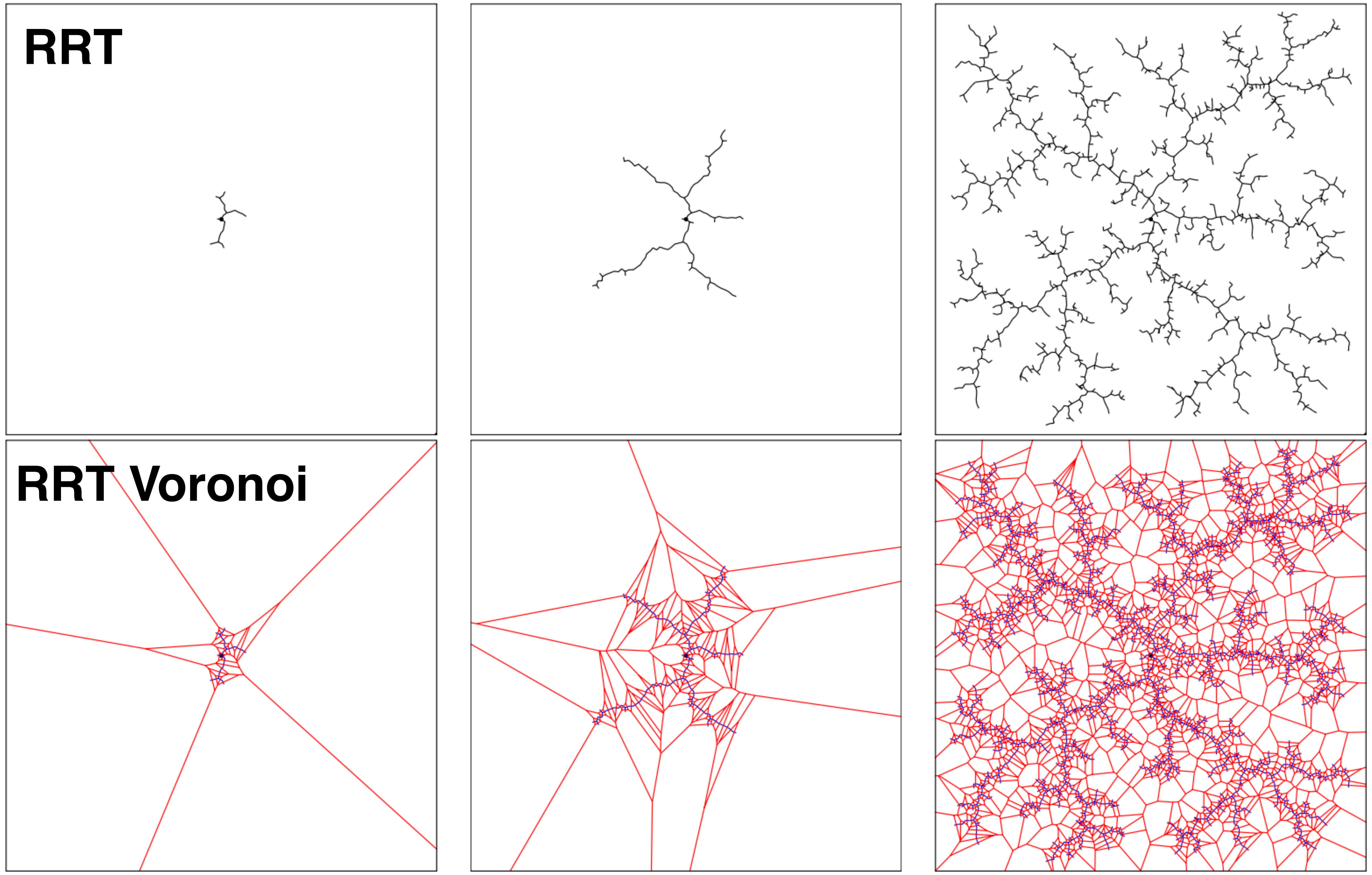
http://www.kuffner.org/james/plan/algorithm.php

Let’s see what happens

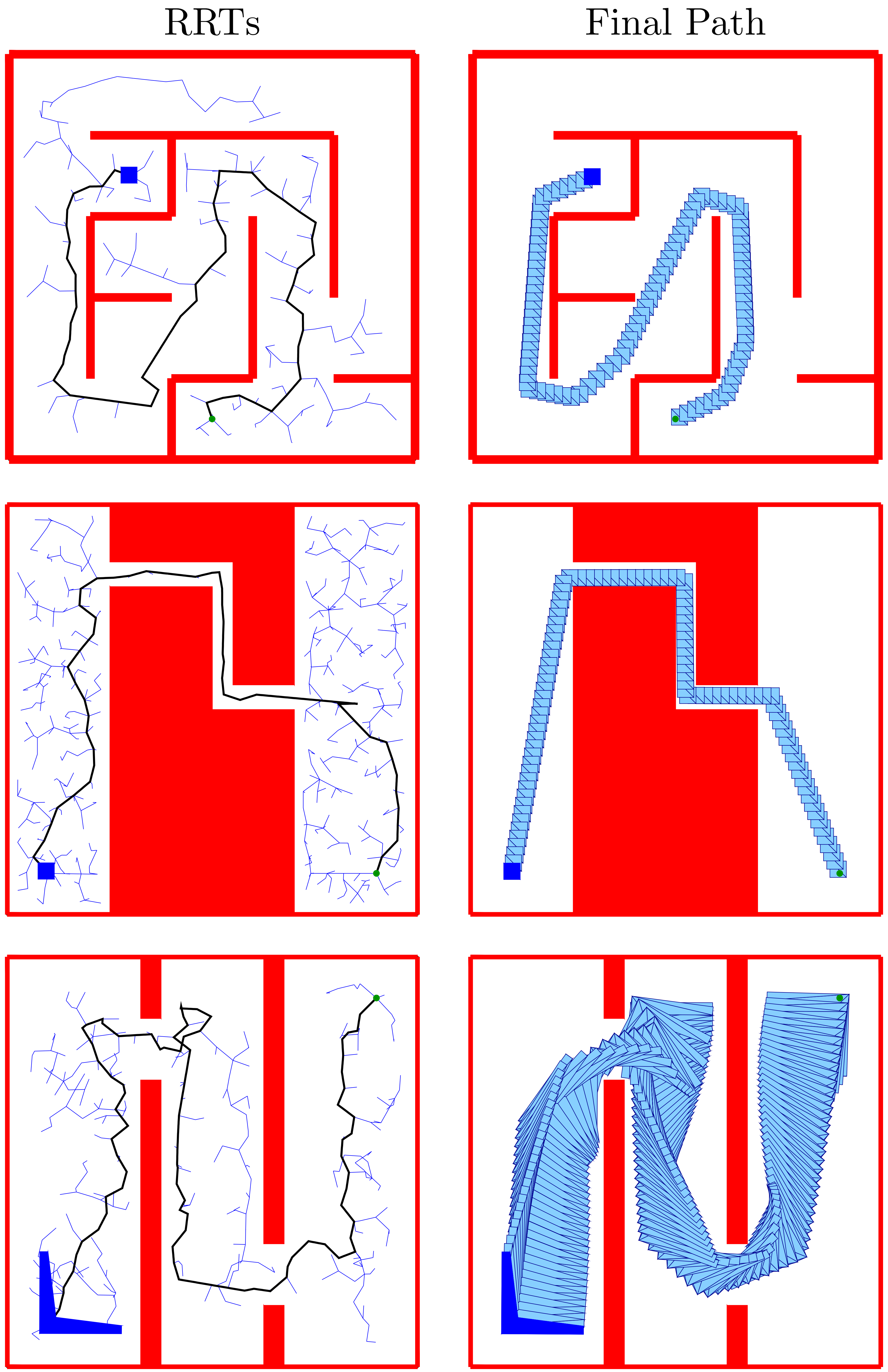


RRT Probabilistic Completeness

- Probability a vertex is selected for extension is proportional to its area in Voronoi diagram
- RRTs converge to a uniform coverage of C-space as the number of samples increases



Examples of RRT-Connect

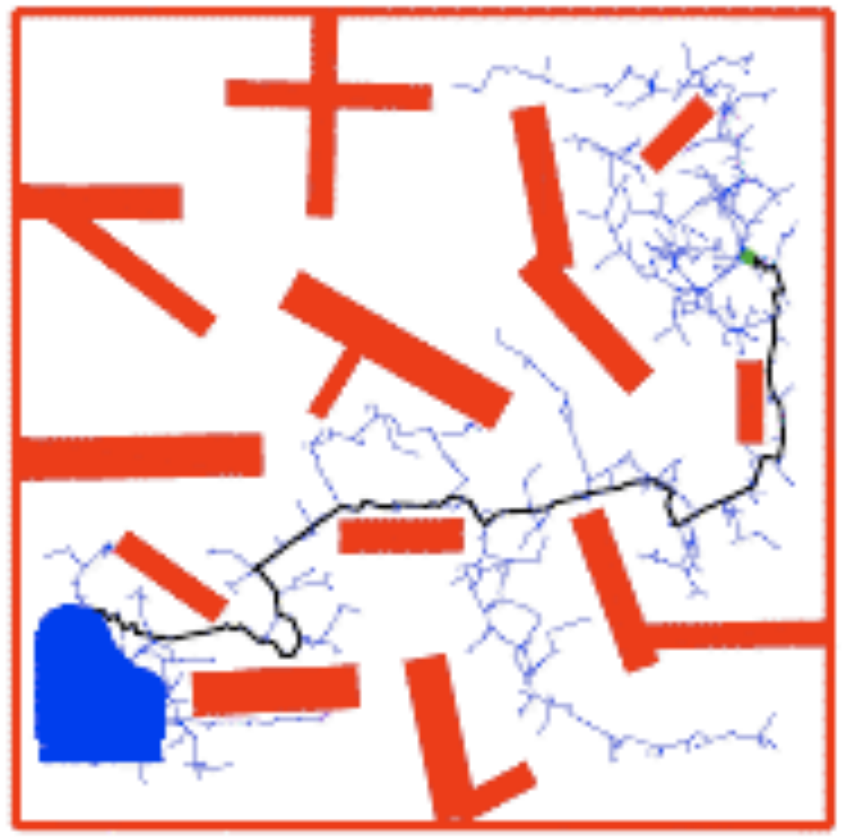


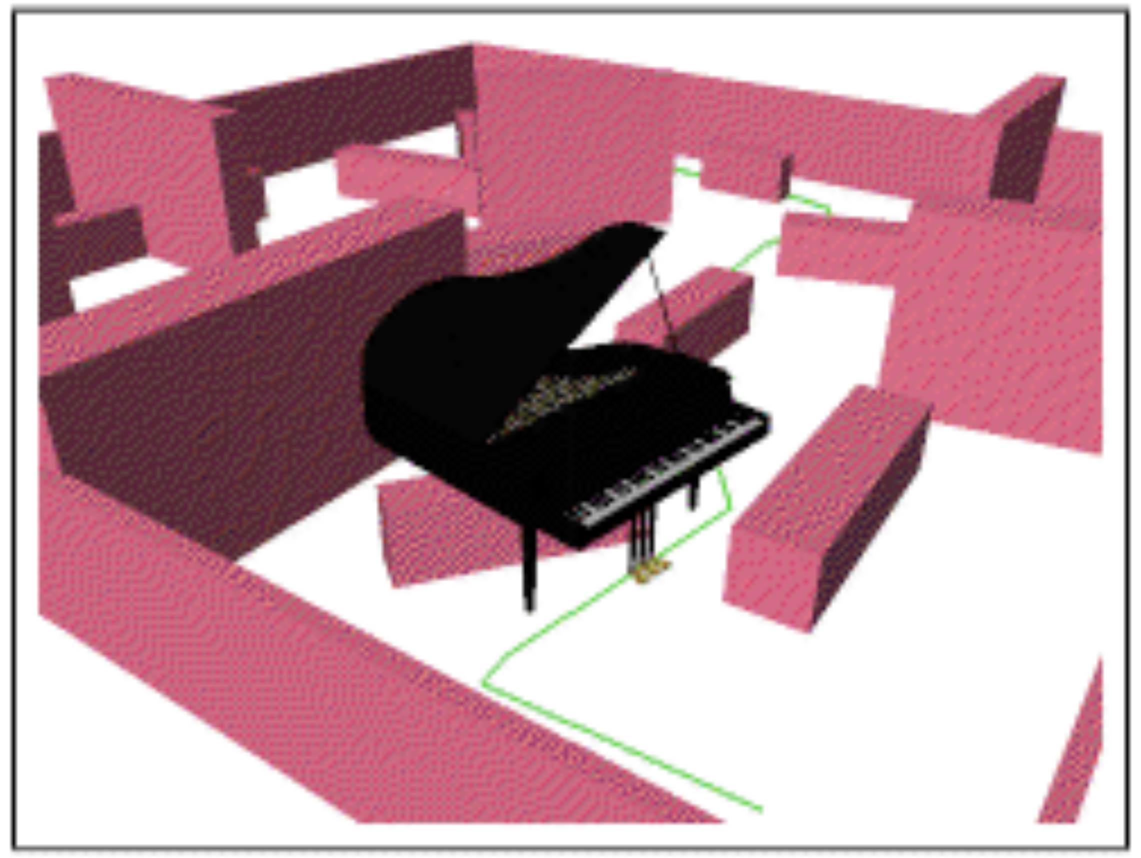
2 DOF maze

2 DOF single passway

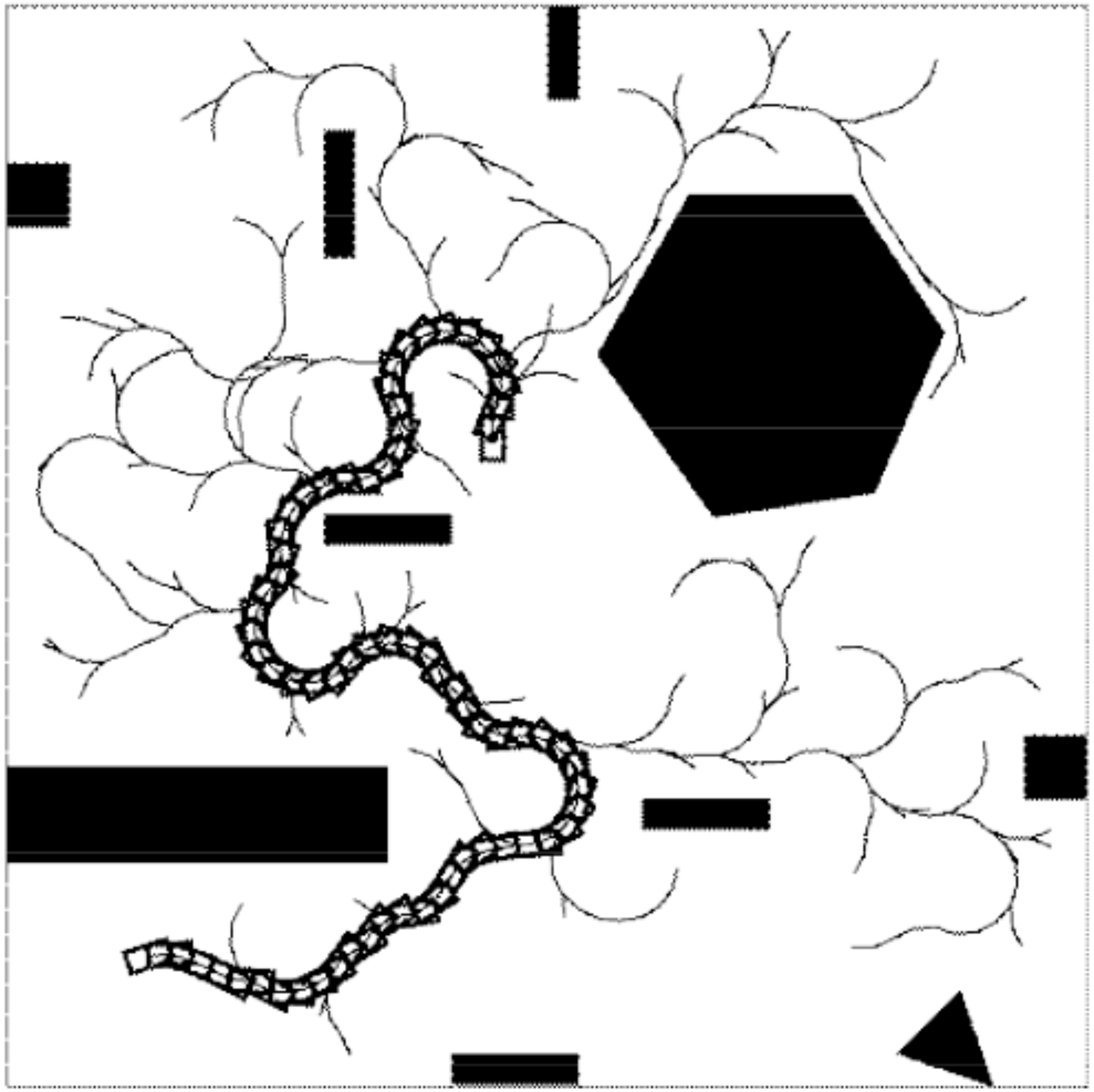
3 DOF single passway
(with non-point geometry)

Piano Mover's Problem

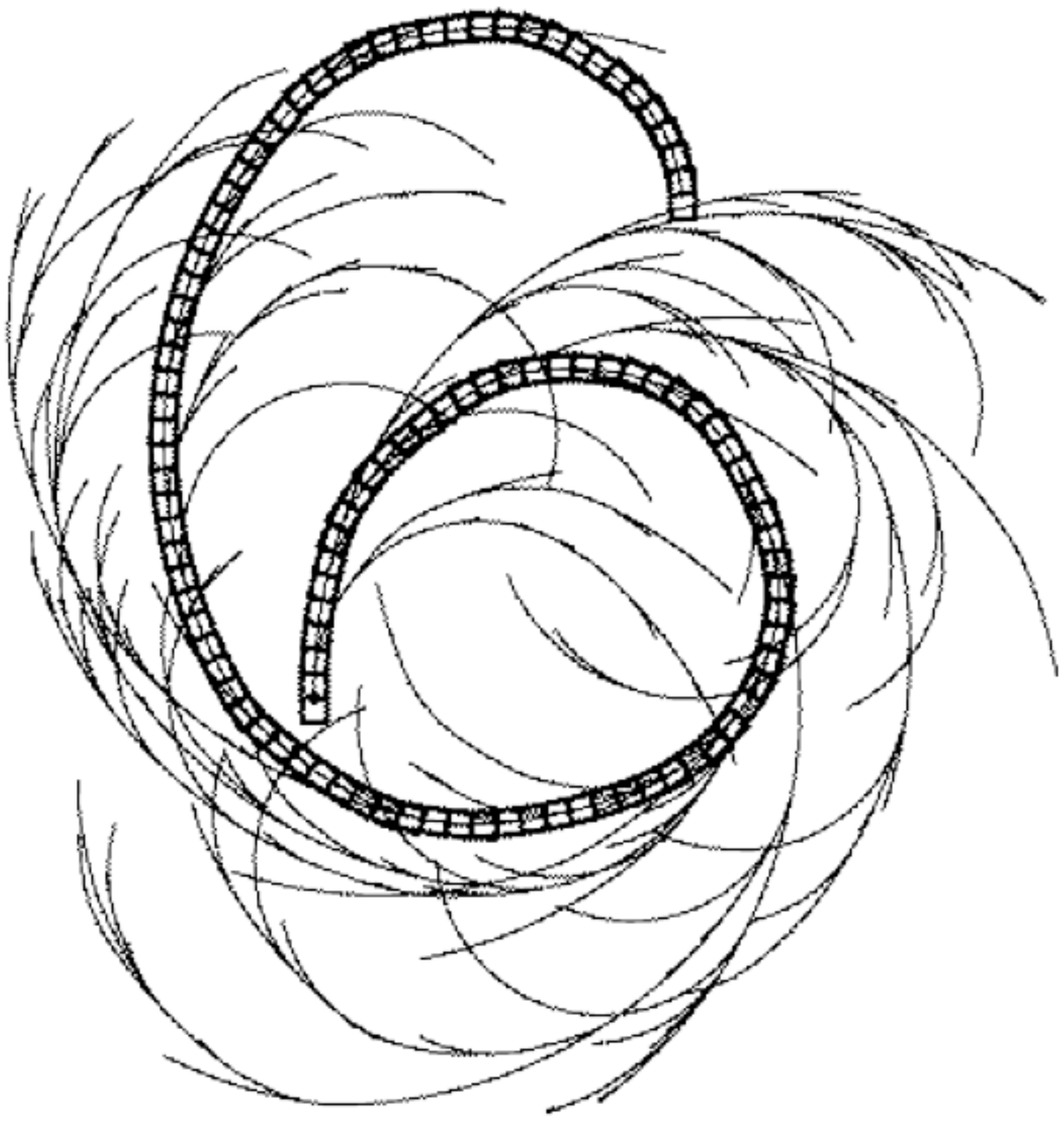




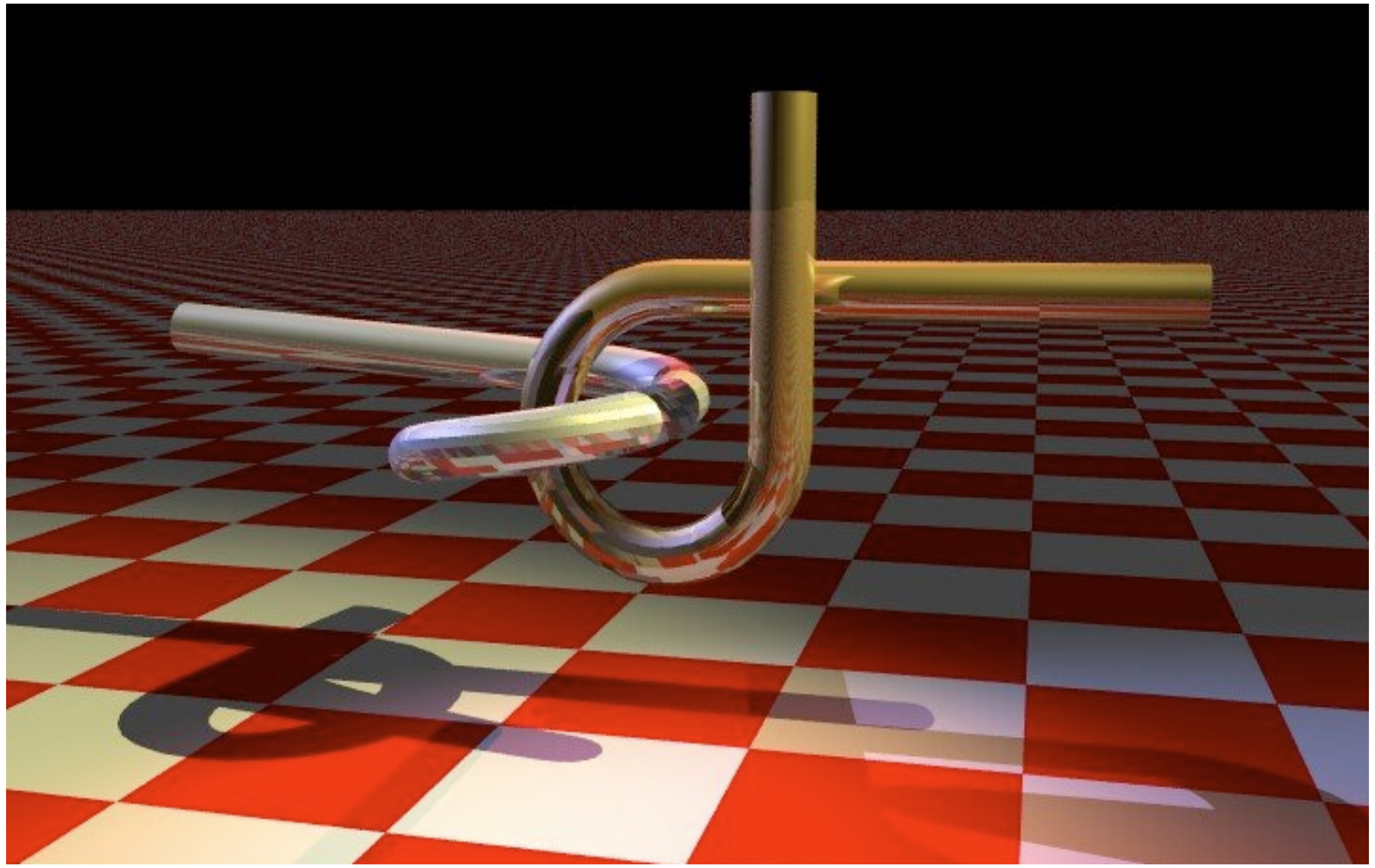
A Car-Like Robot

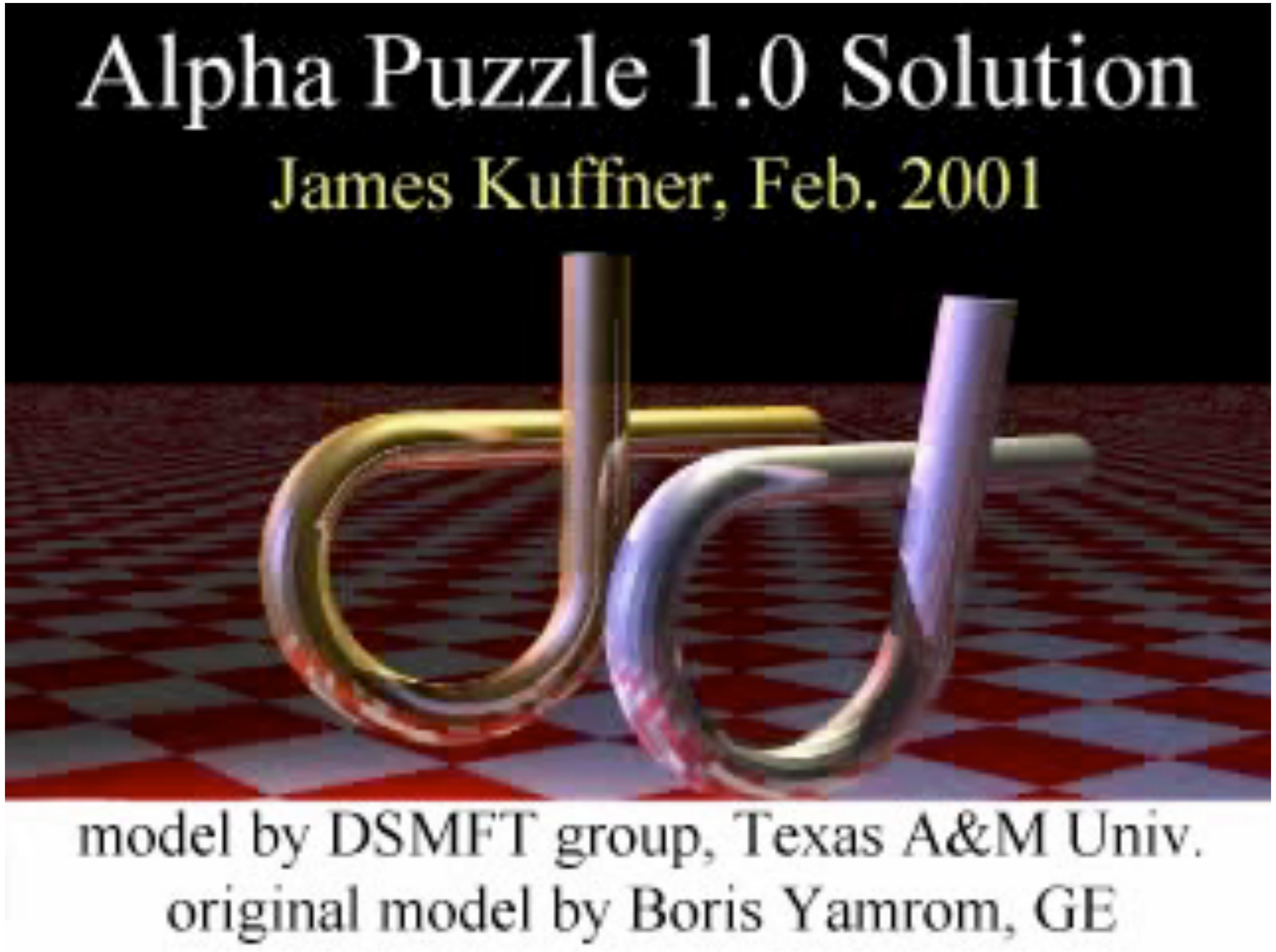


A Right-Turn Only Car

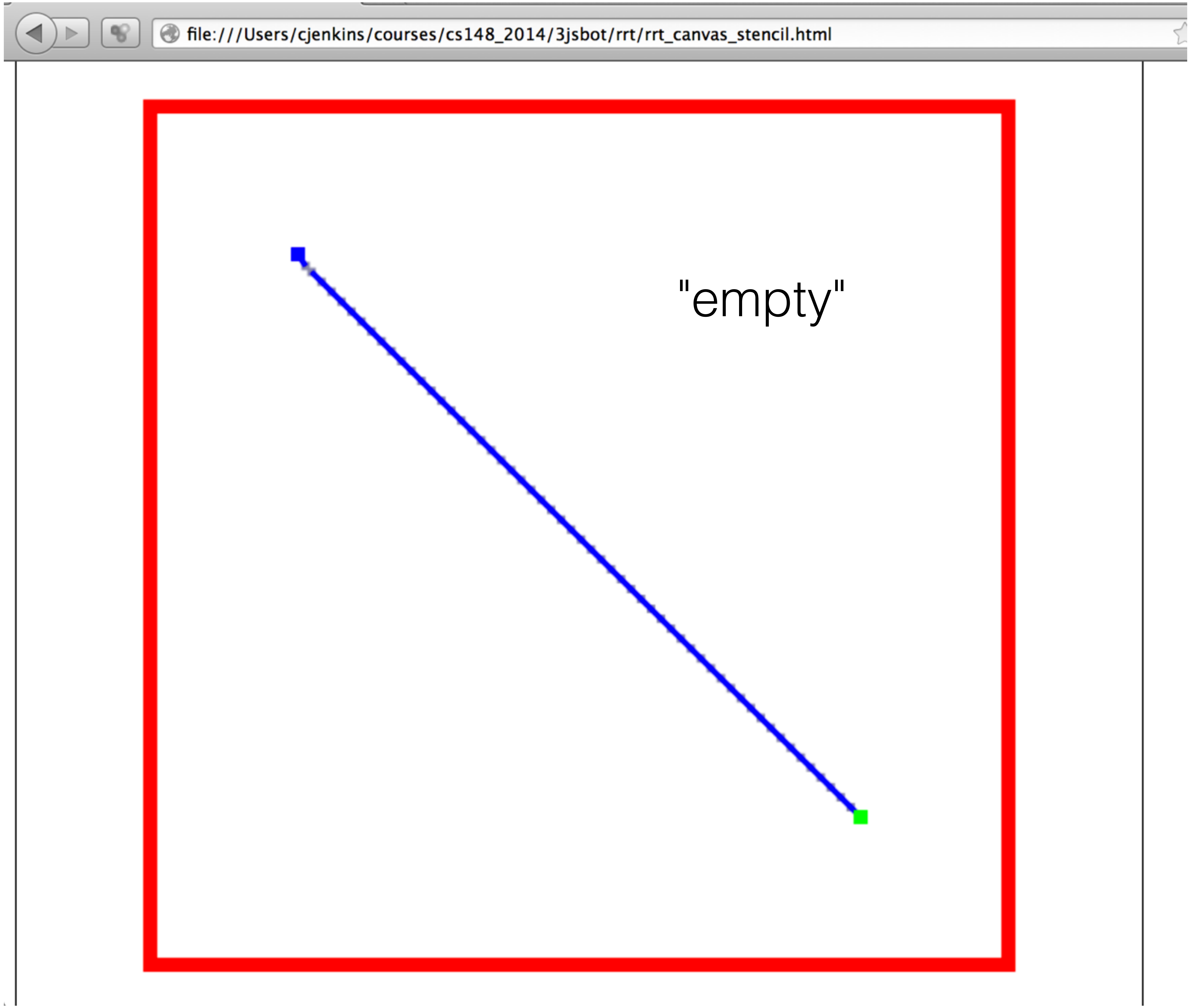


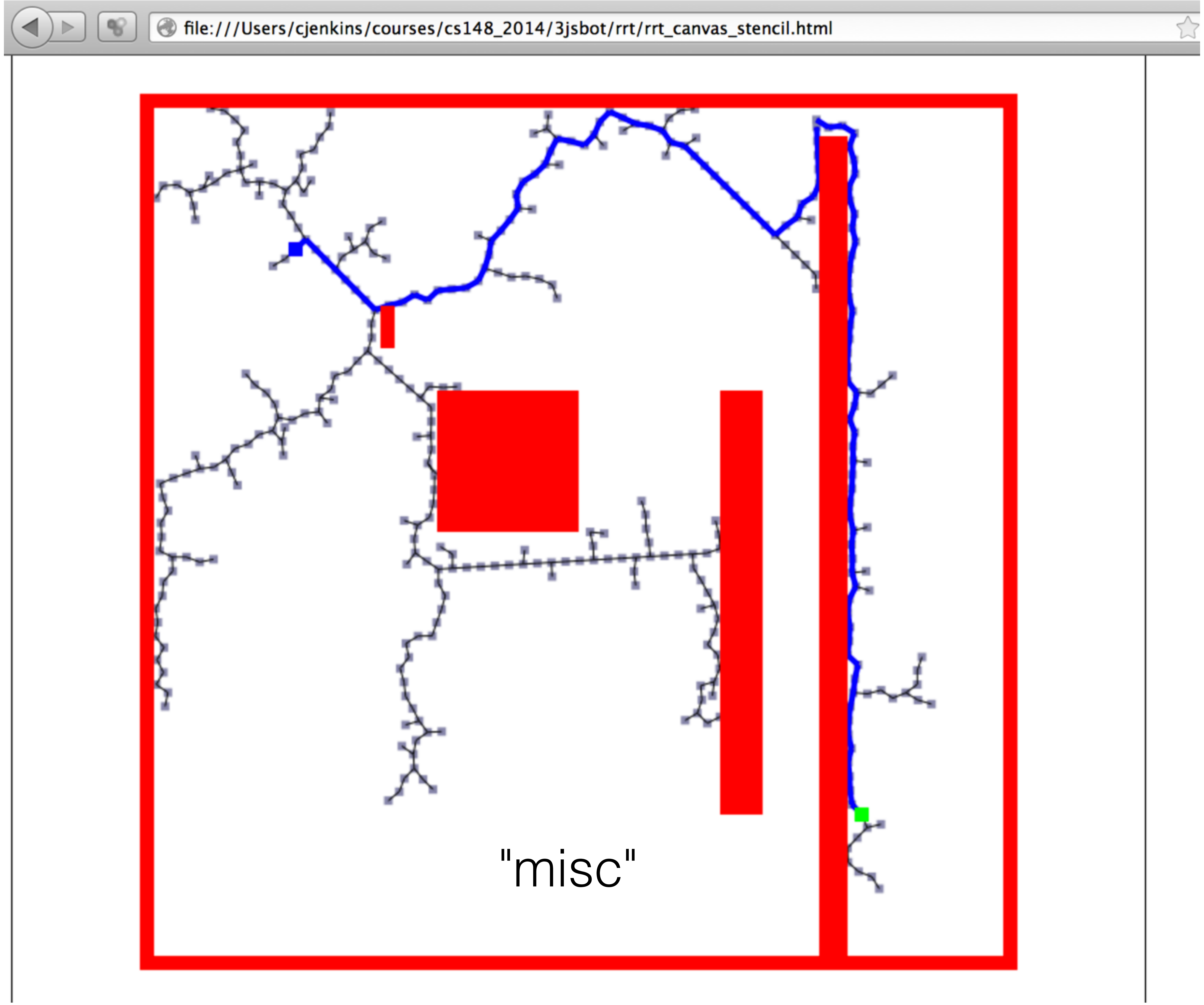
Alpha Puzzle

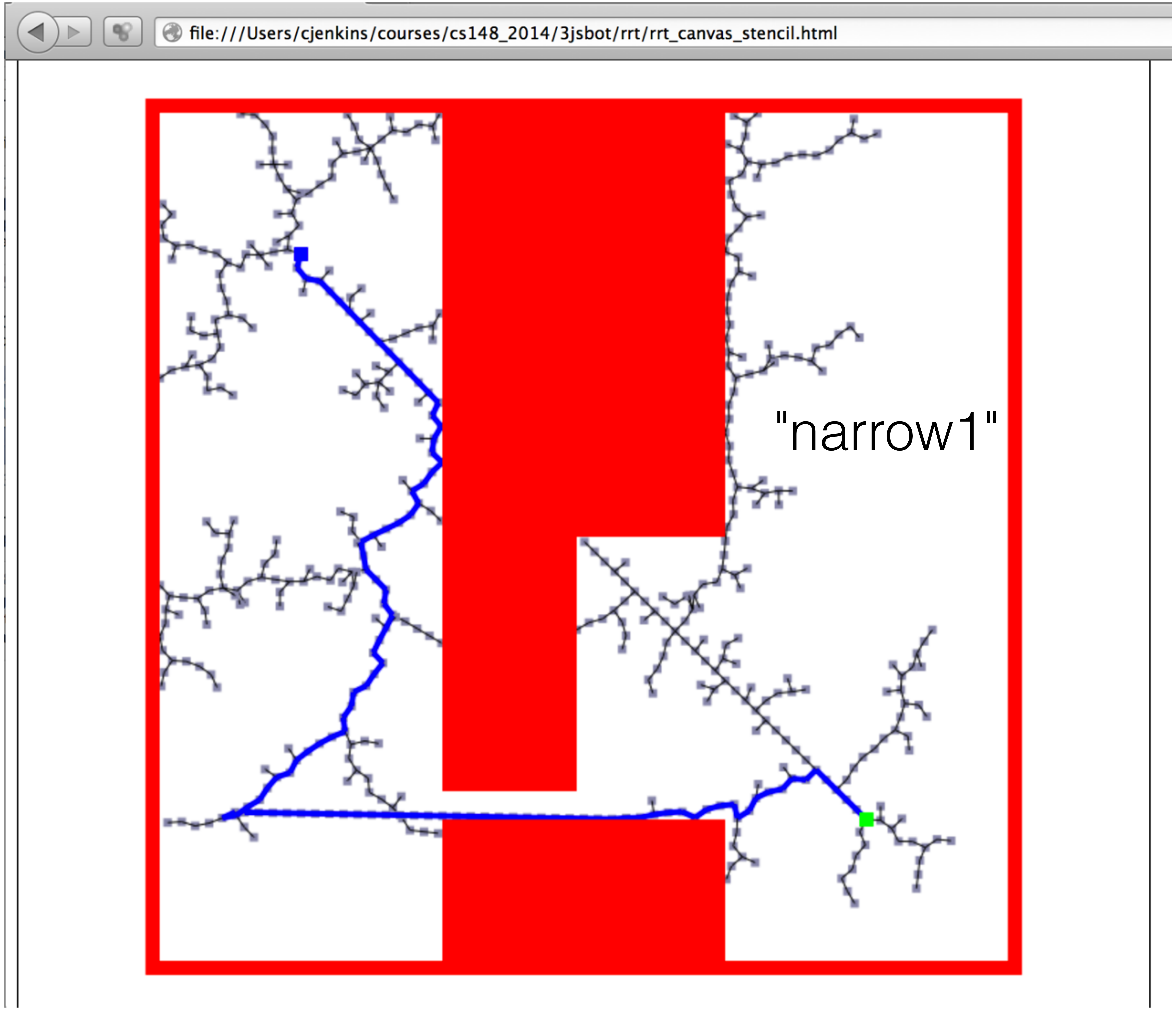


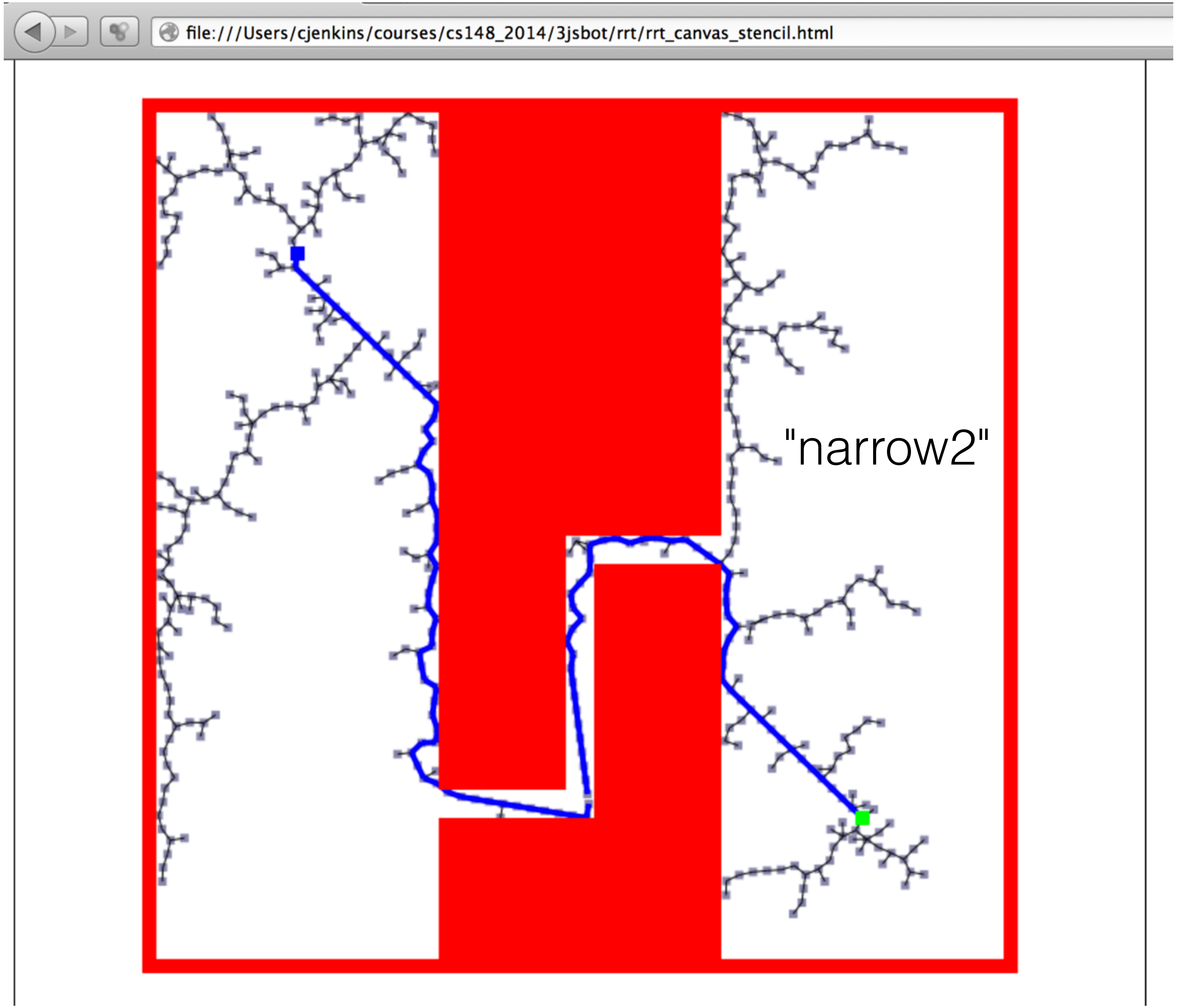


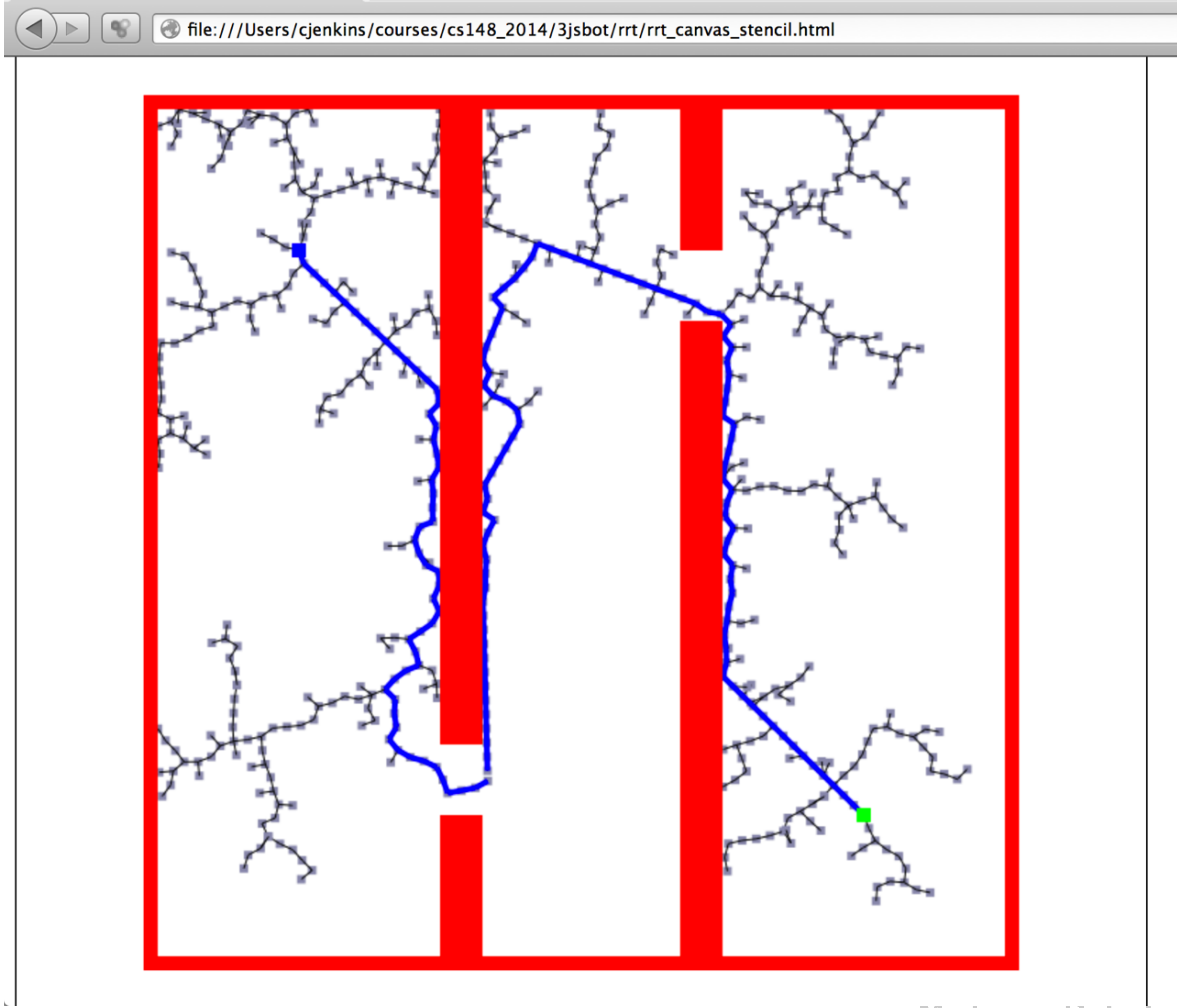
Canvas Stencil Examples


file:///Users/cjenkins/courses/cs148_2014/3jsbot/rrt/rrt_canvas_stencil.html
"empty"


file:///Users/cjenkins/courses/cs148_2014/3jsbot/rrt/rrt_canvas_stencil.html
"misc"

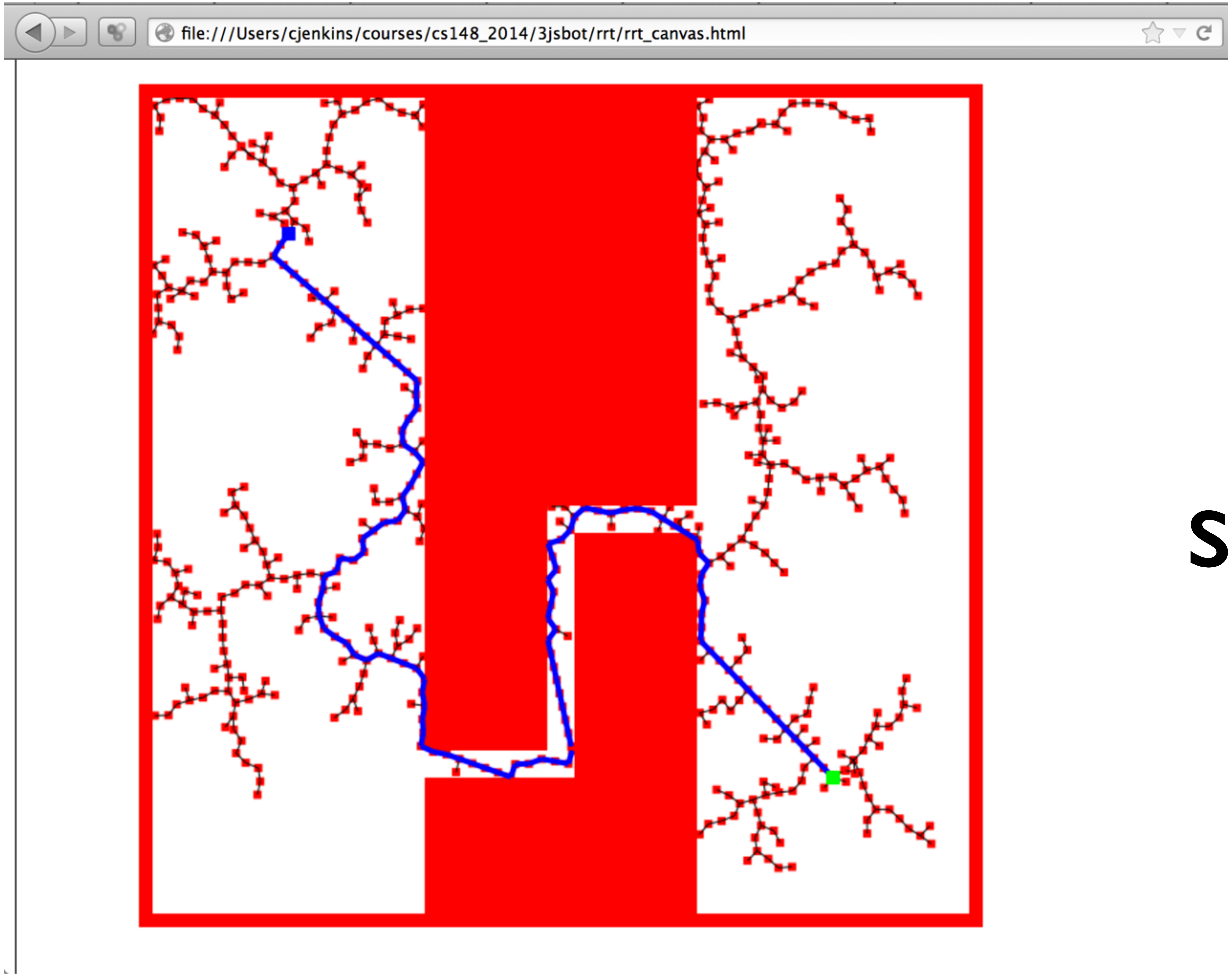

file:///Users/cjenkins/courses/cs148_2014/3jsbot/rrt/rrt_canvas_stencil.html
"narrow1"


file:///Users/cjenkins/courses/cs148_2014/3jsbot/rrt/rrt_canvas_stencil.html
"narrow2"



"three_sections"

RRTs can take a lot of time...



Is there a simpler way?

Potential fields

follow your
potential