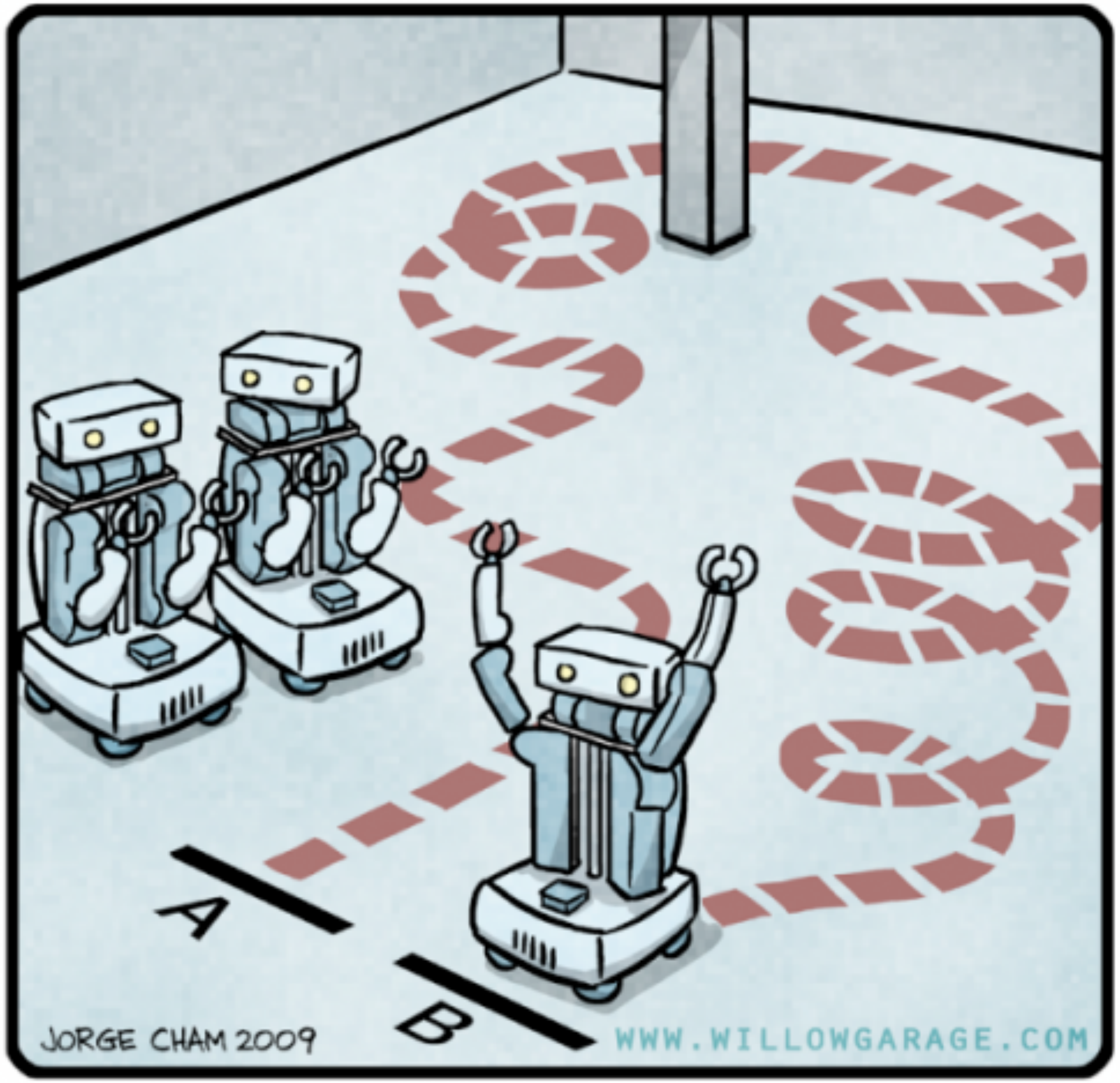
R.O.B.O.T. Comics



"HIS PATH-PLANNING MAY BE SUB-OPTIMAL, BUT IT'S GOT FLAIR."

Path planning

the best way to get from A to B

EECS 367
Intro. to Autonomous Robotics

ROB 320
Robot Operating Systems

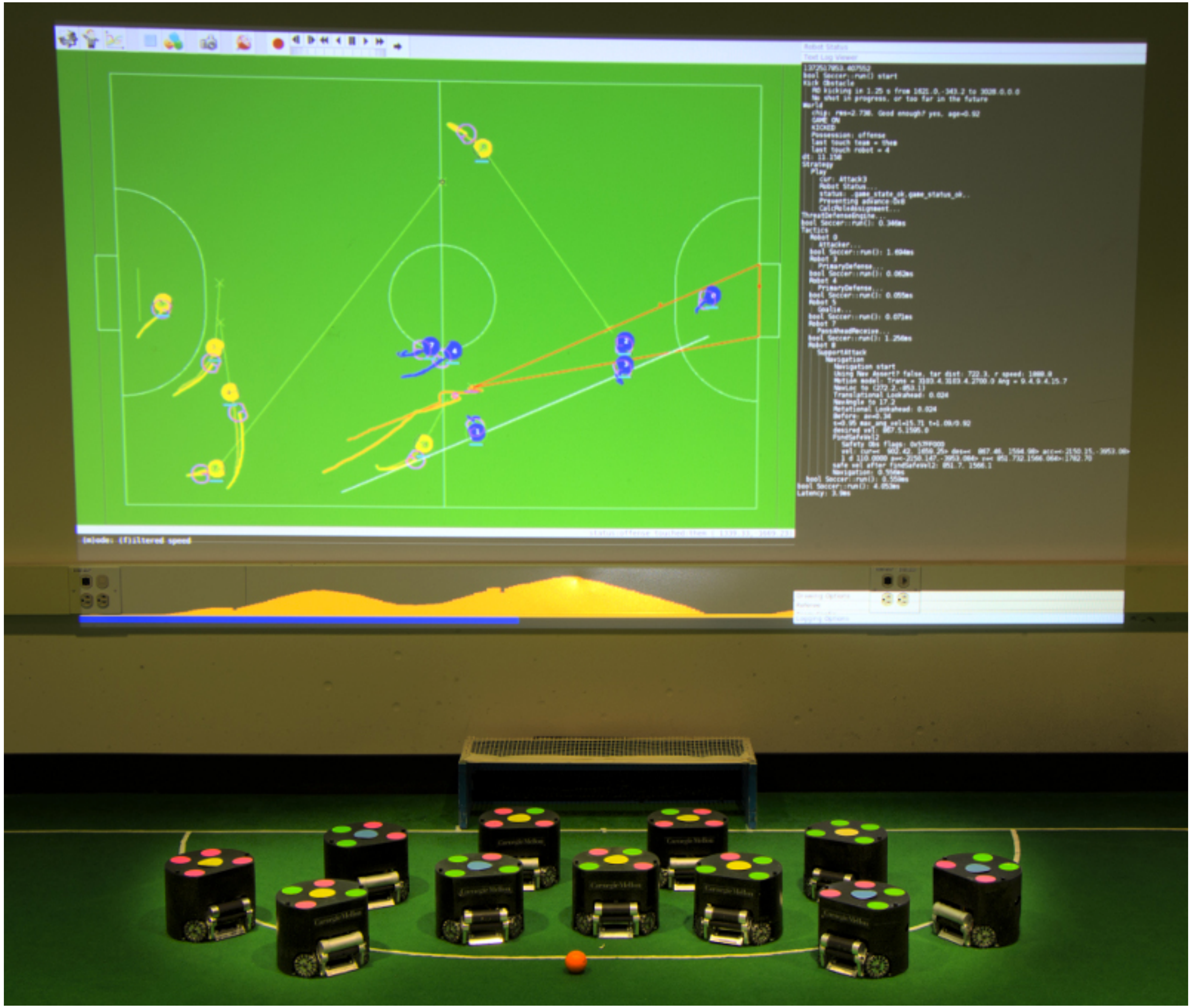
Winter 2022

CMDragons
RoboCup Small
2006

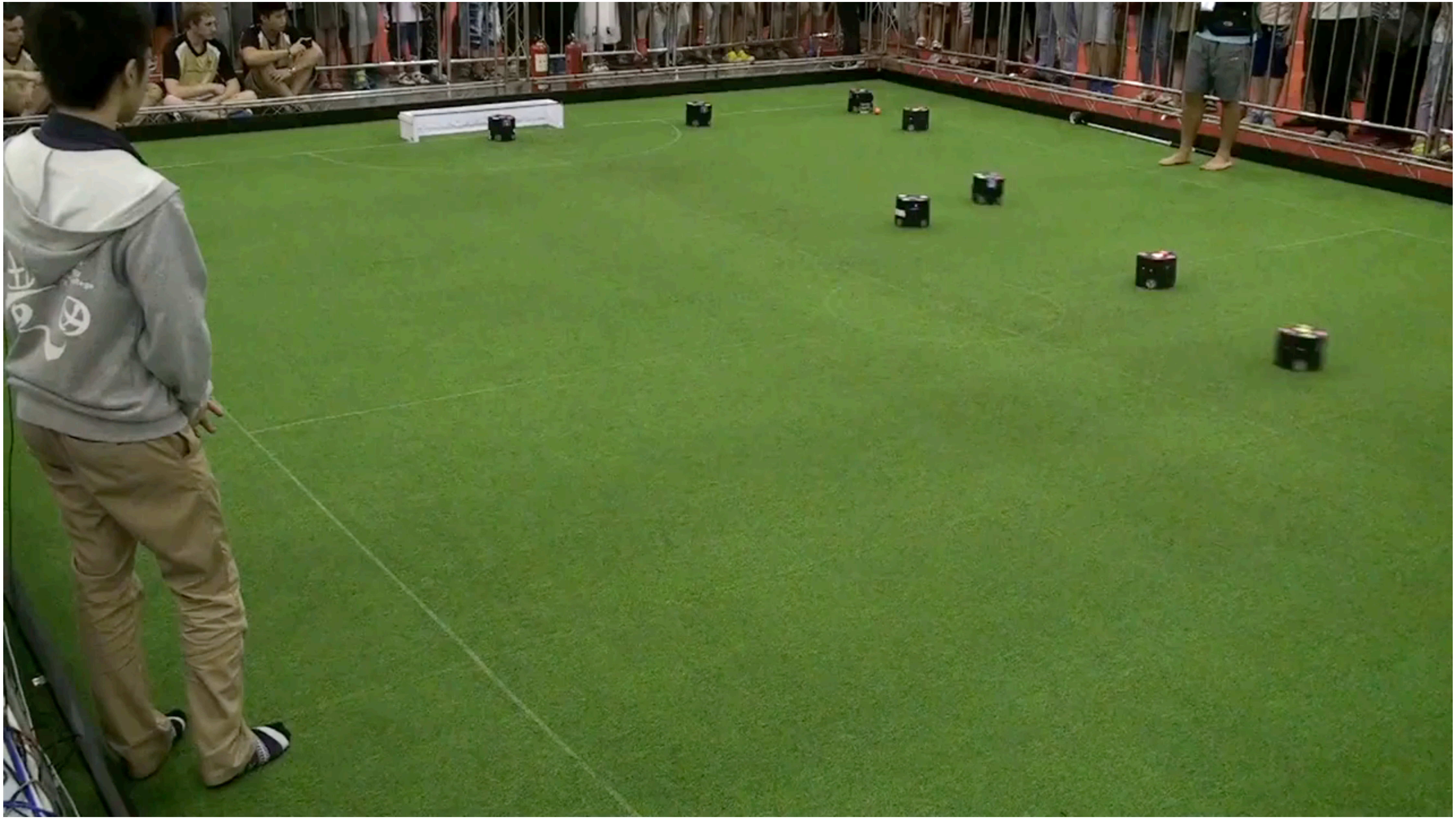
https://youtu.be/-Y4H3Sox_4I

CMDragons
RoboCup Small
2006

https://youtu.be/-Y4H3Sox_4I

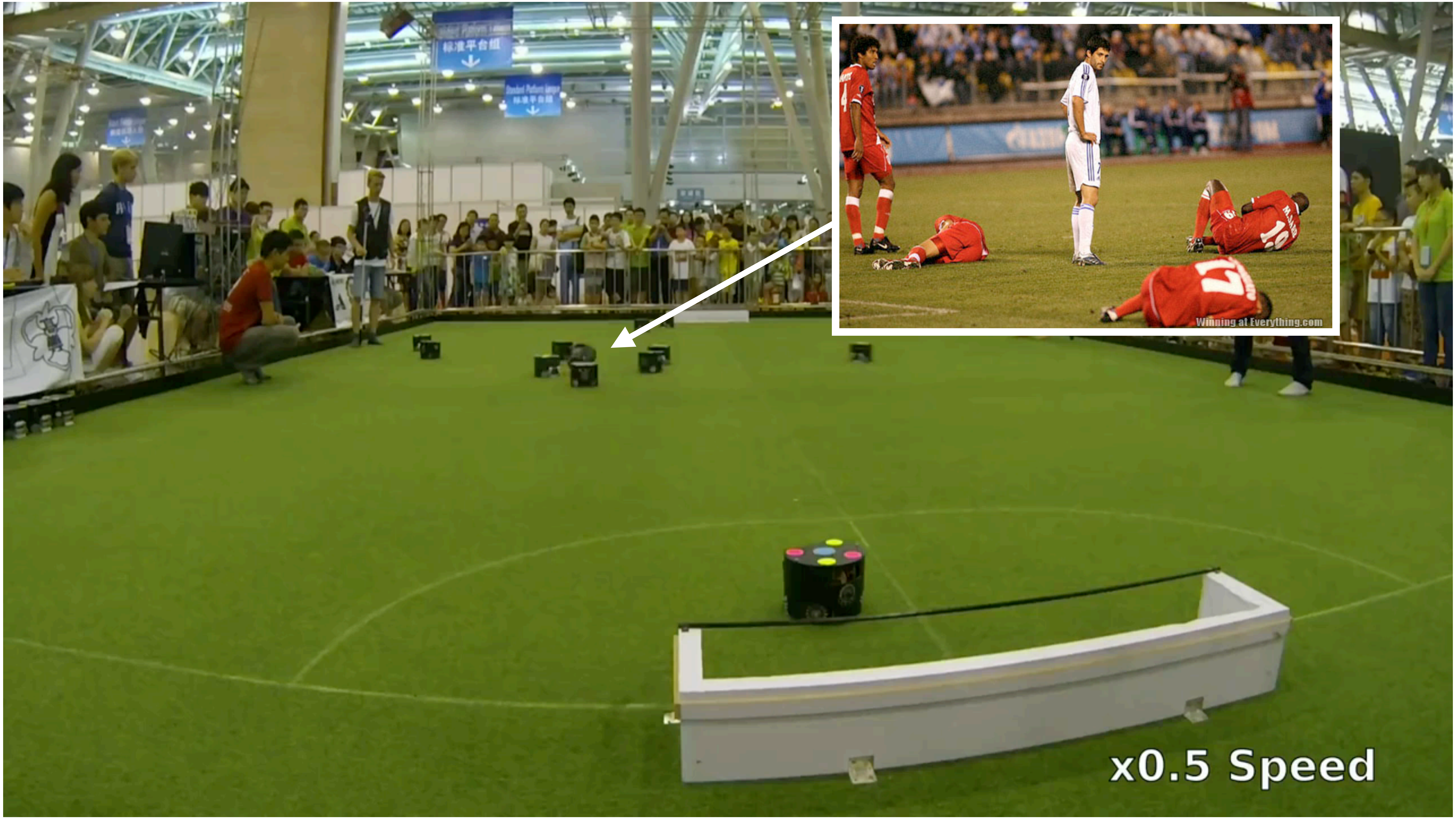


CMDragons - http://www.cs.cmu.edu/~robosoccer/small/



CMDragons 2015 Pass-ahead Goal

CMDragons 2015 slow-motion multi-pass goal



CMDragons 2015 slow-motion multi-pass goal

Manuela Veloso: RoboCup's Champion

This roboticist has transformed robot soccer into a global phenomenon

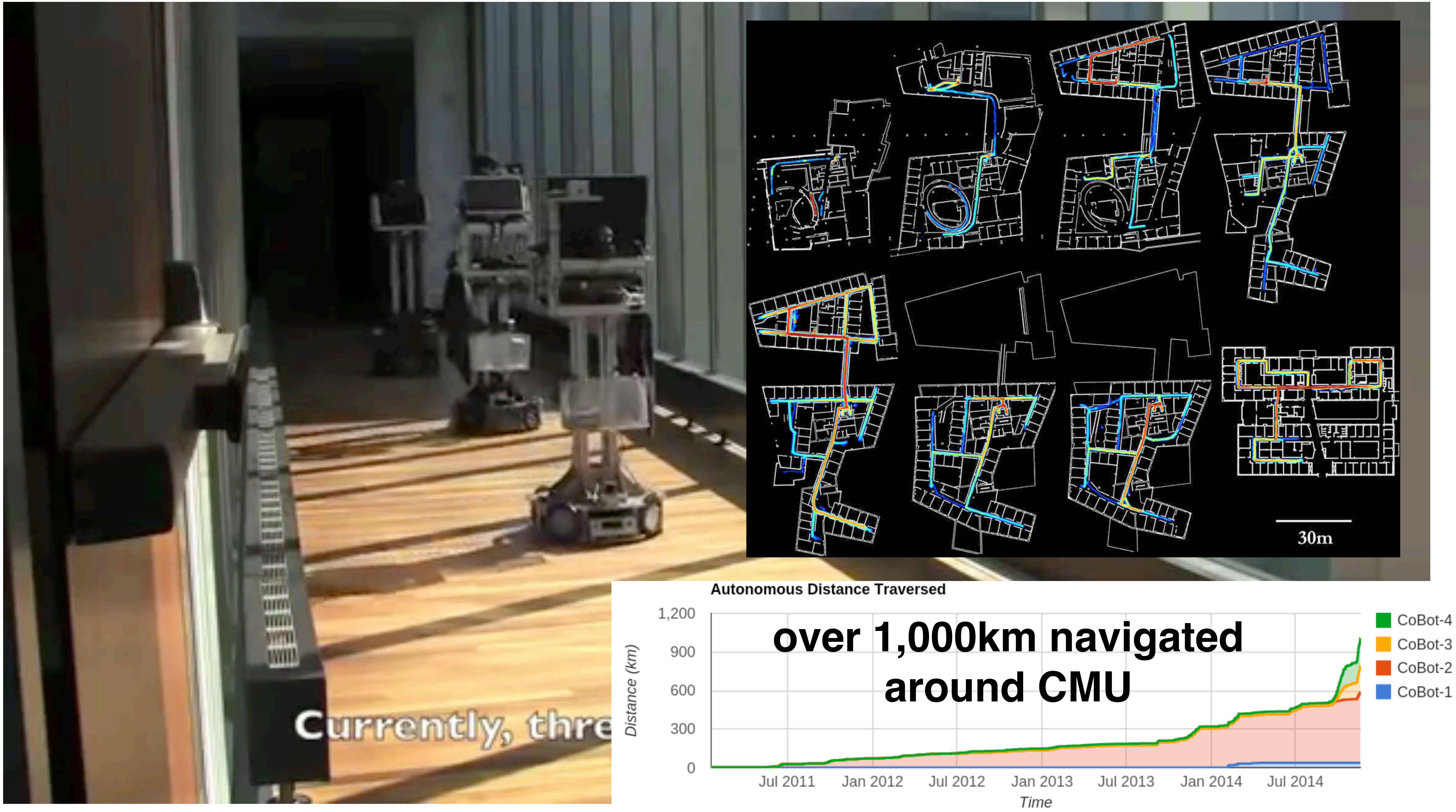
By **Prachi Patel**

Stepping out of the elevator on the seventh floor of Carnegie Mellon University's Gates Center for Computer Science, I'm greeted by an ungainly yet courteous robot. It guides me to the office of Manuela Veloso, who beams at the bot like a proud parent. Veloso then punches a few buttons to send it off to her laboratory a few corridors away.

Photo: Ross Mantle

Veloso, a computer science professor at CMU, in Pittsburgh, has worked for over two decades to develop such autonomous mobile robots. She believes that humans and robots will one day coexist, and my robot escort, named CoBot (for Collaborative Robot), is one of her contributions to that future.

http://www.cs.cmu.edu/~coral/projects/cobot/

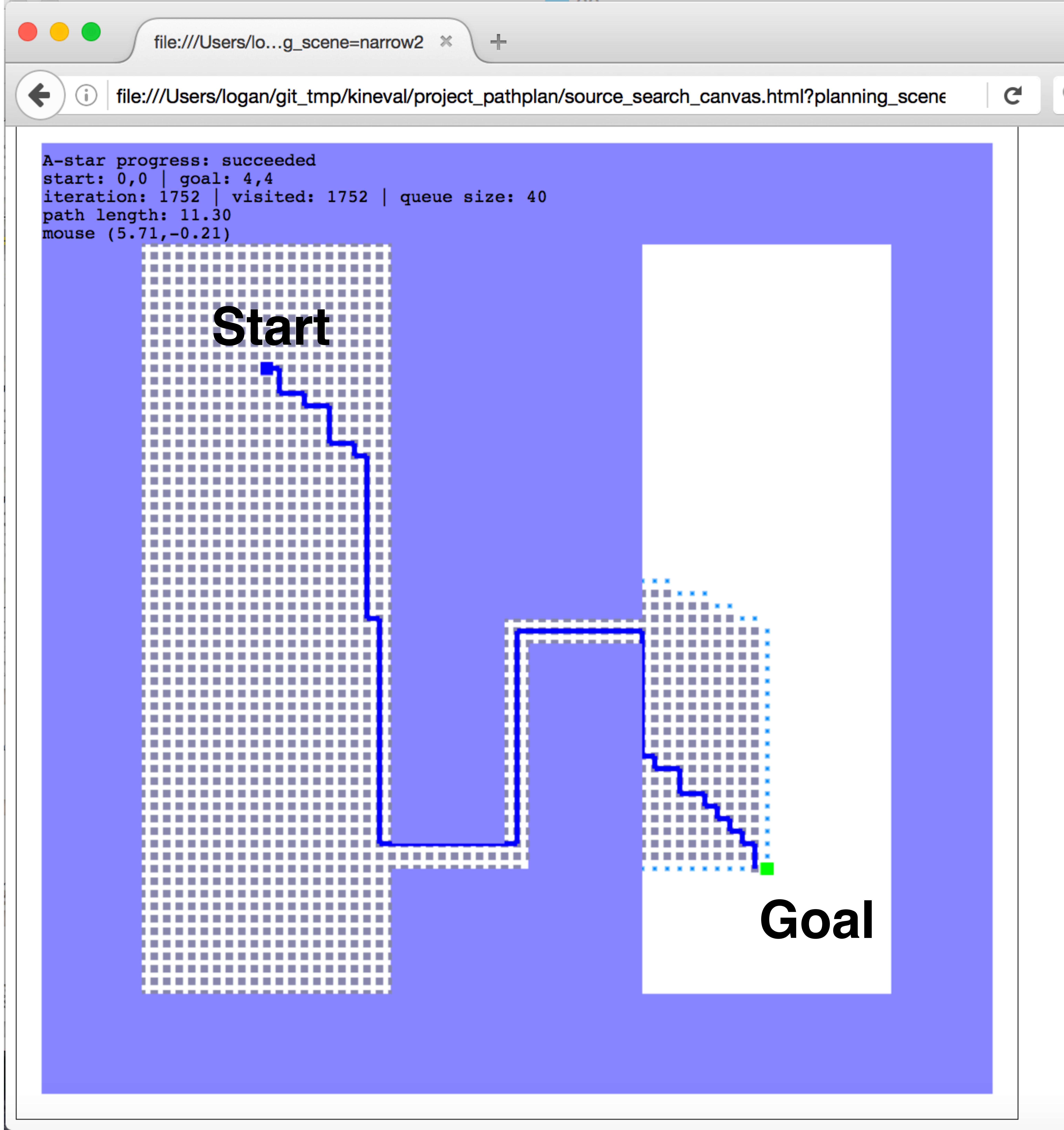


http://www.cs.cmu.edu/~coral/projects/cobot/

https://www.joydeepb.com/research.html

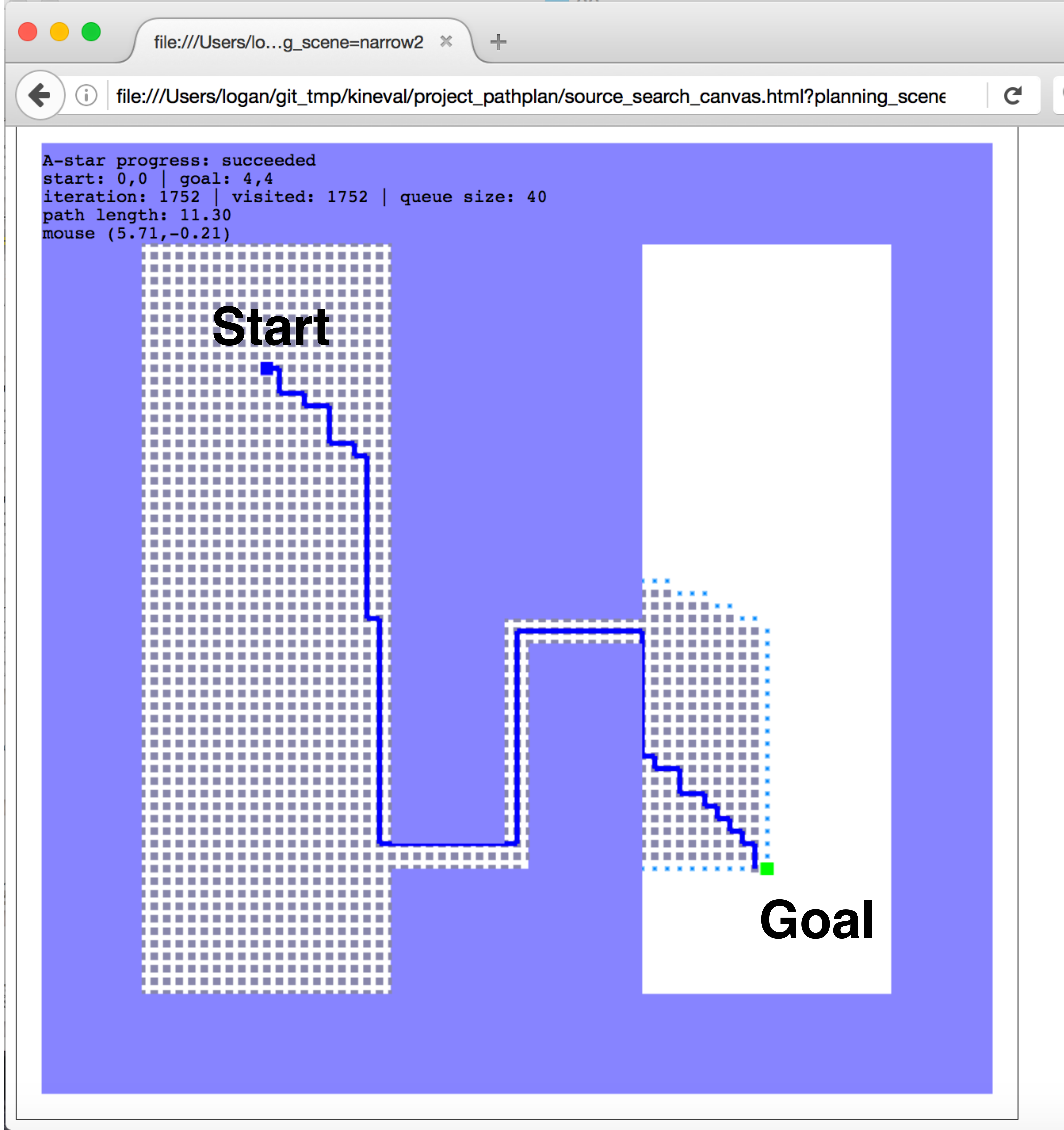
Project 1:
2D Path Planning

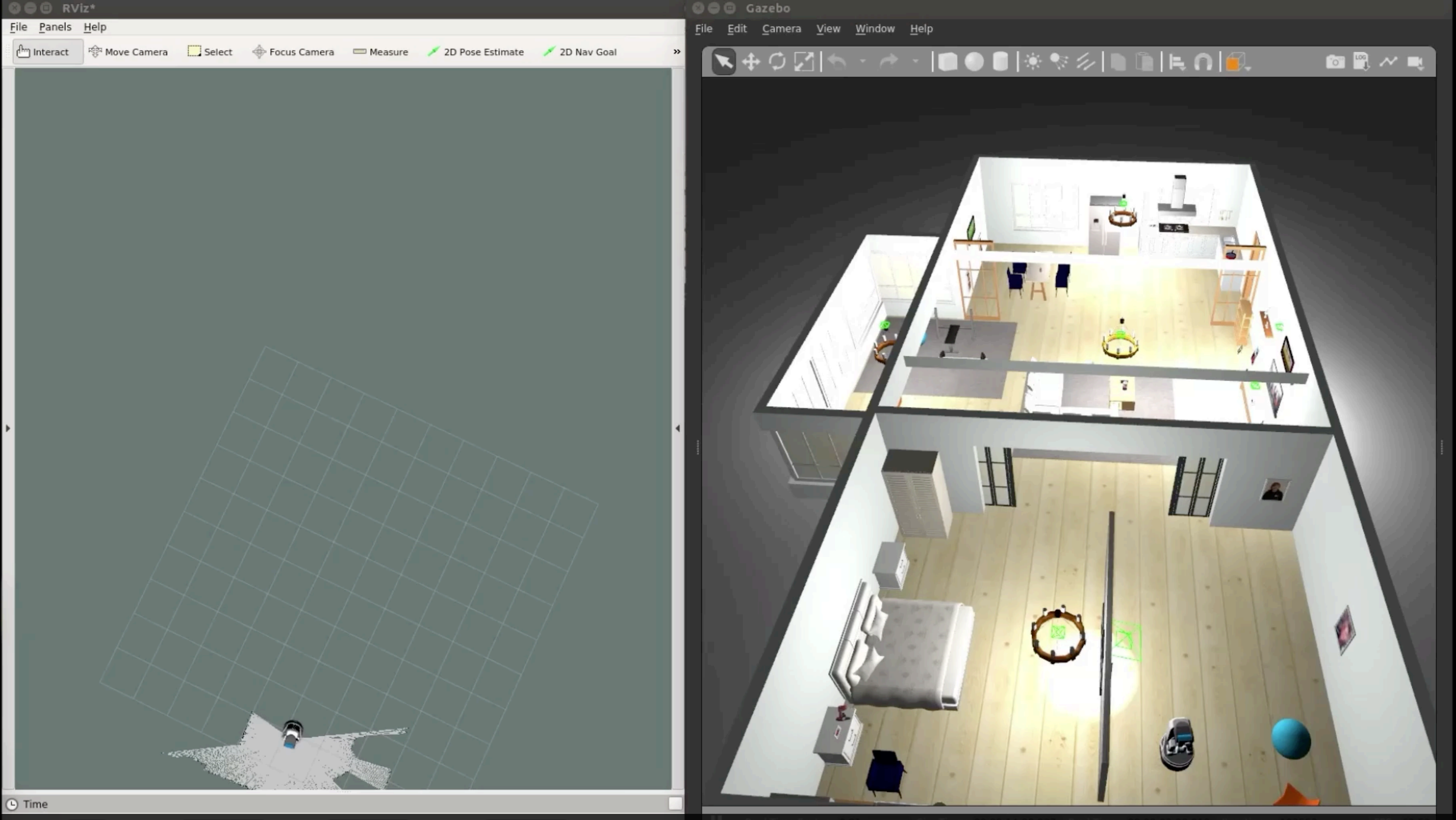
- A-star algorithm for search in a given 2D world
- Implement in JavaScript/ HTML5
- Heap data structure for priority queue
- Submit through your git repository



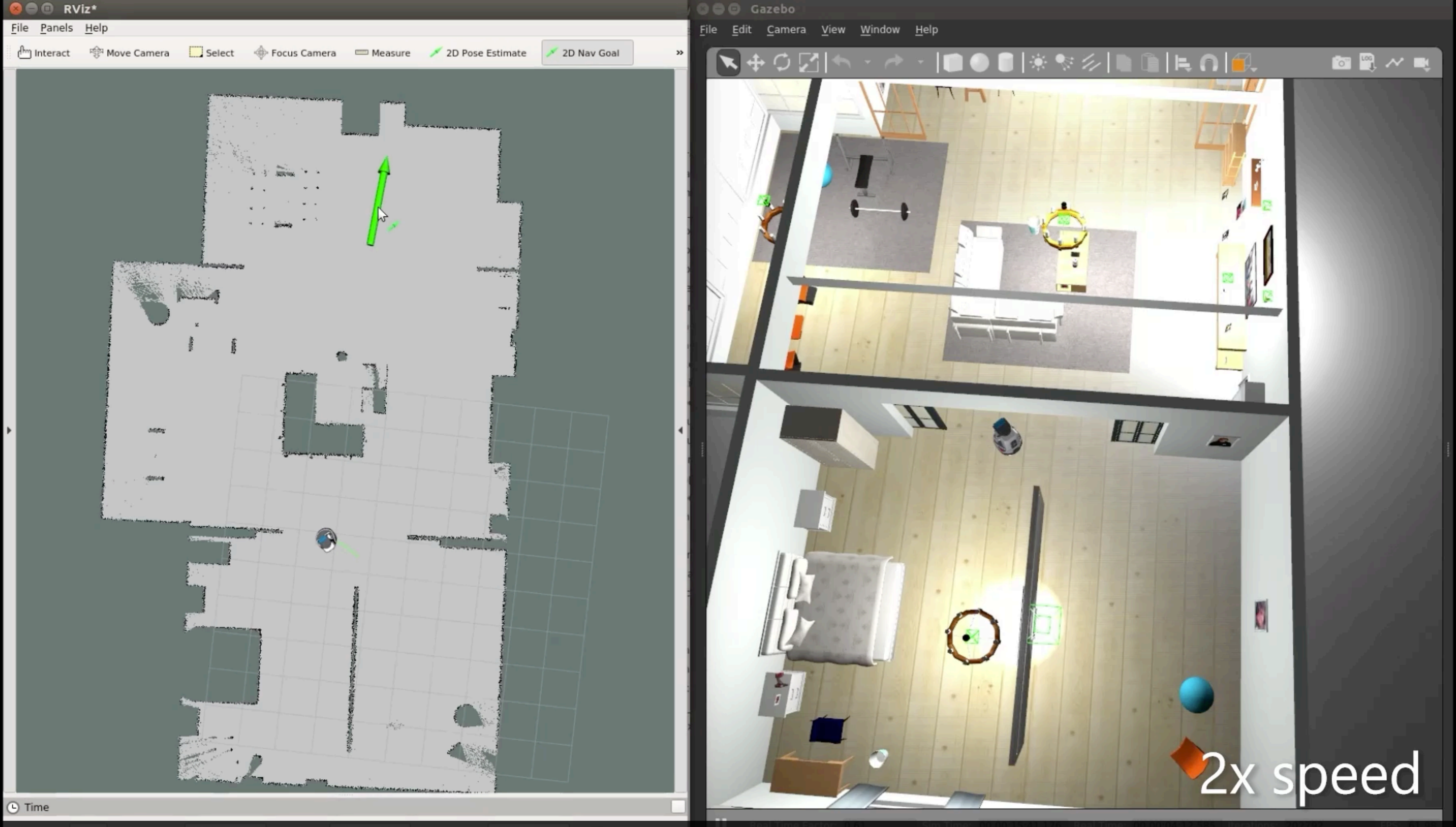
Path Planning

- The robot knows:
  - **Localization**: where it is now
  - **Goal**: where it needs to go
  - **Map**: where it will hit something
- Infer:
  - **Path**: Collision-free sequence of locations to follow to goal





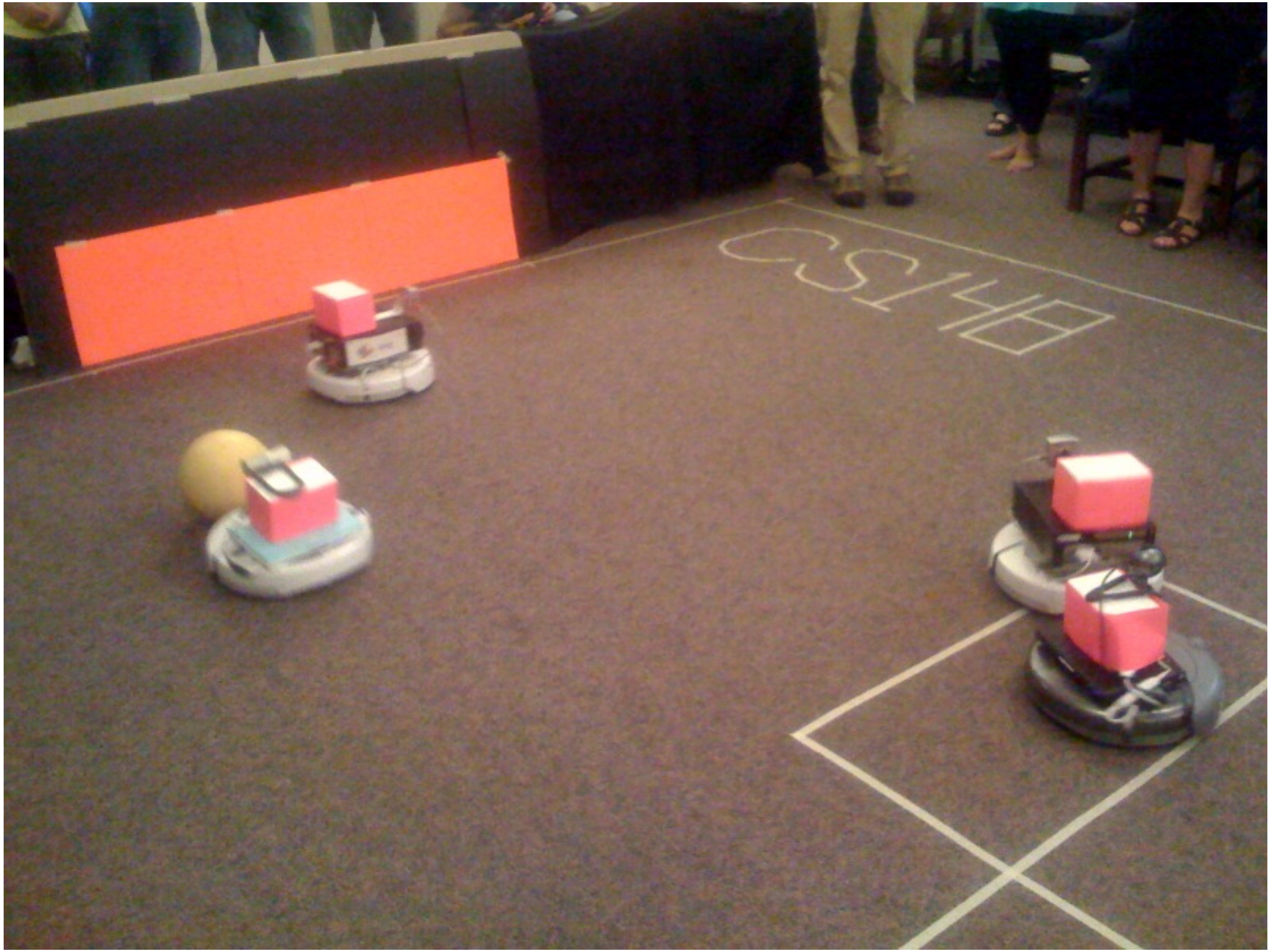
Localization and Mapping - Alphonsus Adu-Bredu - https://youtu.be/wH0QhWgtmuA



Autonomous Navigation - Alphonsus Adu-Bredu - https://youtu.be/wH0QhWgtmuA

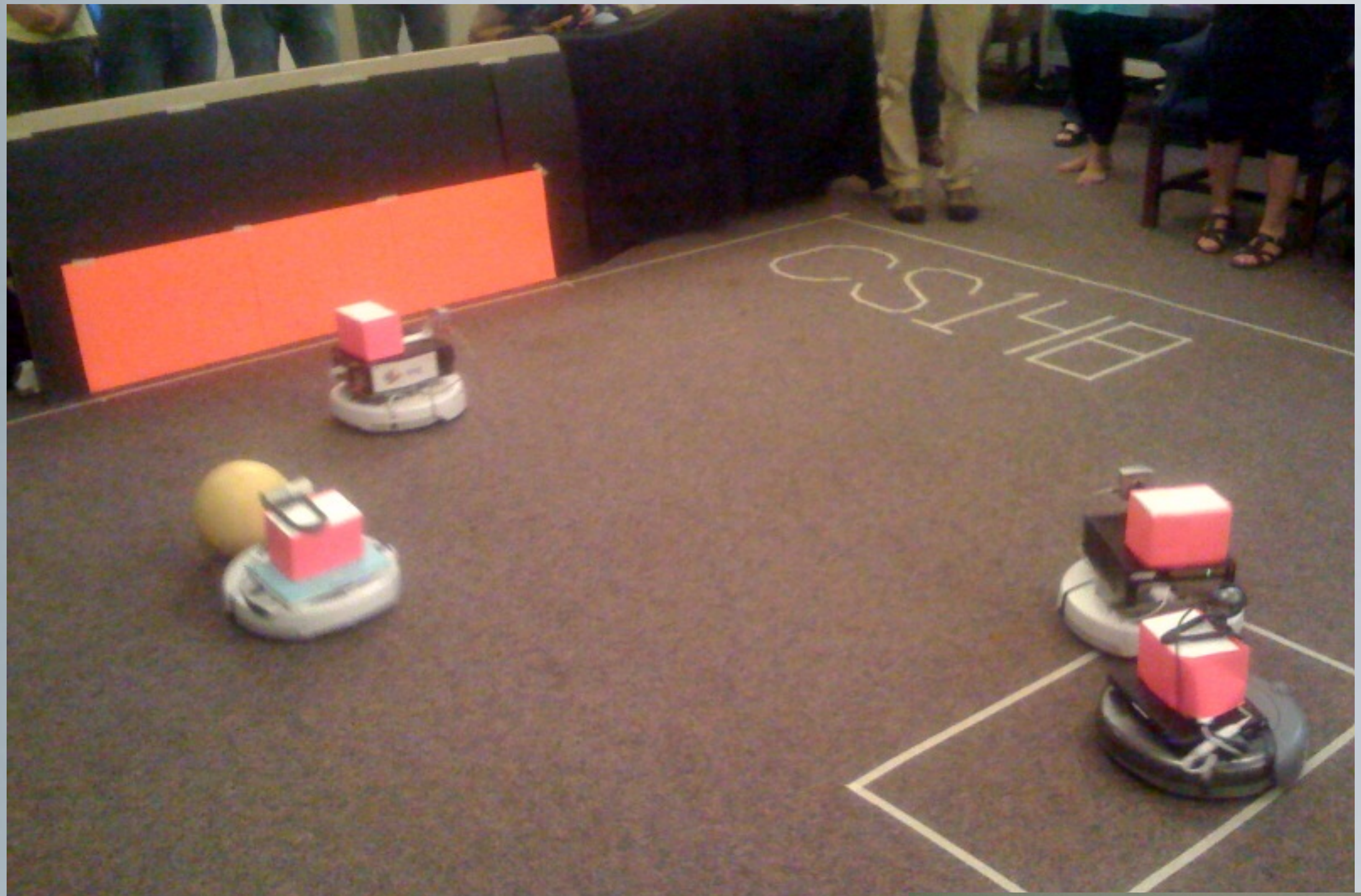
How do we get from A to B?

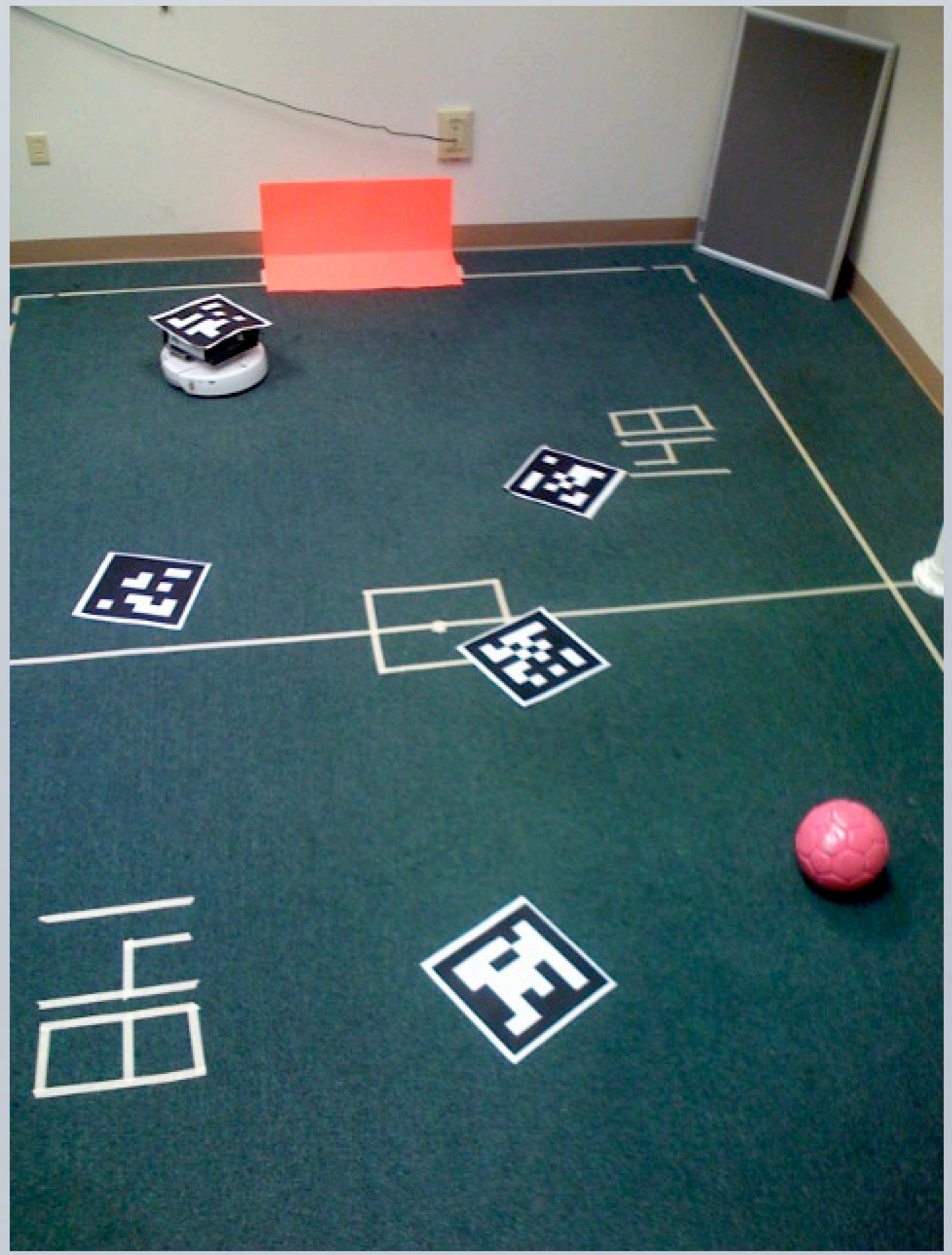
Going back to robot soccer…

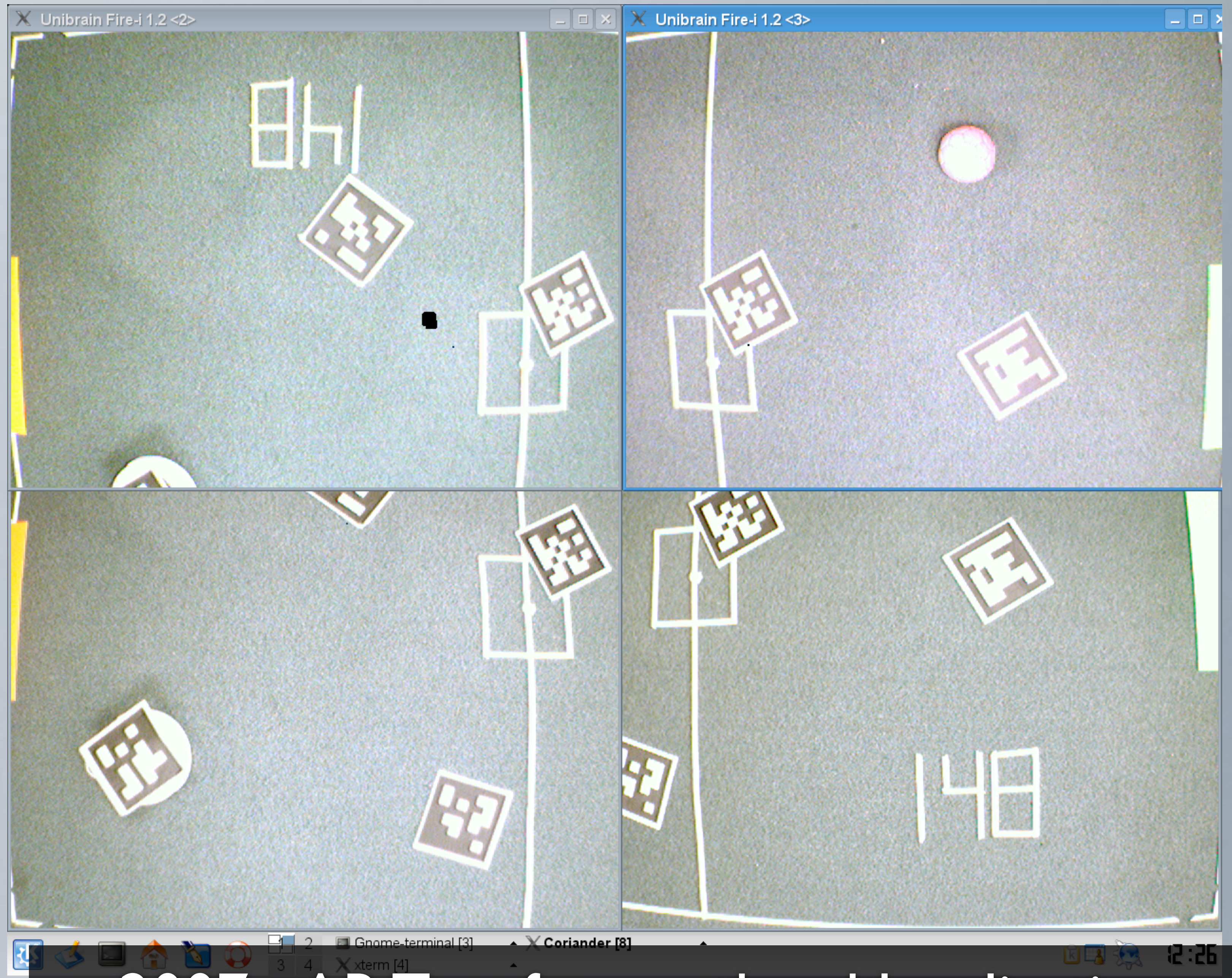


Brown CS148 Promotional Video 2009 - https://youtu.be/bsvUQ5Kp2Q4

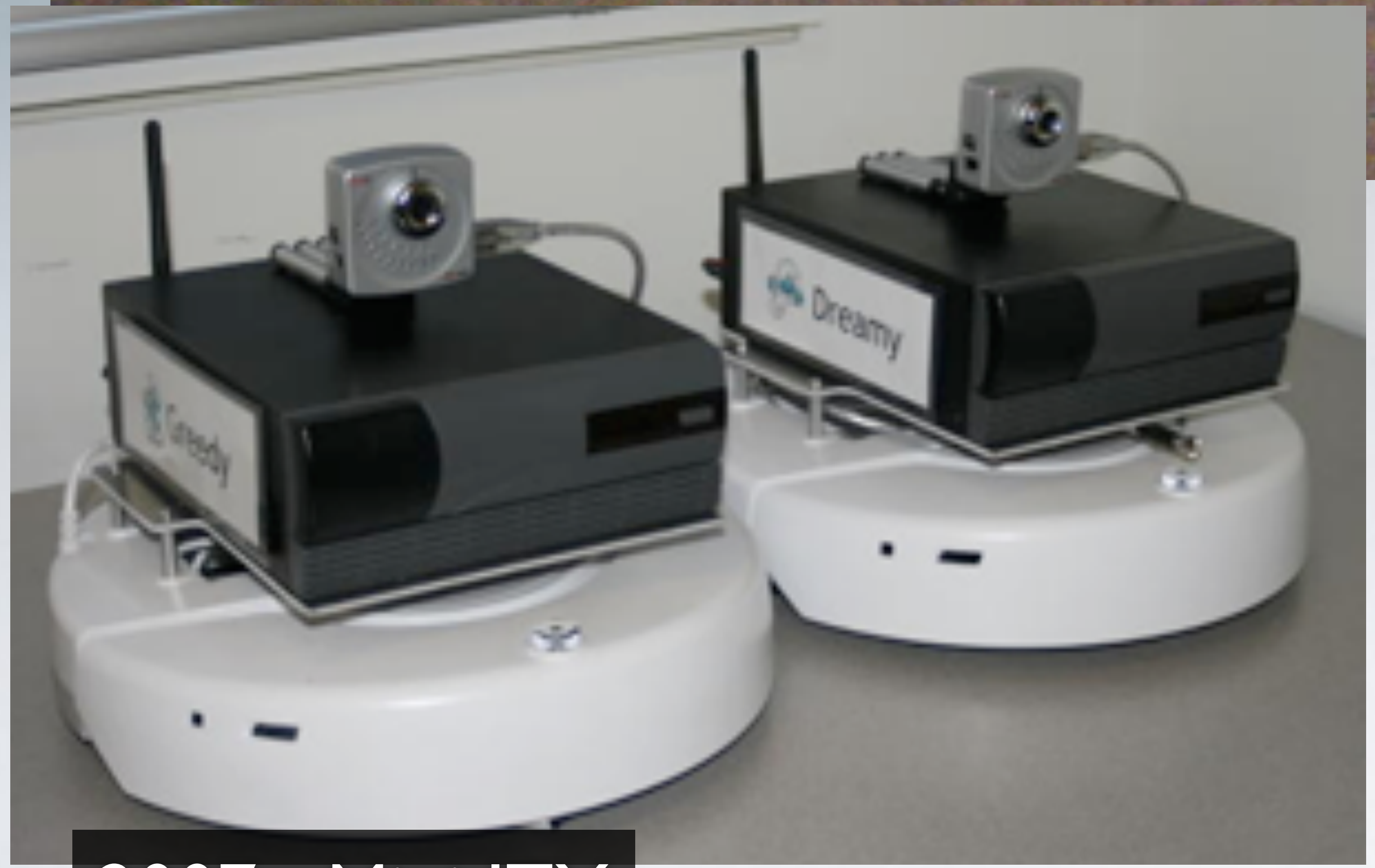
2007-10: SOCCER WITH IROBOT CREATE







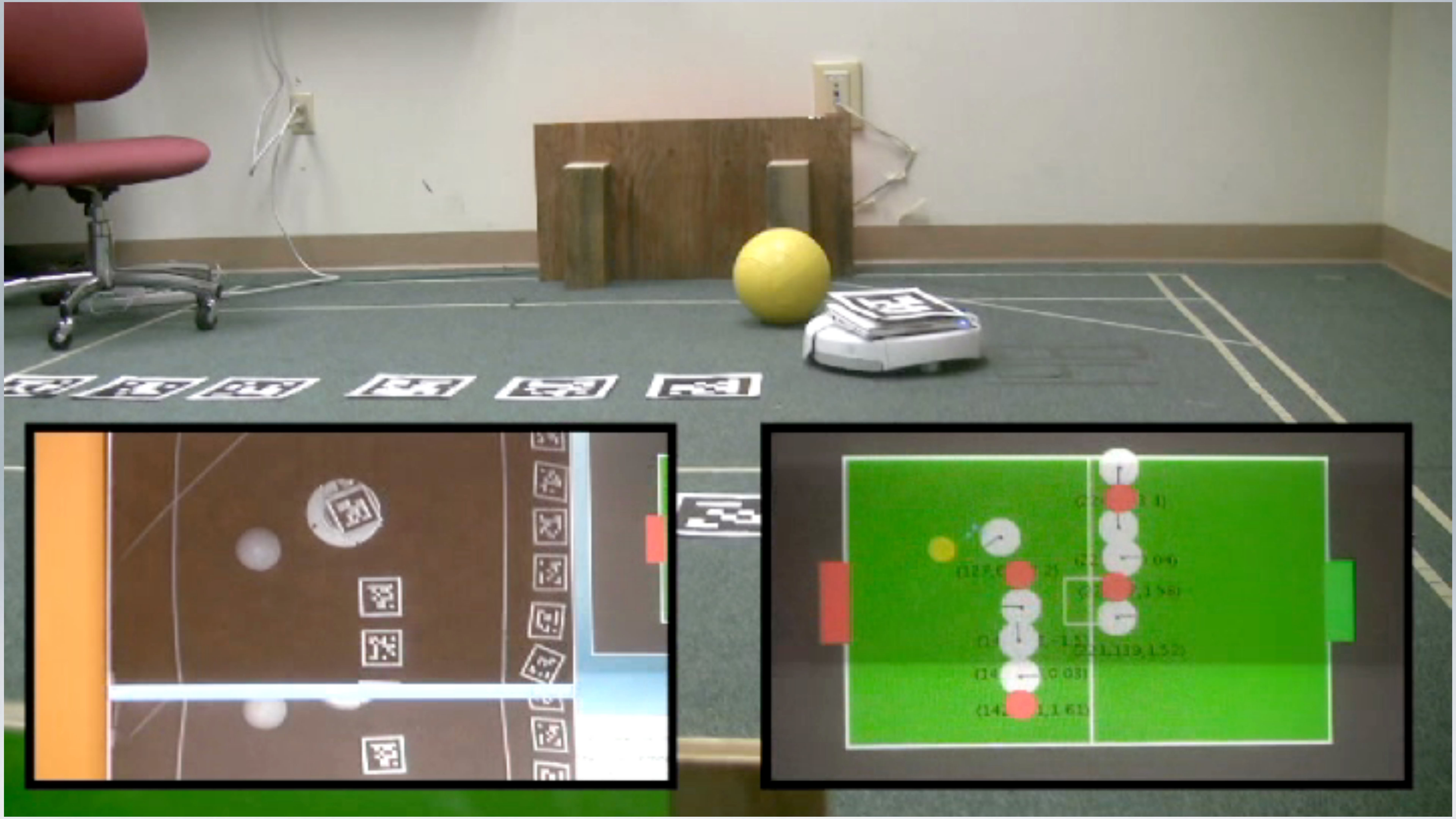
2007 - AR Tags for overhead localization

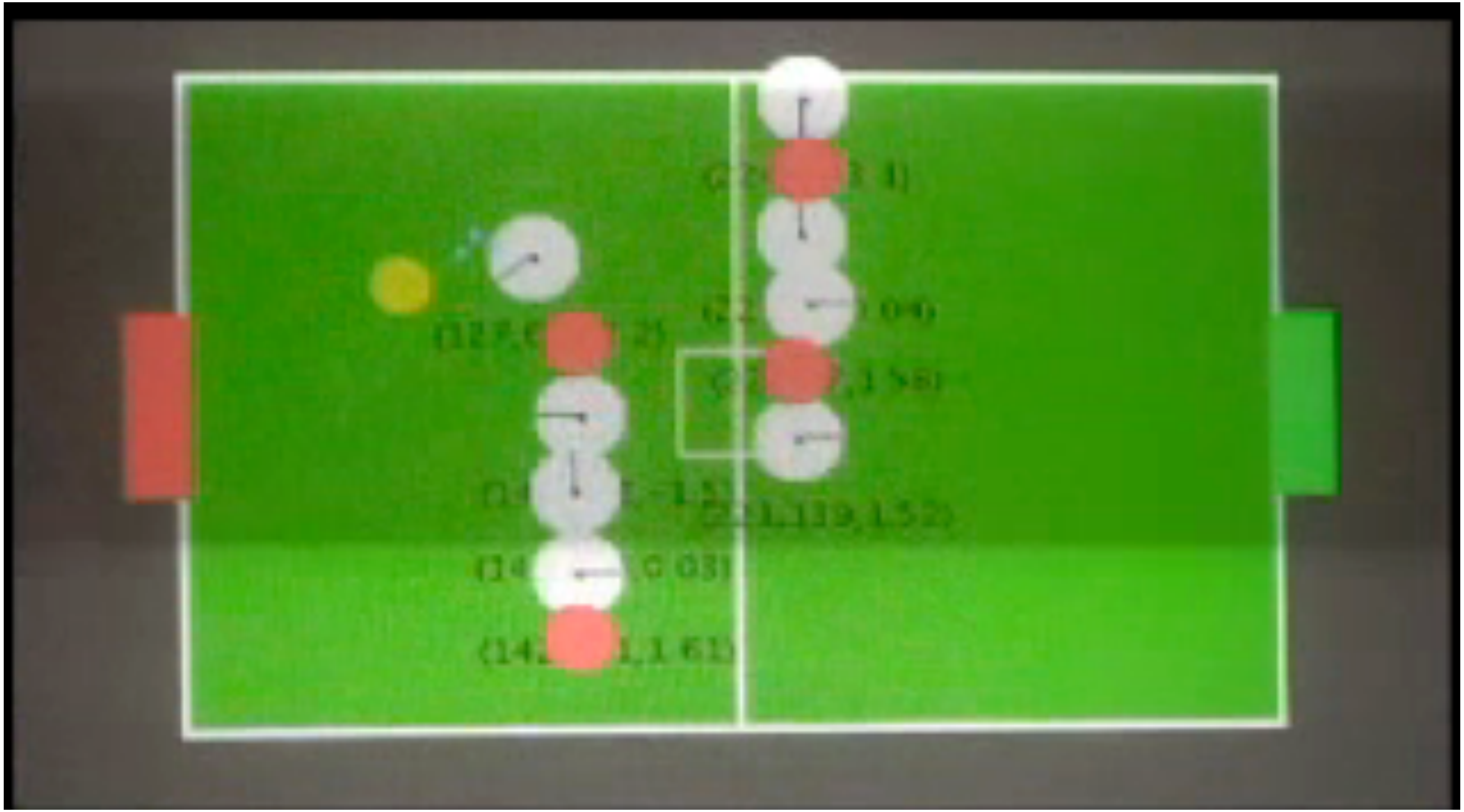


2007 - Mini ITX

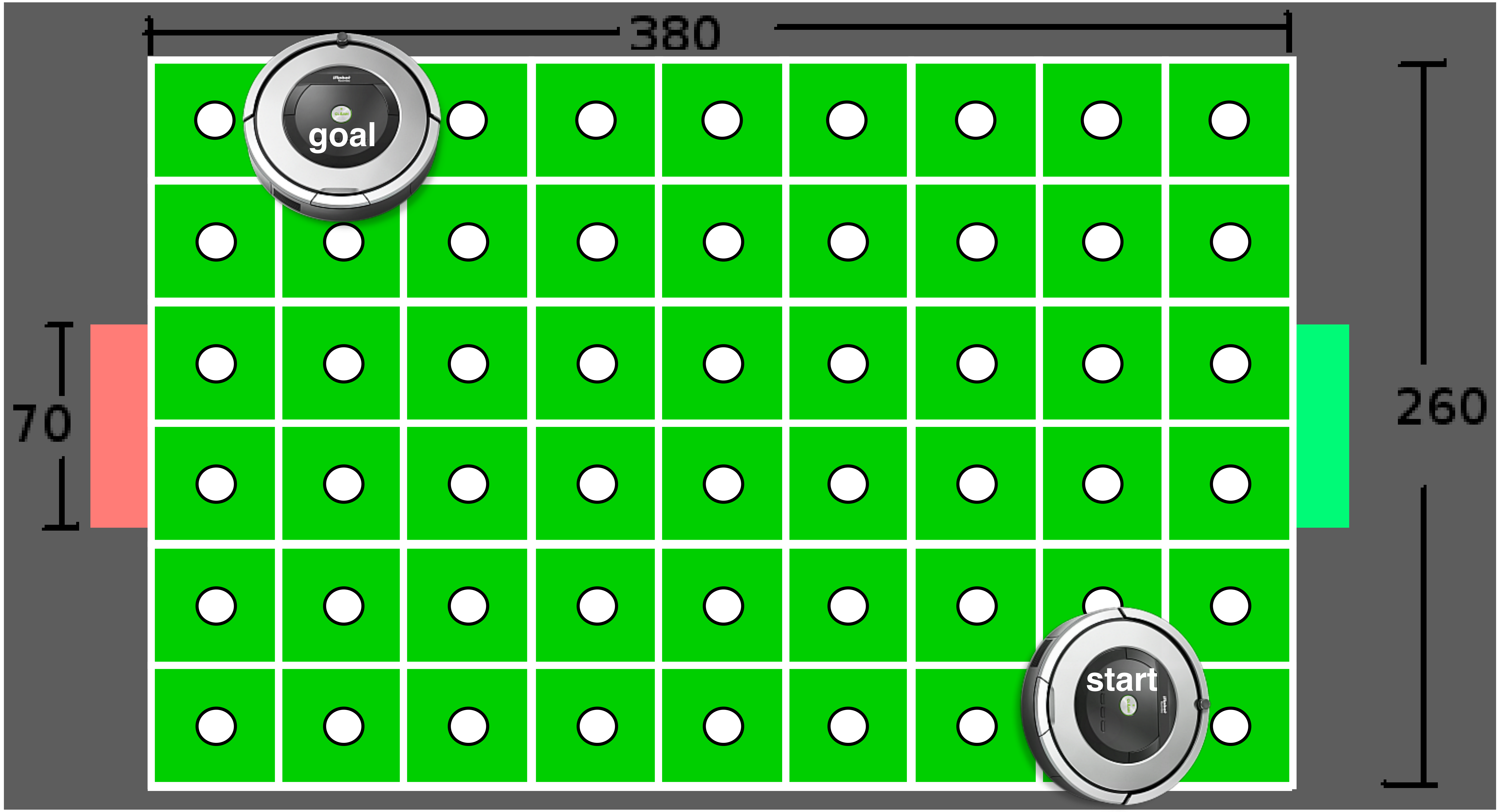
2009 - Asus EEE

youtube.com/watch?v=88zR6IC7S0g



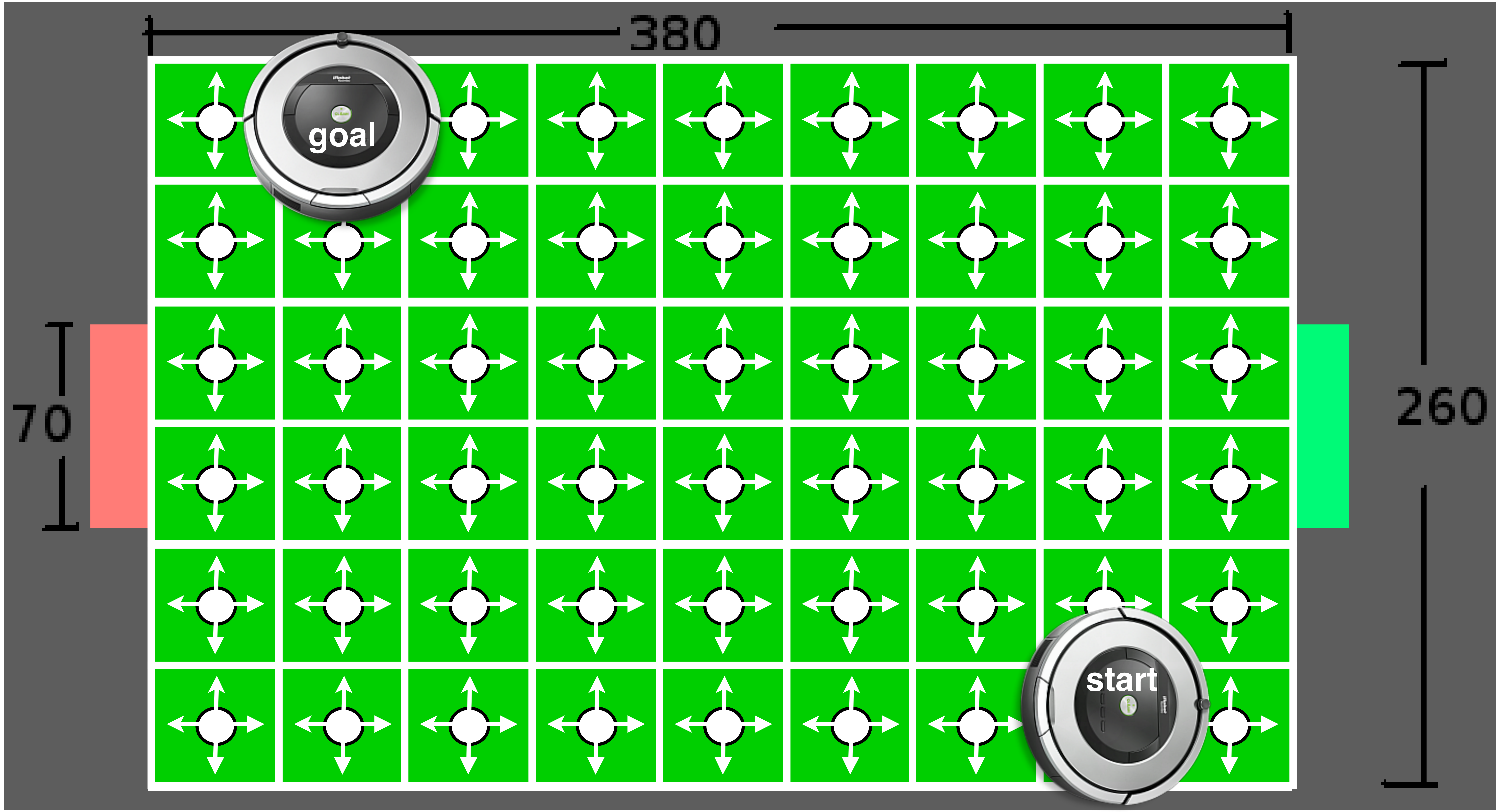


Consider all possible poses as uniformly distributed array of cells in a graph



Consider all possible poses as uniformly distributed array of cells in a graph

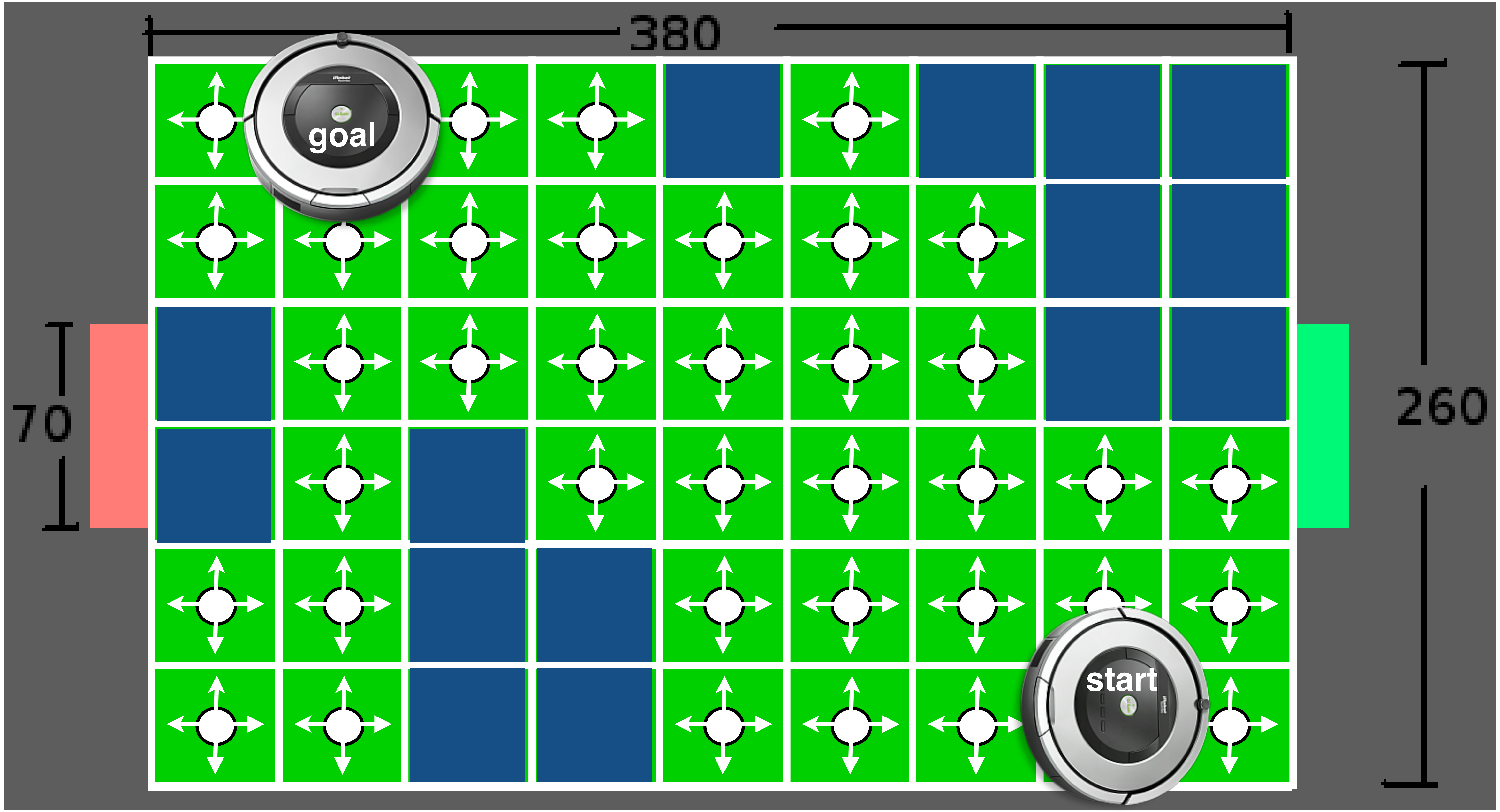
Edges connect adjacent cells, weighted by distance



Consider all possible poses as uniformly distributed array of cells in a graph

Edges connect adjacent cells, weighted by distance

Cells are invalid where its associated robot pose results in a collision

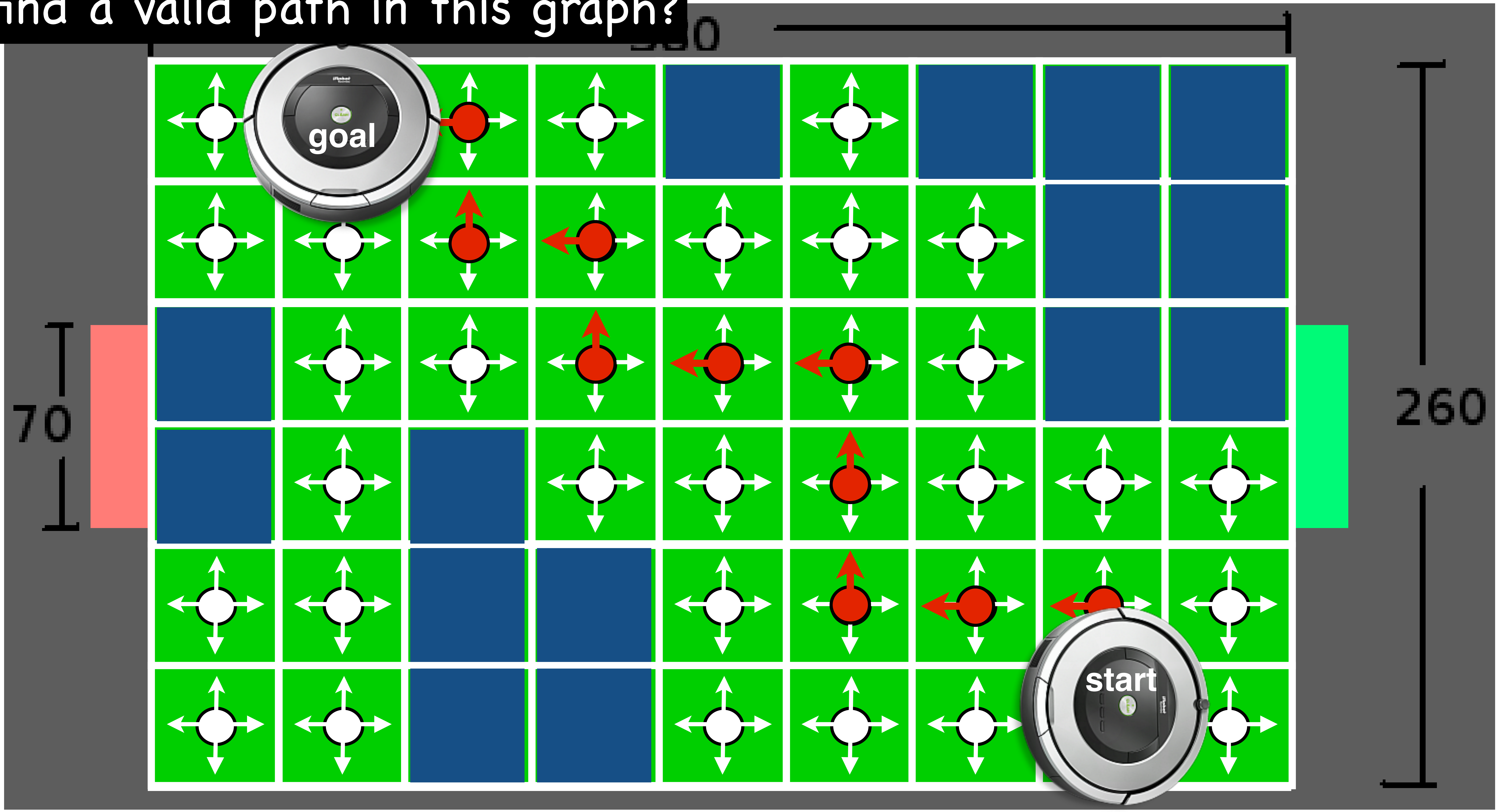


Consider all possible poses as uniformly distributed array of cells in a graph

Edges connect adjacent cells, weighted by distance

Cells are invalid where its associated robot pose results in a collision

How to find a valid path in this graph?



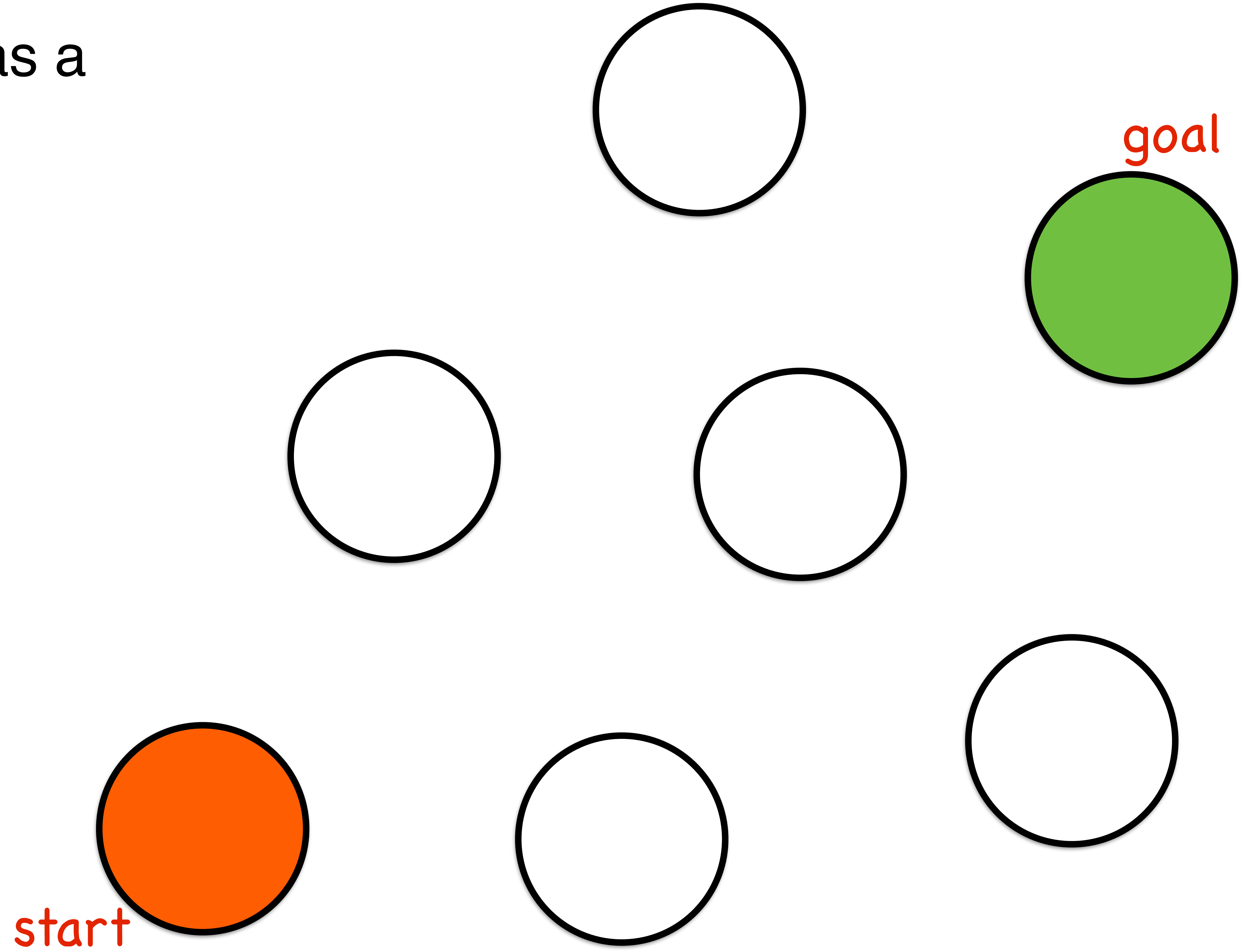
Approaches to motion planning

- Bug algorithms: Bug[0-2], Tangent Bug
- **Graph Search (fixed graph)**
  - **Depth-first, Breadth-first, Dijkstra, A-star, Greedy best-first**
- Sampling-based Search (build graph):
  - Probabilistic Road Maps, Rapidly-exploring Random Trees
- Optimization (local search):
  - Gradient descent, potential fields, Wavefront

Consider a simple search graph

Consider a simple search graph

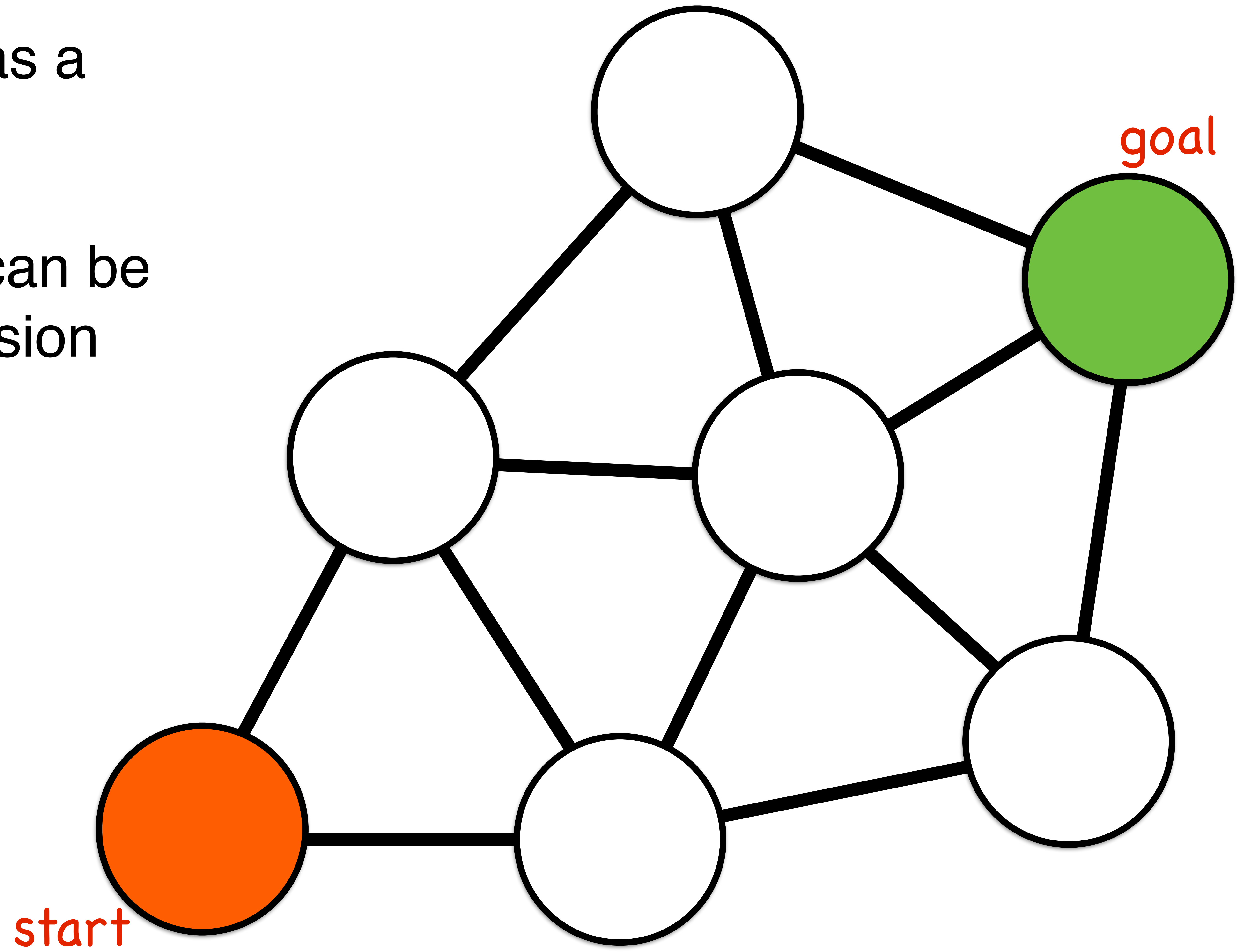
Consider each possible robot pose as a node V_i in a graph $G(V,E)$



Consider a simple search graph

Consider each possible robot pose as a node V_i in a graph $G(V,E)$

Graph edges E connect poses that can be reliably moved between without collision

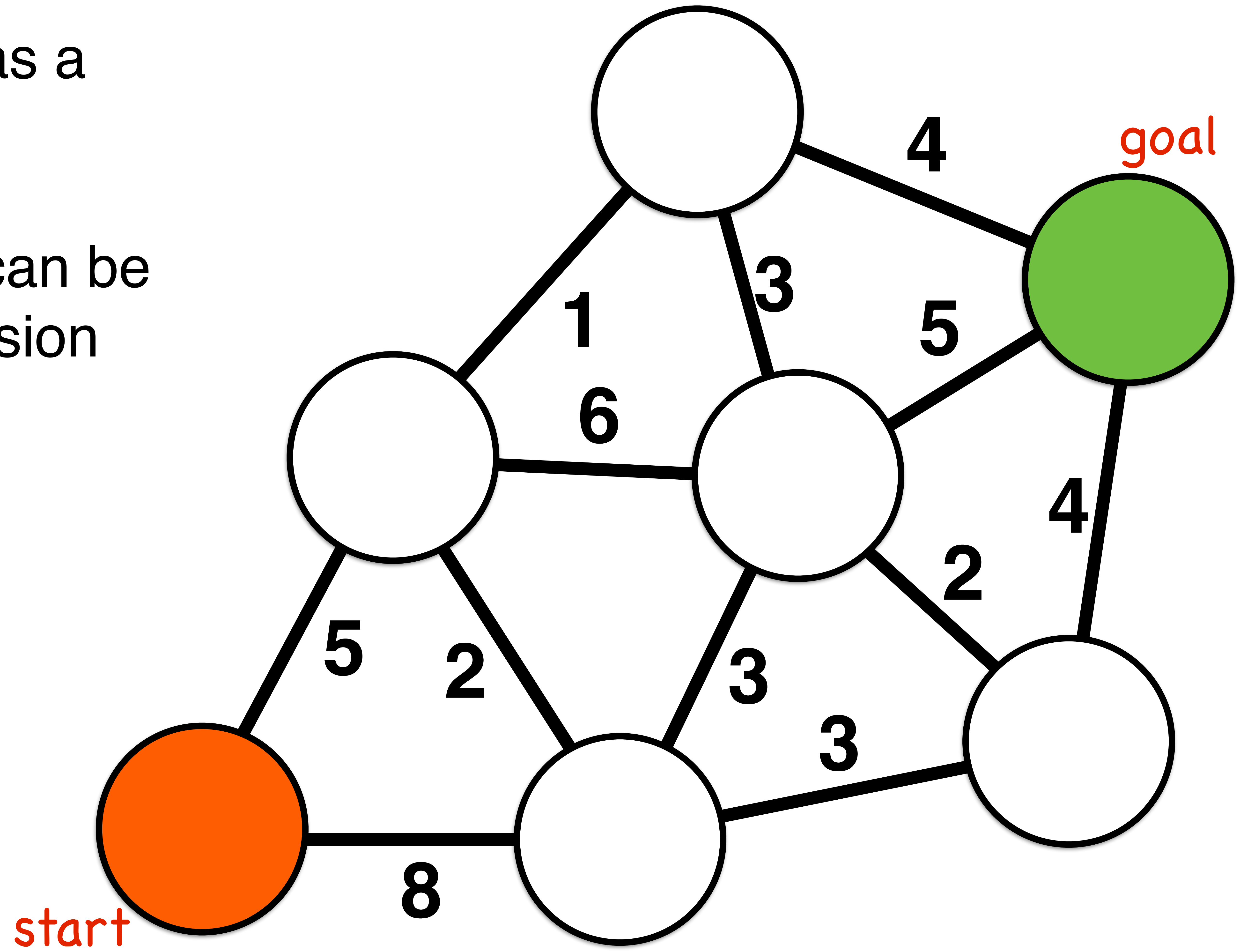


Consider a simple search graph

Consider each possible robot pose as a node V_i in a graph $G(V,E)$

Graph edges E connect poses that can be reliably moved between without collision

Edges have a cost for traversal



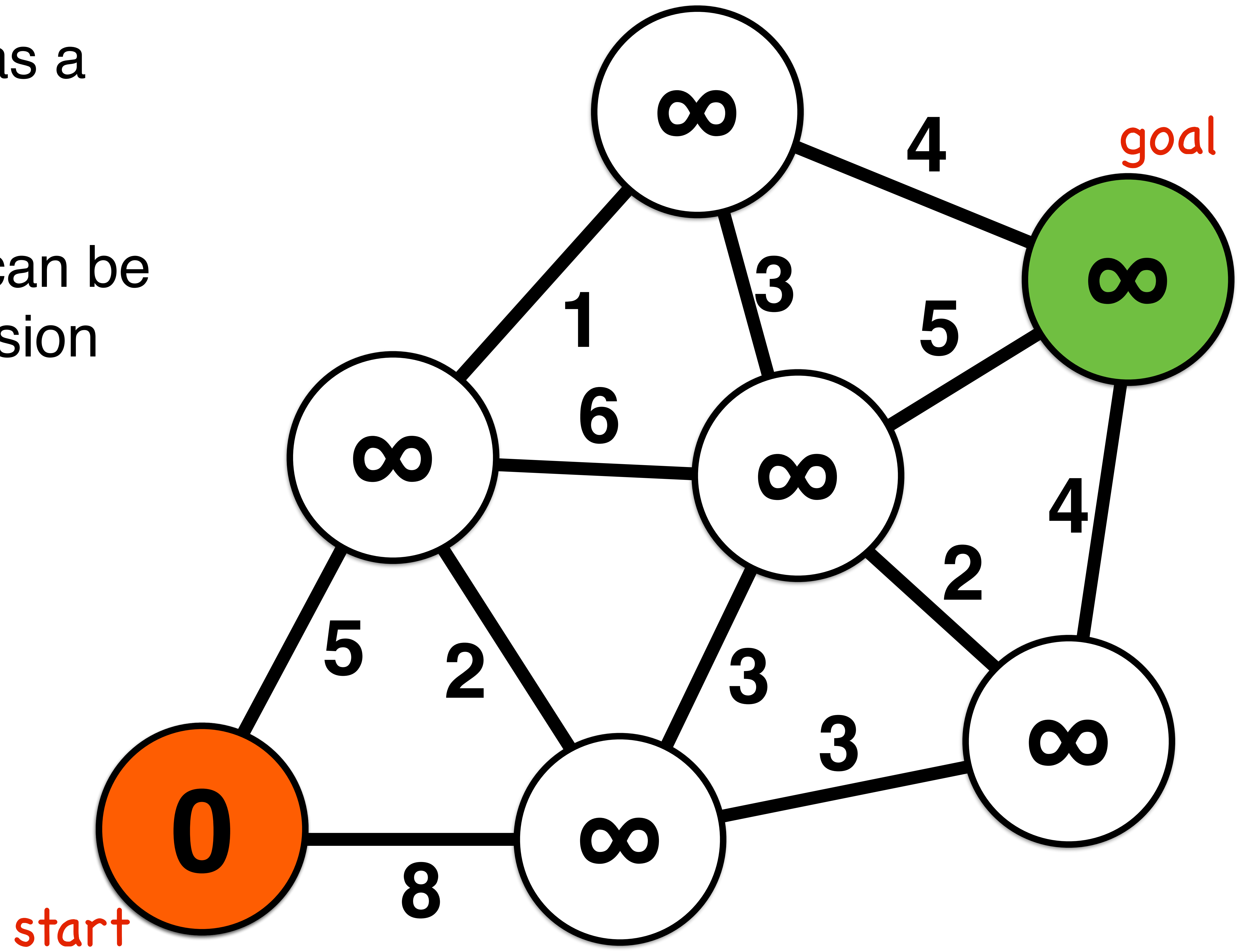
Consider a simple search graph

Consider each possible robot pose as a node V_i in a graph $G(V,E)$

Graph edges E connect poses that can be reliably moved between without collision

Edges have a cost for traversal

Each node maintains the **distance** traveled from start as a scalar cost



Consider a simple search graph

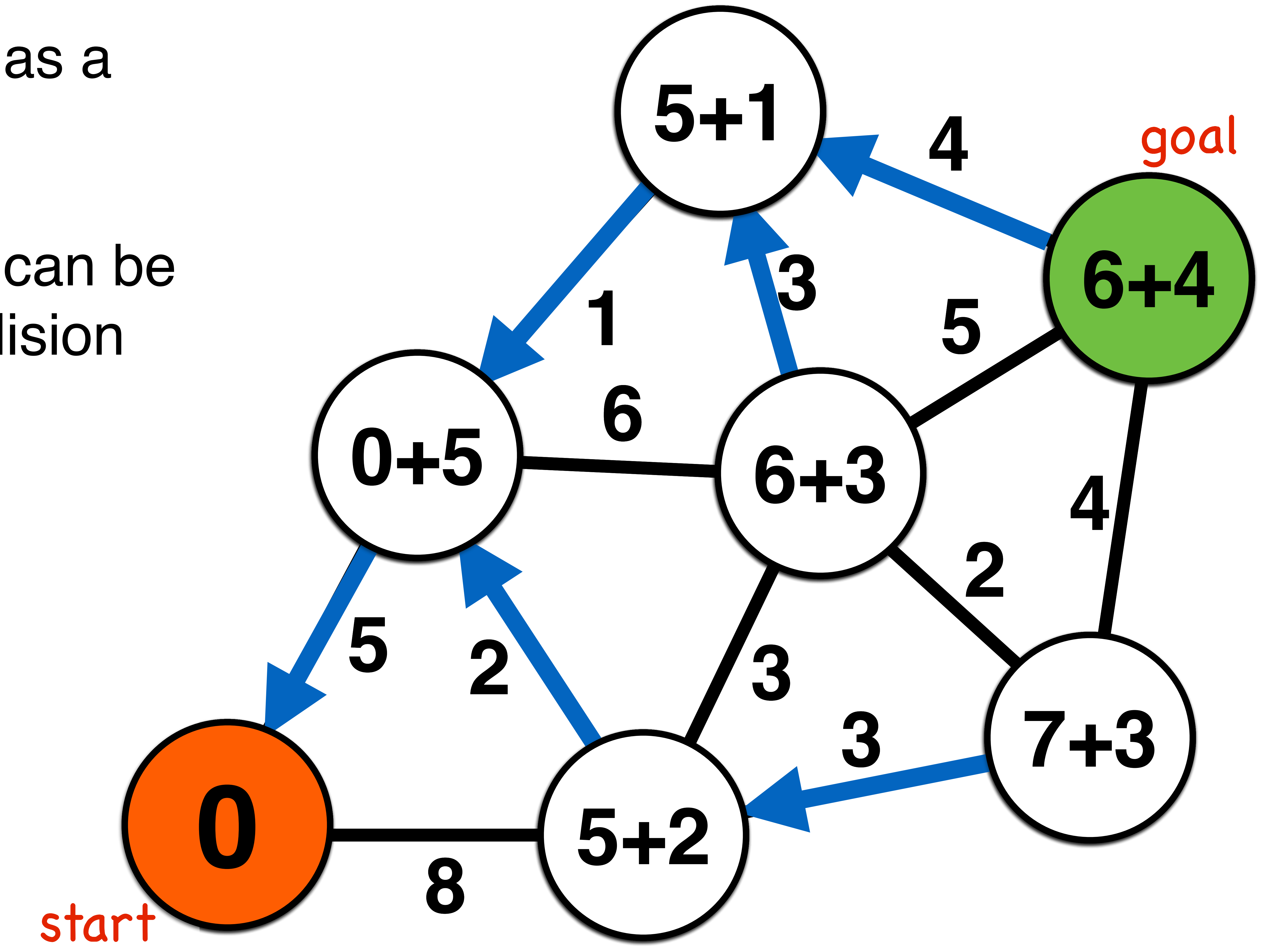
Consider each possible robot pose as a node V_i in a graph $G(V,E)$

Graph edges E connect poses that can be reliably moved between without collision

Edges have a cost for traversal

Each node maintains the **distance** traveled from start as a scalar cost

Each node has a **parent** node that specifies its route to the start node

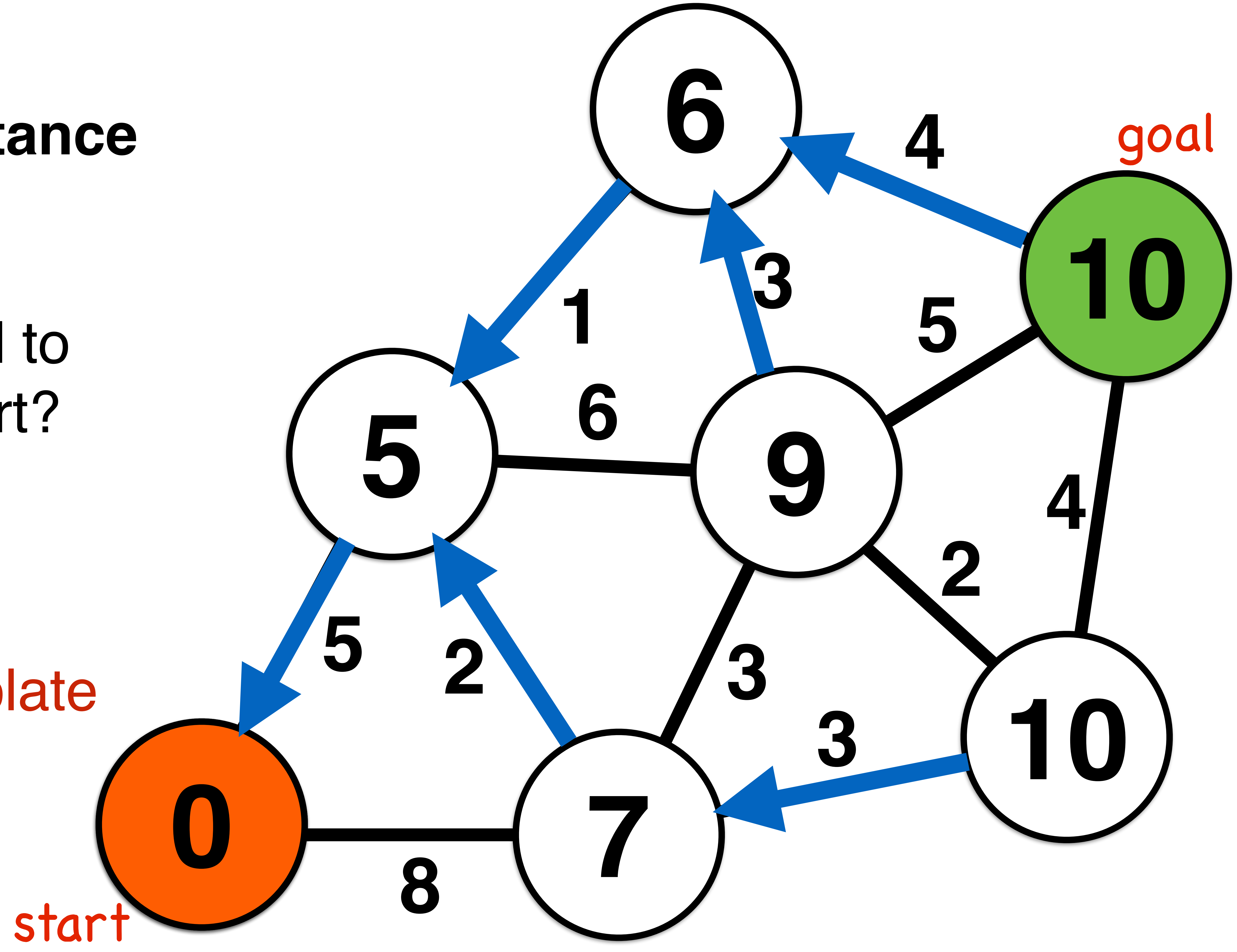


Path Planning as Graph Search

Which route is best to optimize **distance** traveled from start?

Which **parent** node should be used to specify route between goal and start?

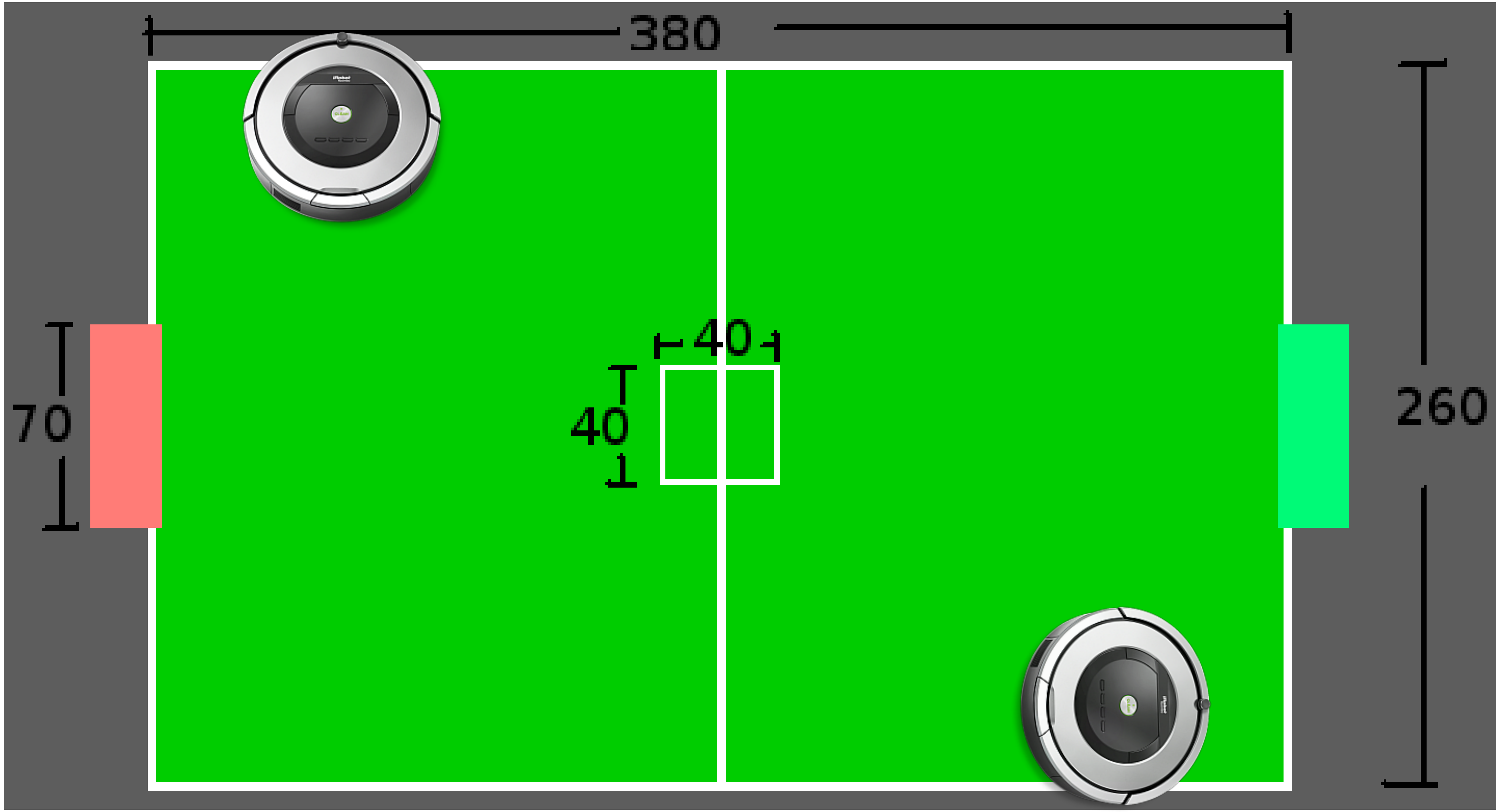
We will use a single algorithm template for our graph search computation



Depth-first search

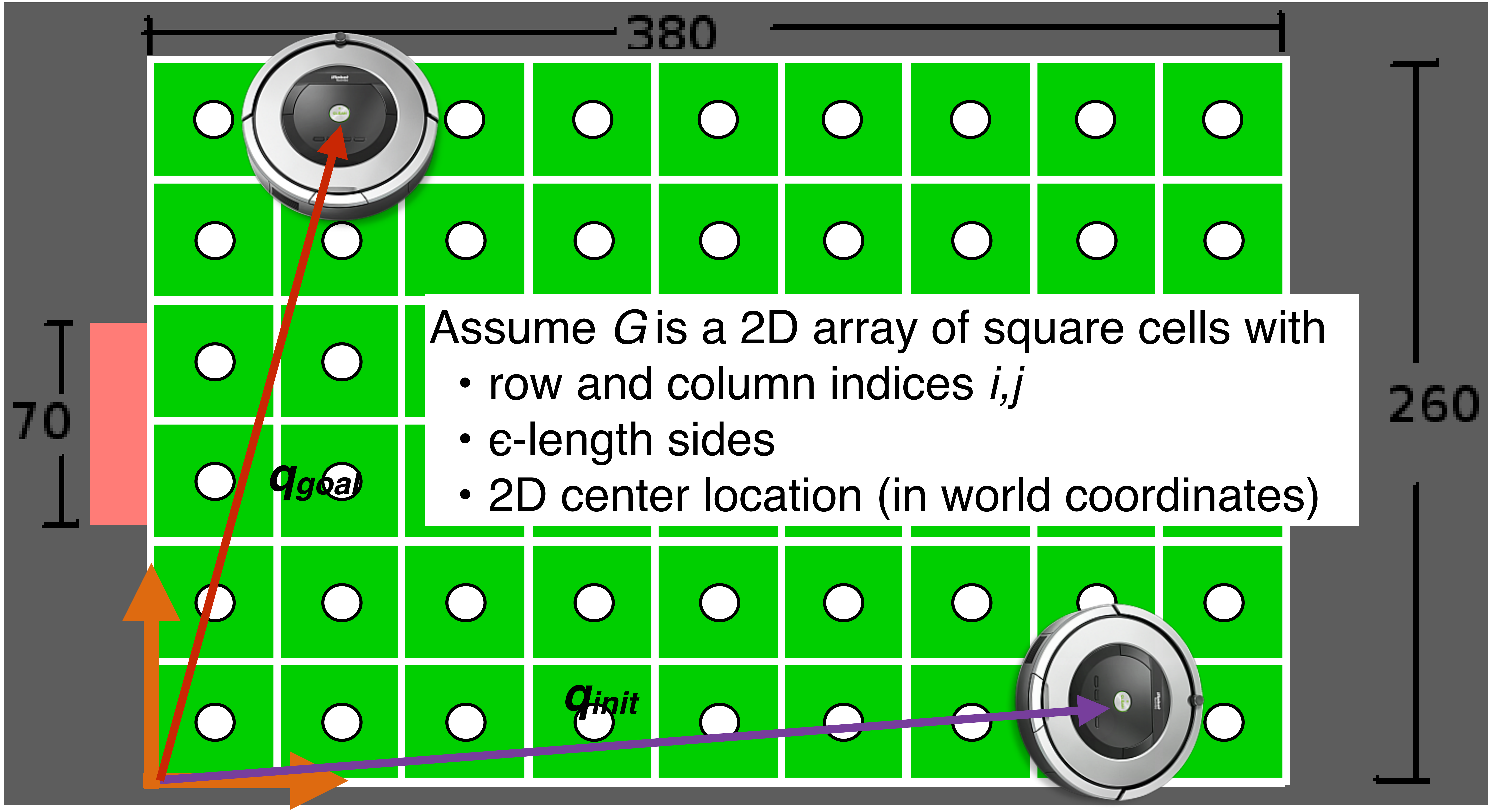
intuition and walkthrough

Depth-first search



Depth-first search

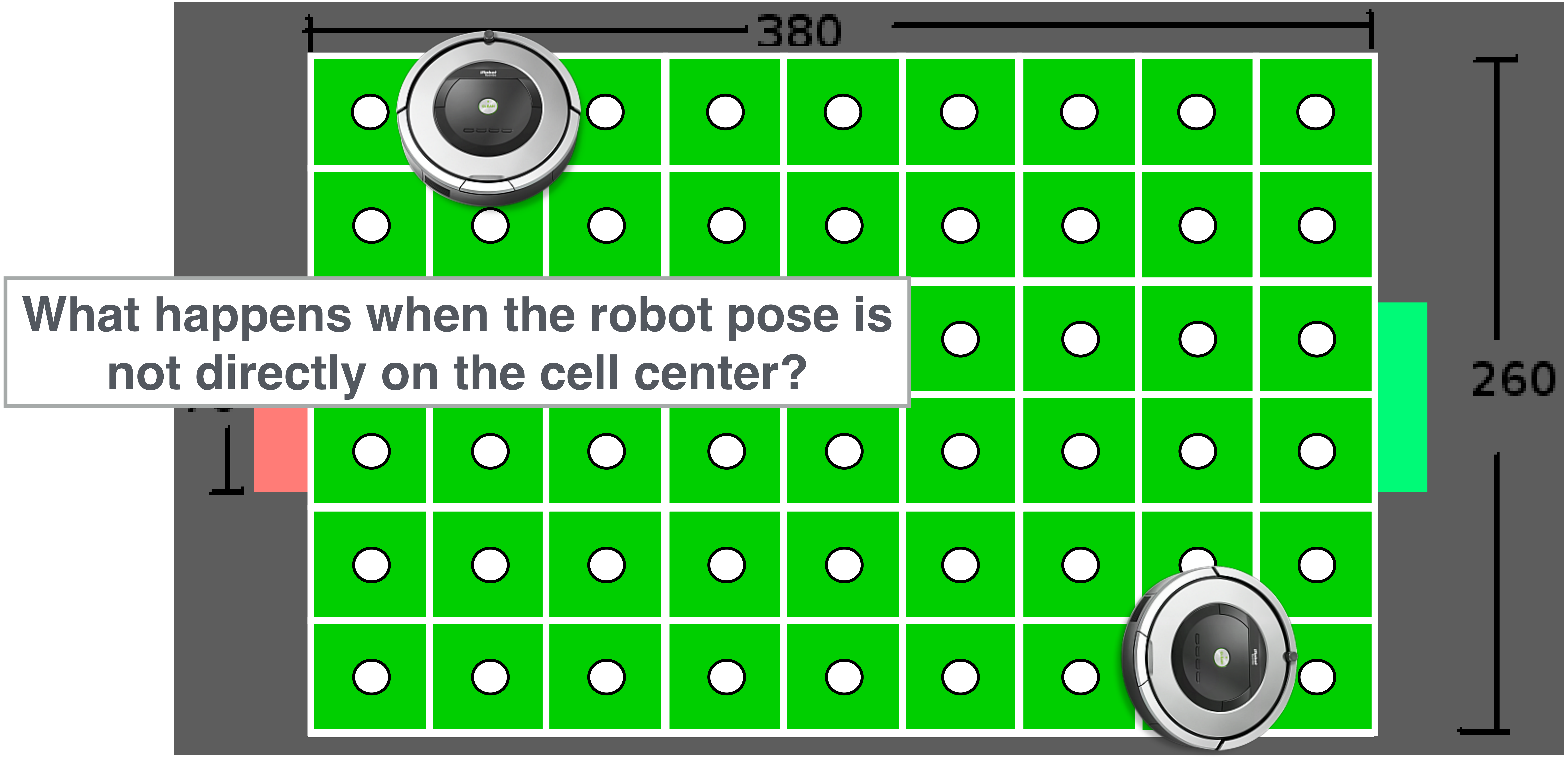
Depth-first search



Assume G is a 2D array of square cells with

- row and column indices i,j
- ϵ-length sides
- 2D center location (in world coordinates)

Depth-first search



Graph Accessibility

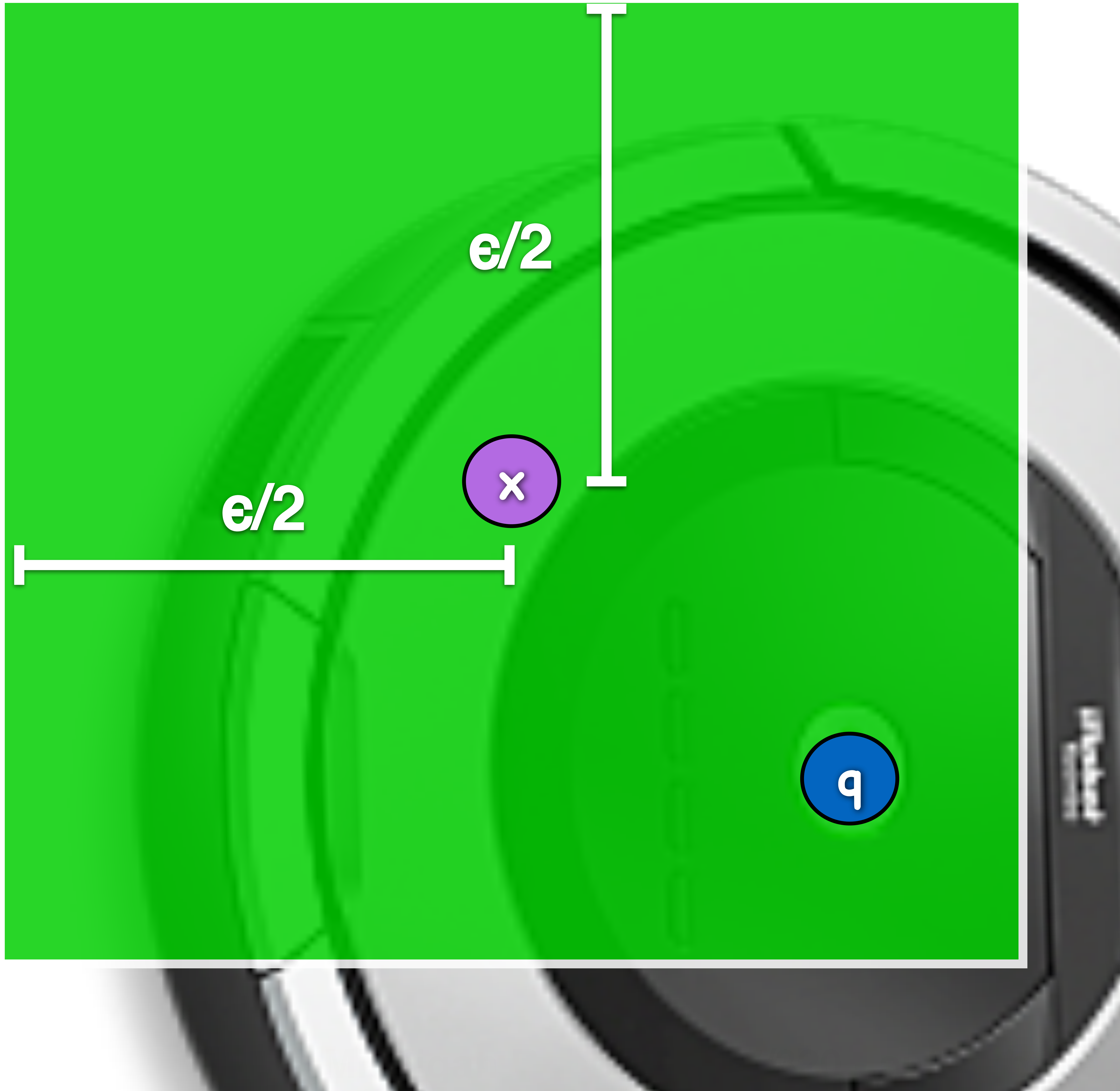
What happens when the robot pose is not directly on the cell center?

Graph Accessibility

A graph node $G_{i,j}$ represents a region of space contained by its cell

Start node: the robot accesses graph G at the cell that contains location $\boldsymbol{q_{init}}$

Goal node: the robot departs graph G at the cell that contains location $\boldsymbol{q_{goal}}$

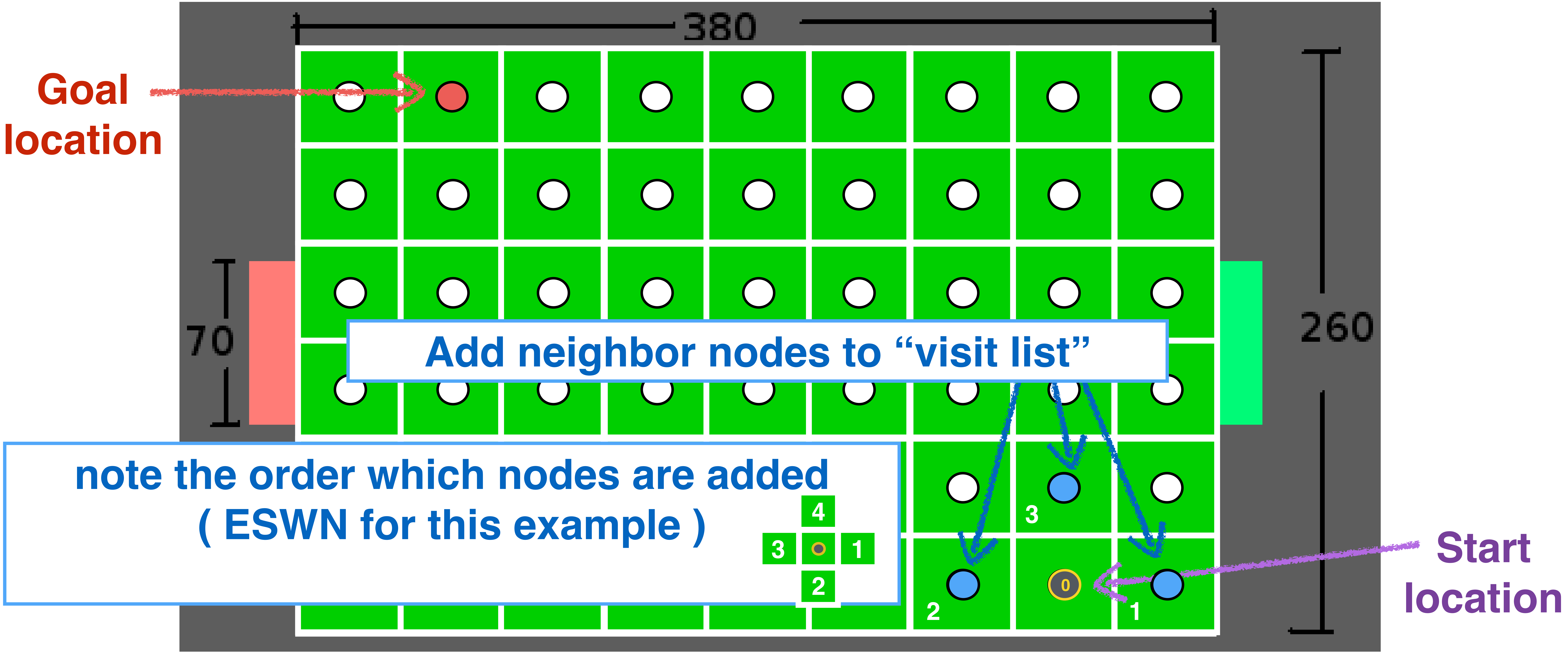


Depth-first search

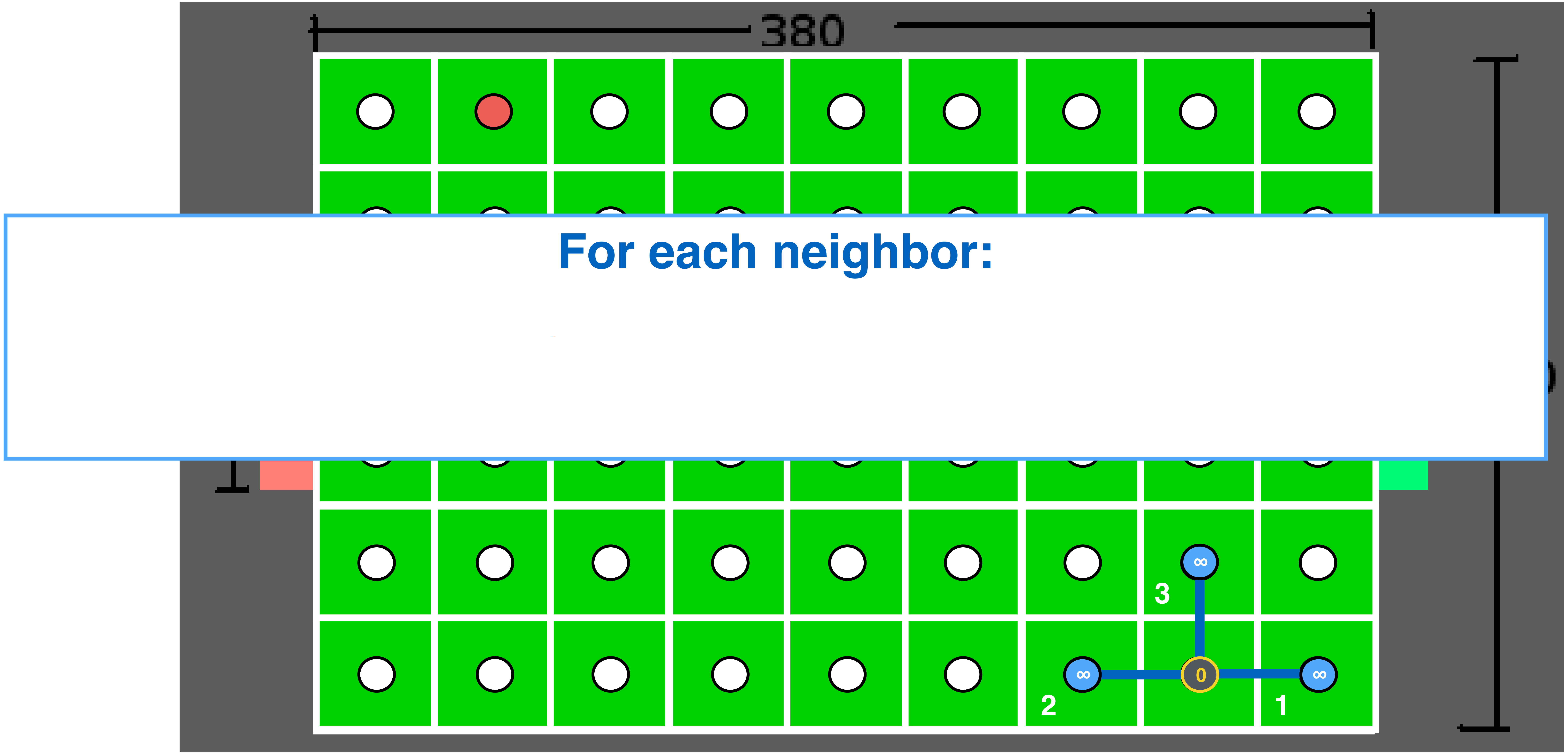
Depth-first search

Depth-first search

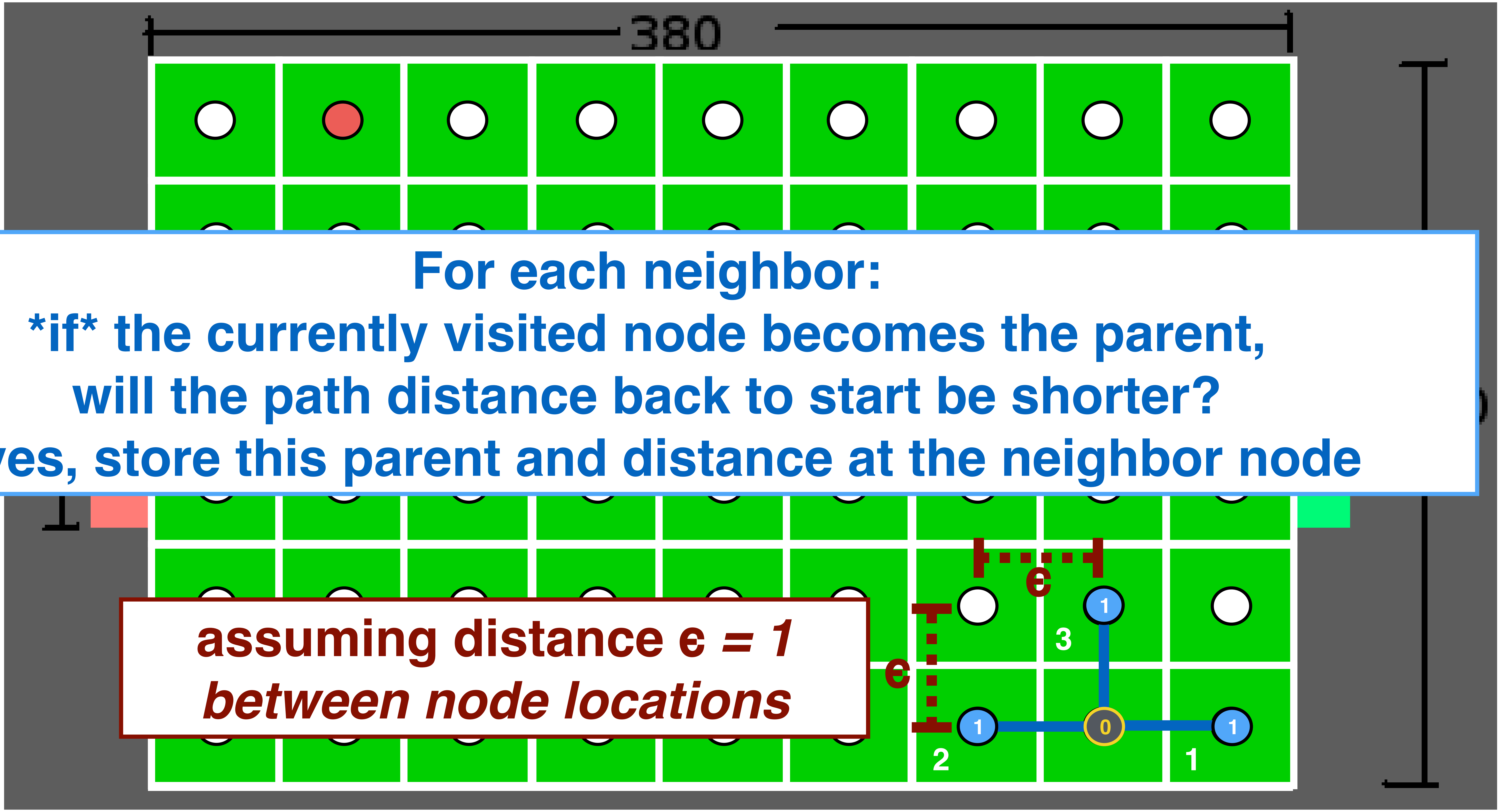
Depth-first search



Depth-first search



Depth-first search

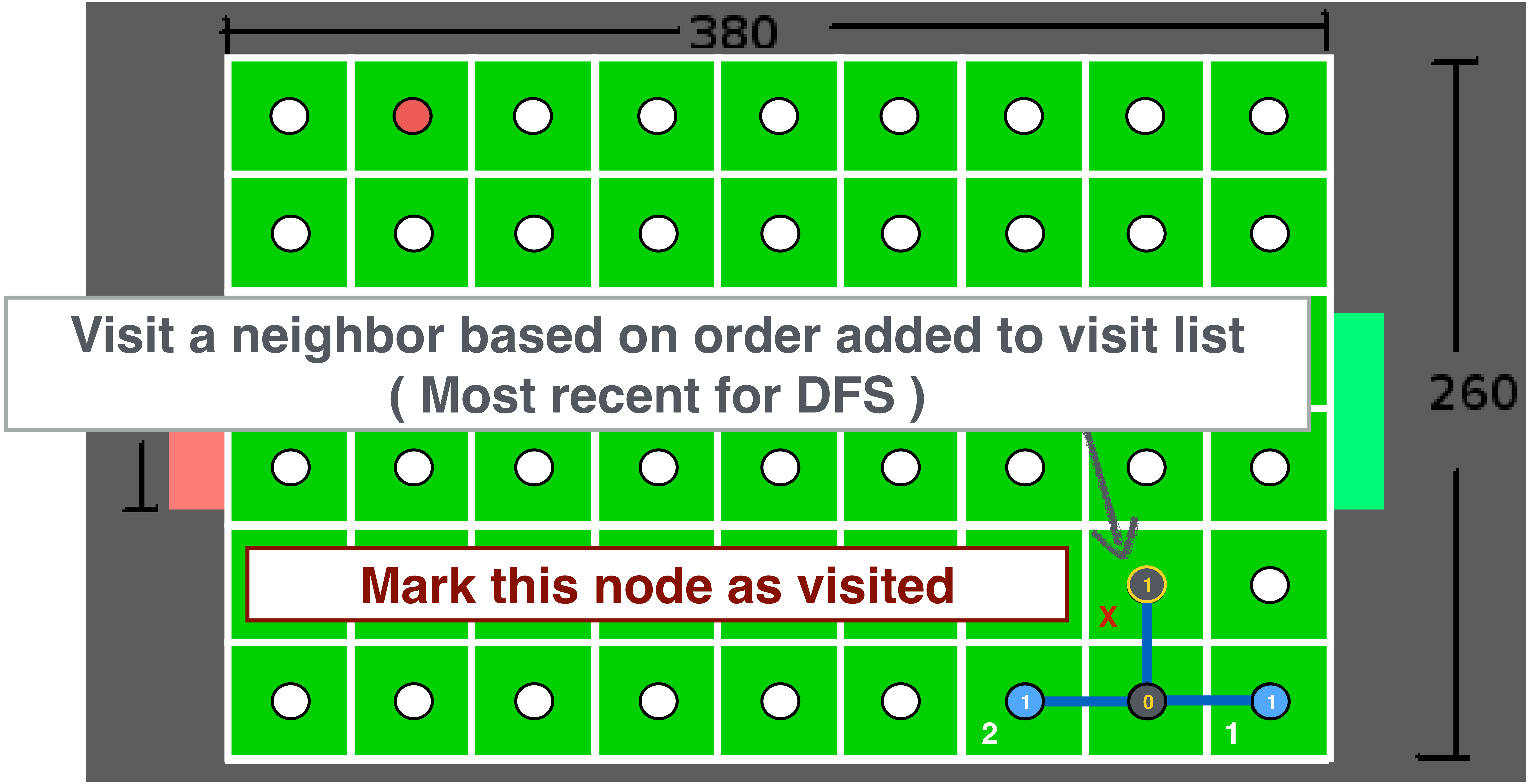


For each neighbor:
if the currently visited node becomes the parent,
will the path distance back to start be shorter?
if yes, store this parent and distance at the neighbor node

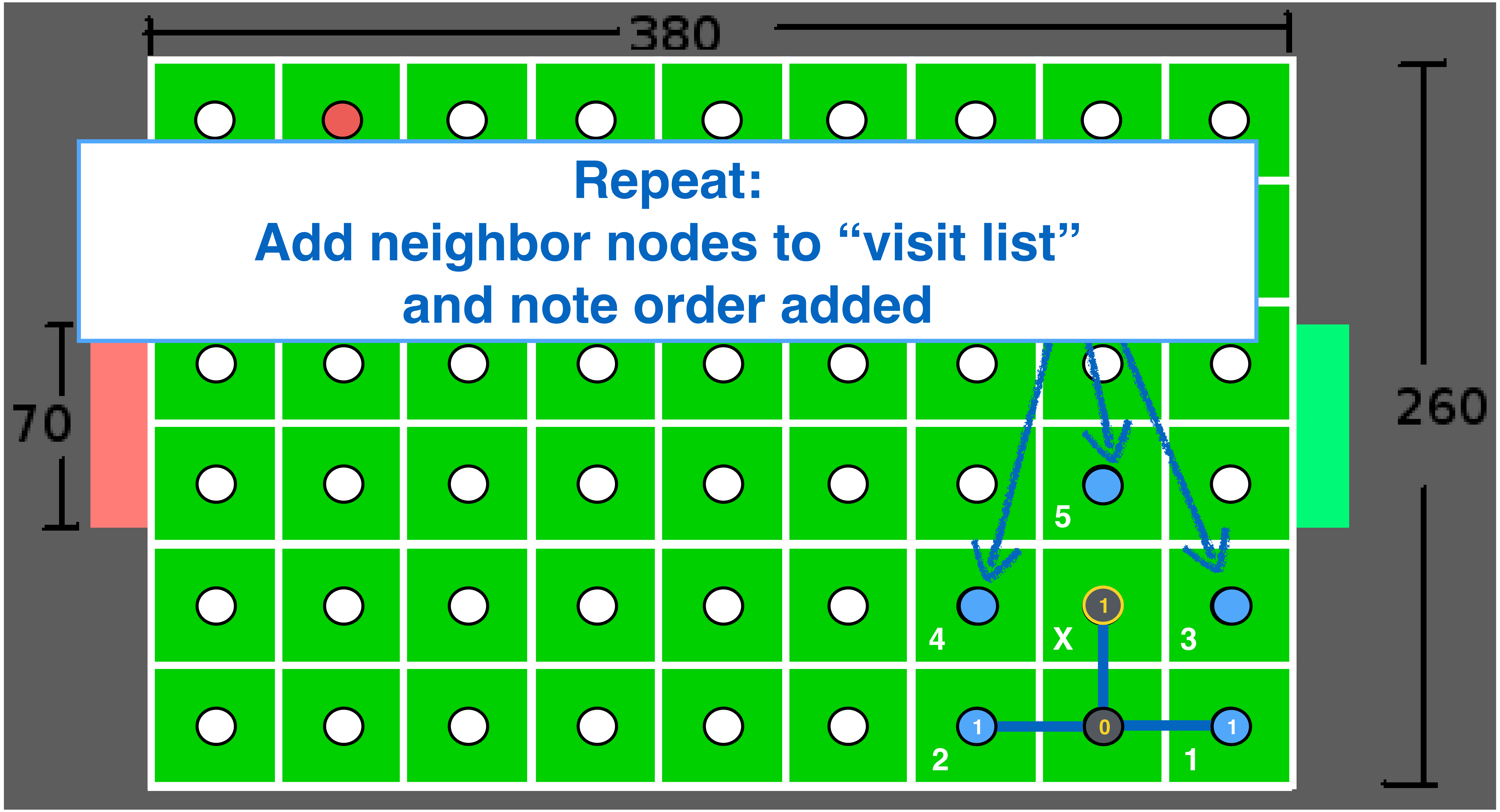
assuming distance ϵ = 1
between node locations

Depth-first search

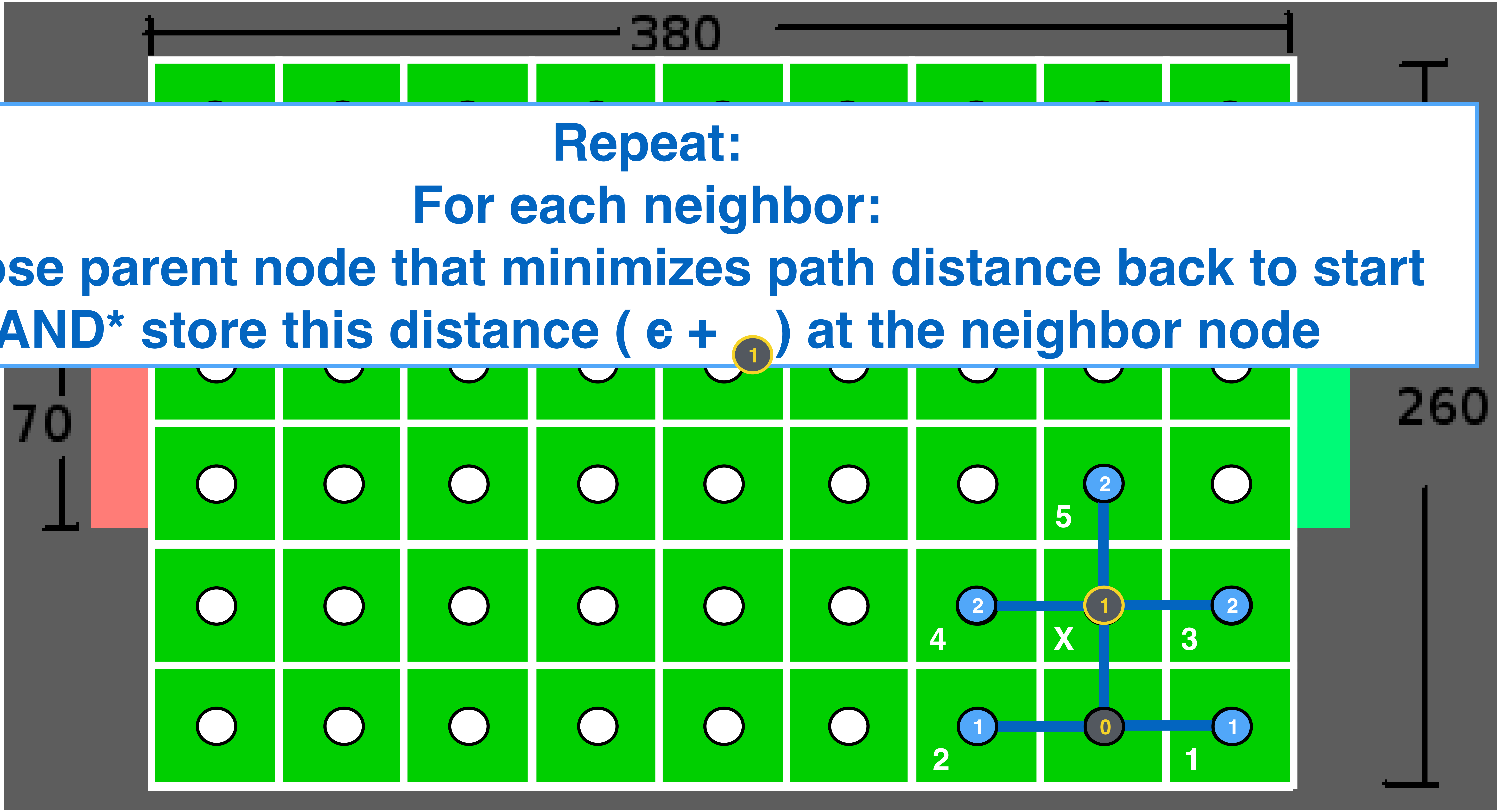
Depth-first search



Depth-first search

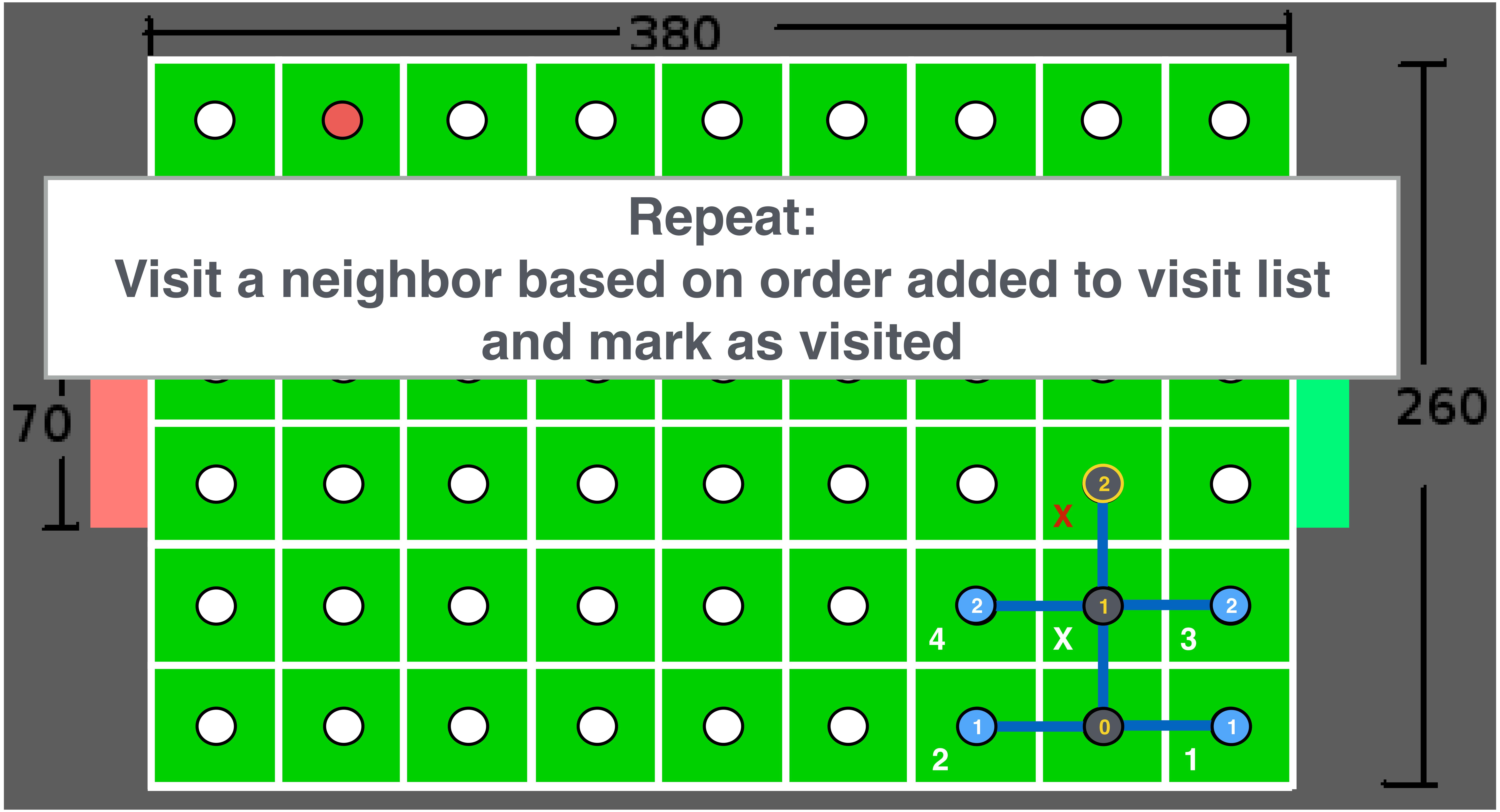


Depth-first search



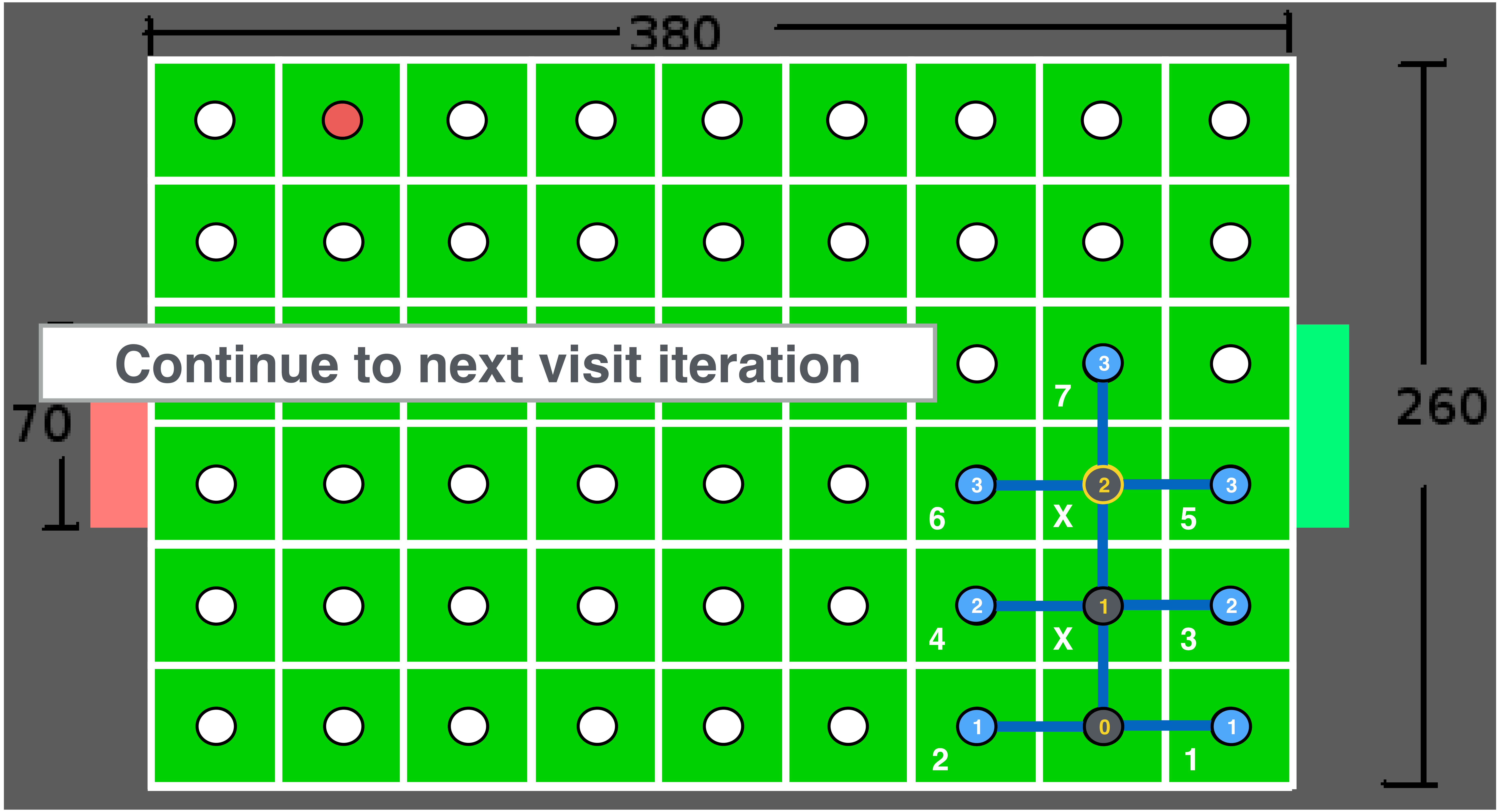
Repeat:
For each neighbor:
choose parent node that minimizes path distance back to start
***AND* store this distance (ϵ + 1) at the neighbor node**

Depth-first search



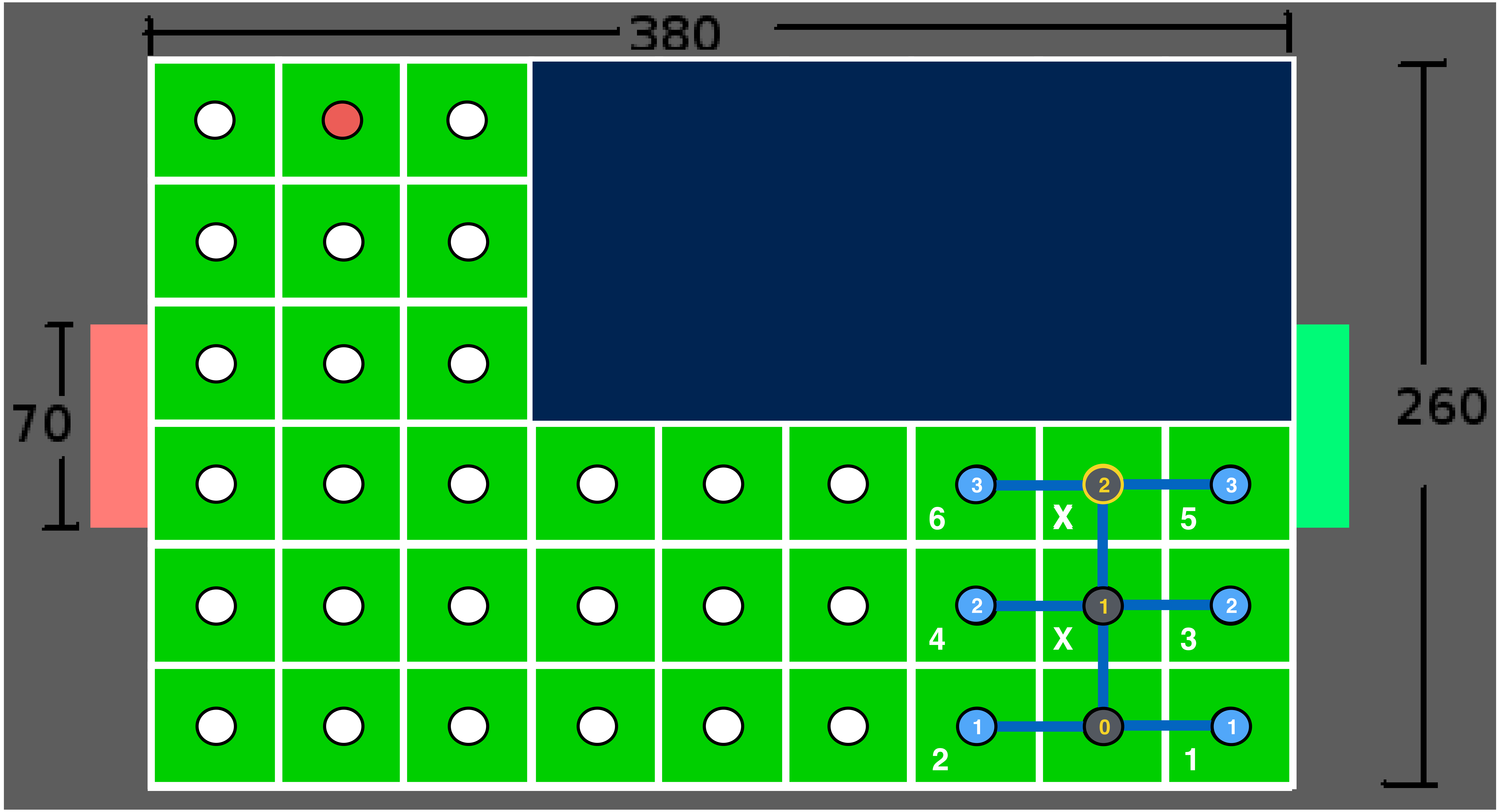
Repeat:
Visit a neighbor based on order added to visit list and mark as visited

Depth-first search

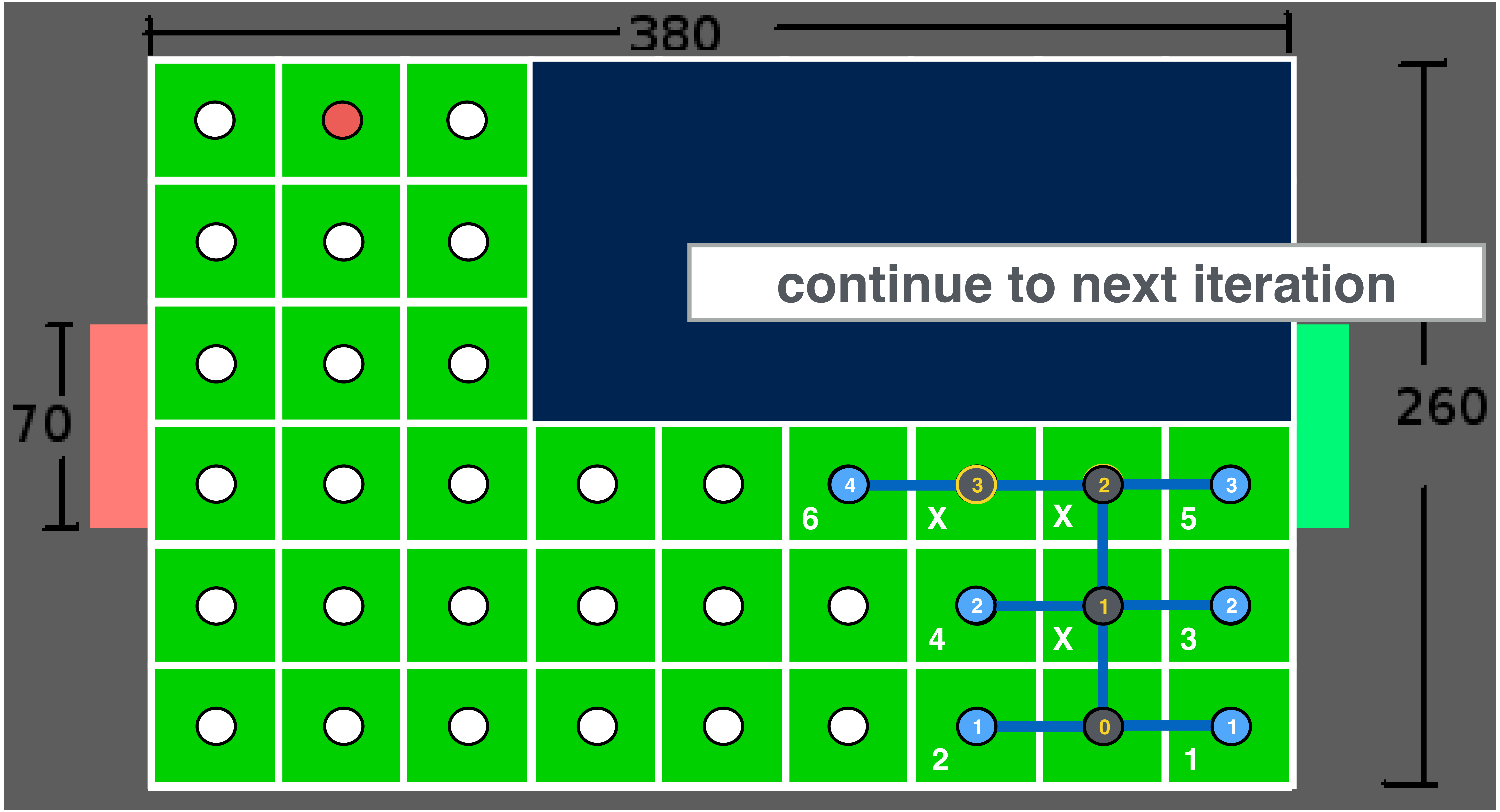


Depth-first search

Depth-first search

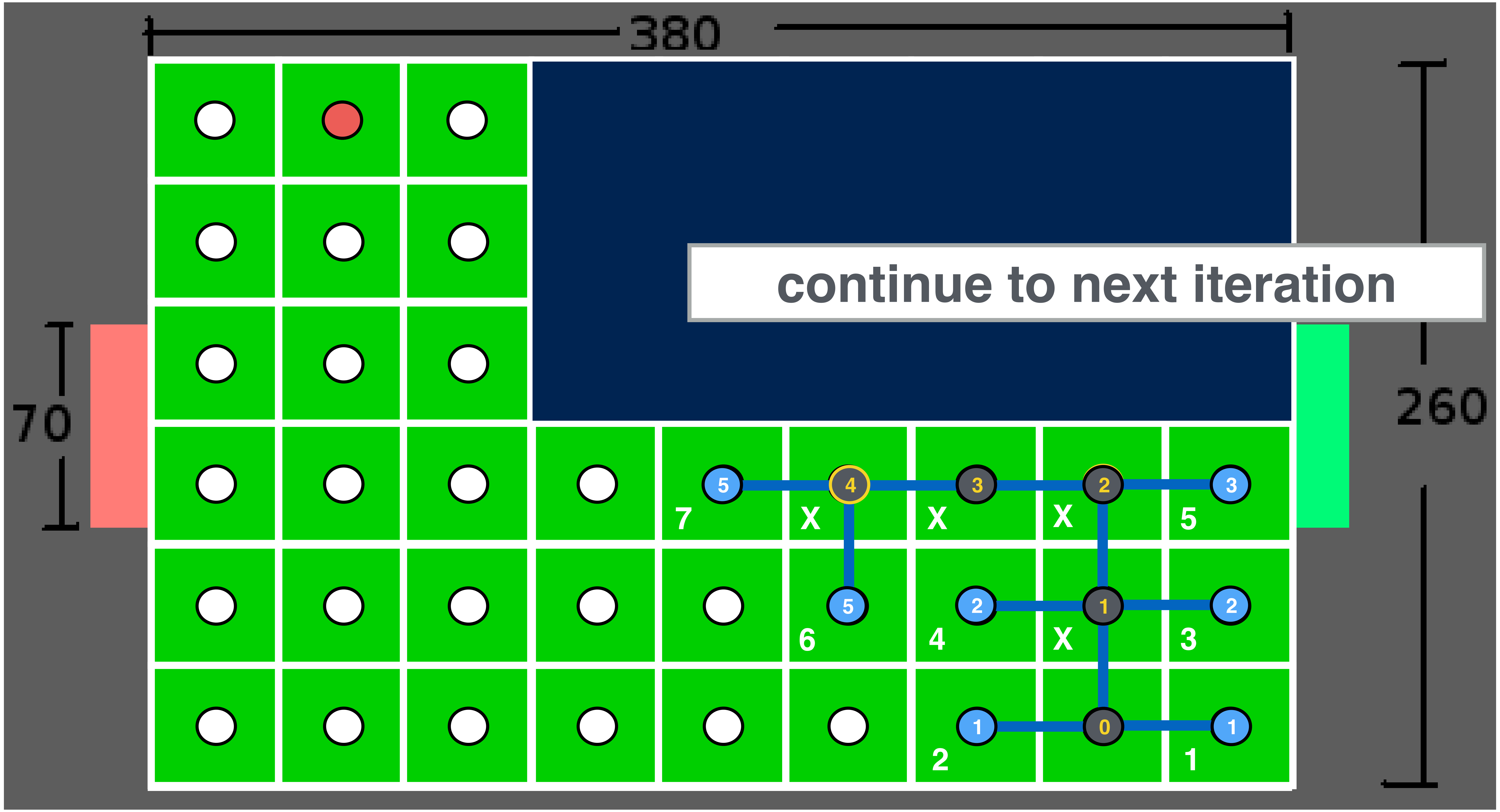


Depth-first search

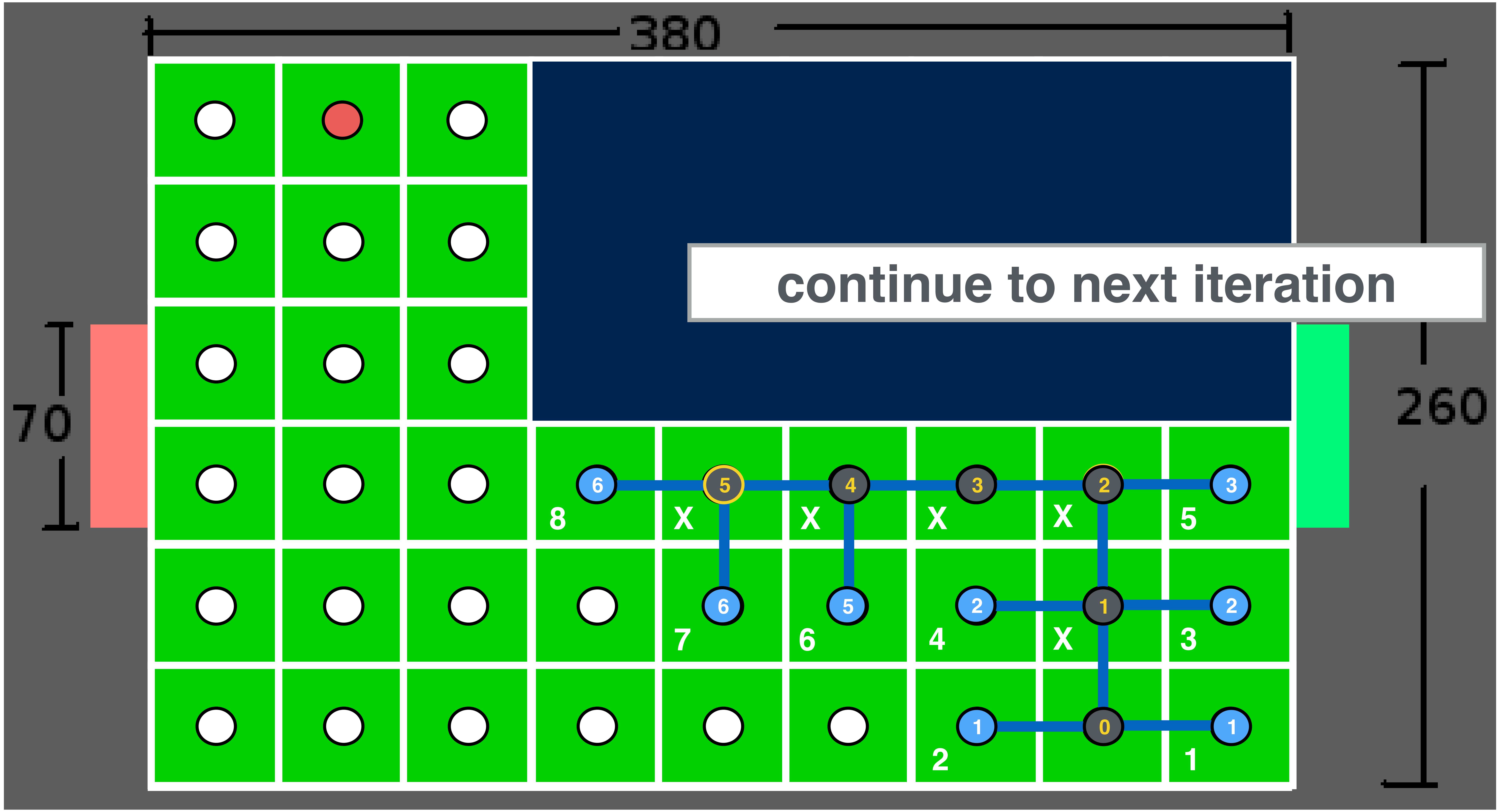


Depth-first search

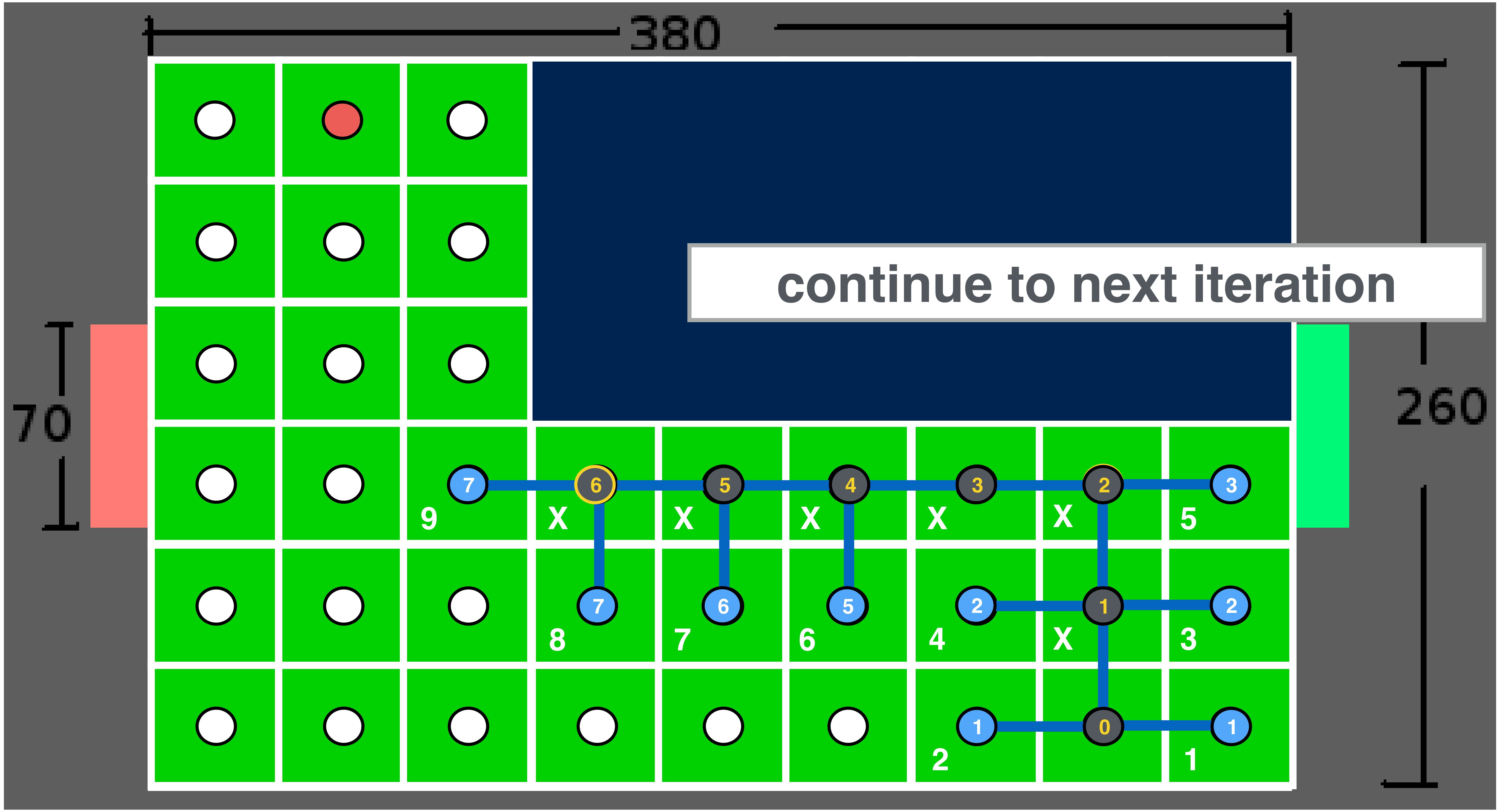
Depth-first search



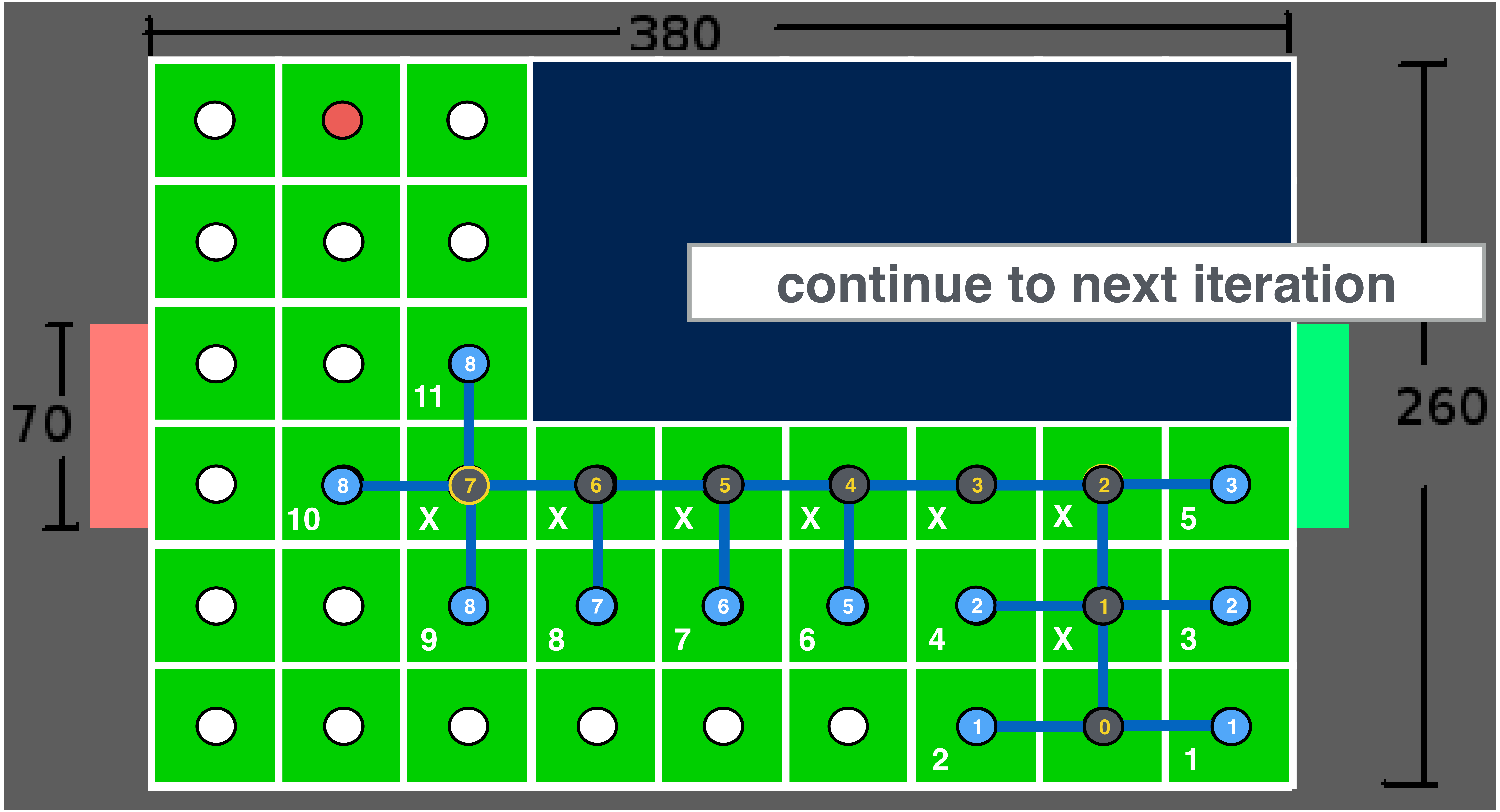
Depth-first search



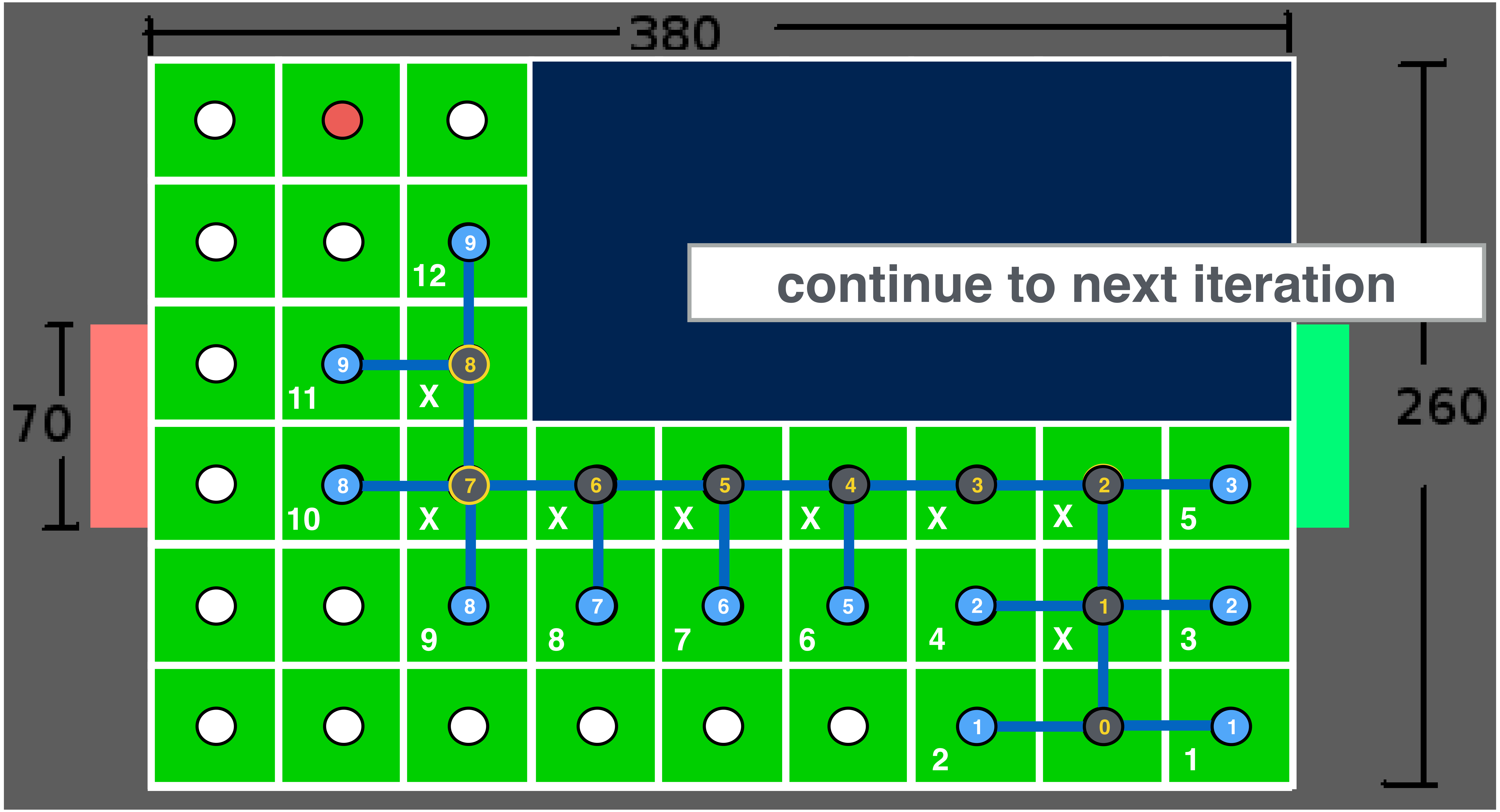
Depth-first search



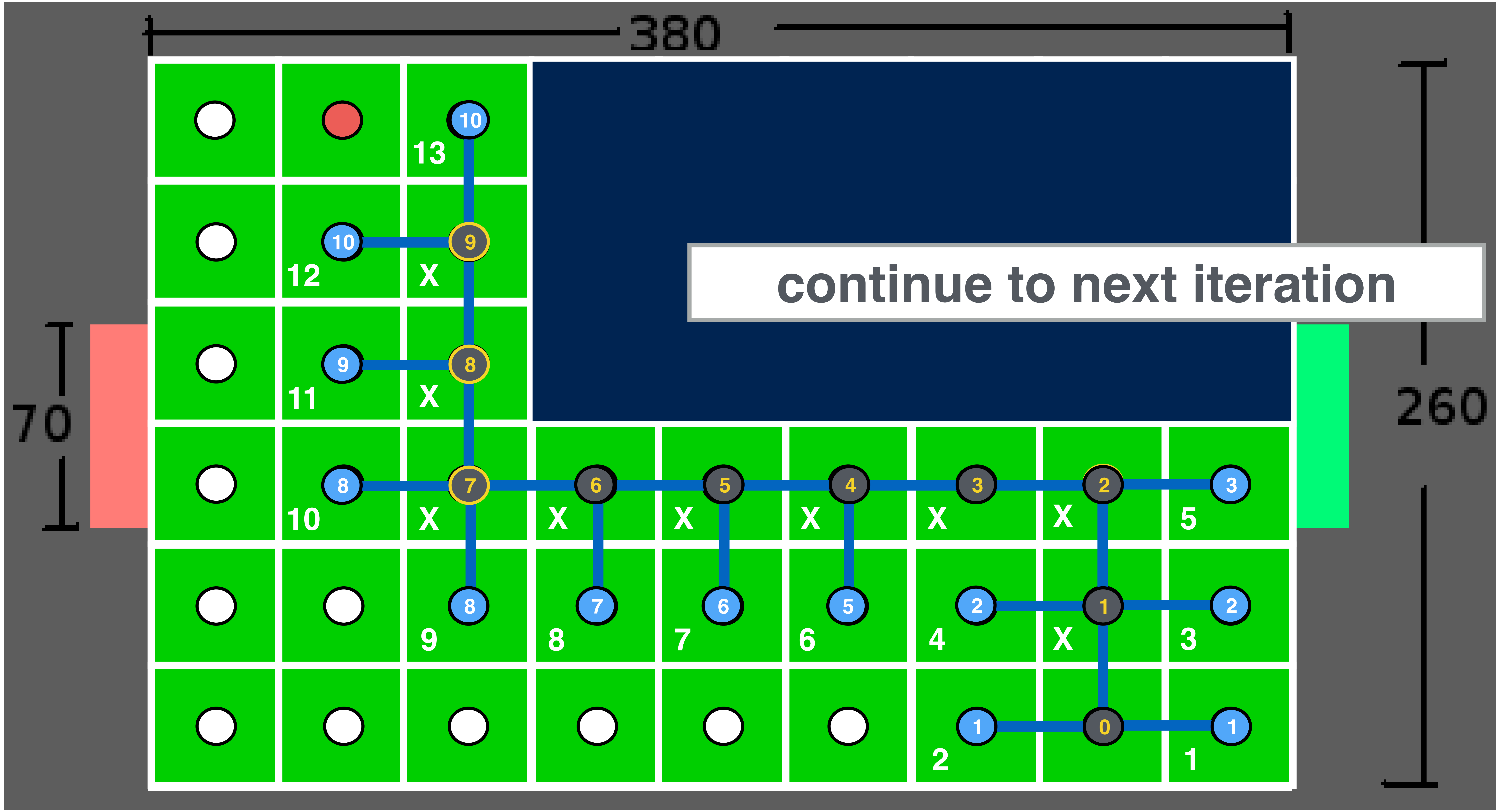
Depth-first search



Depth-first search

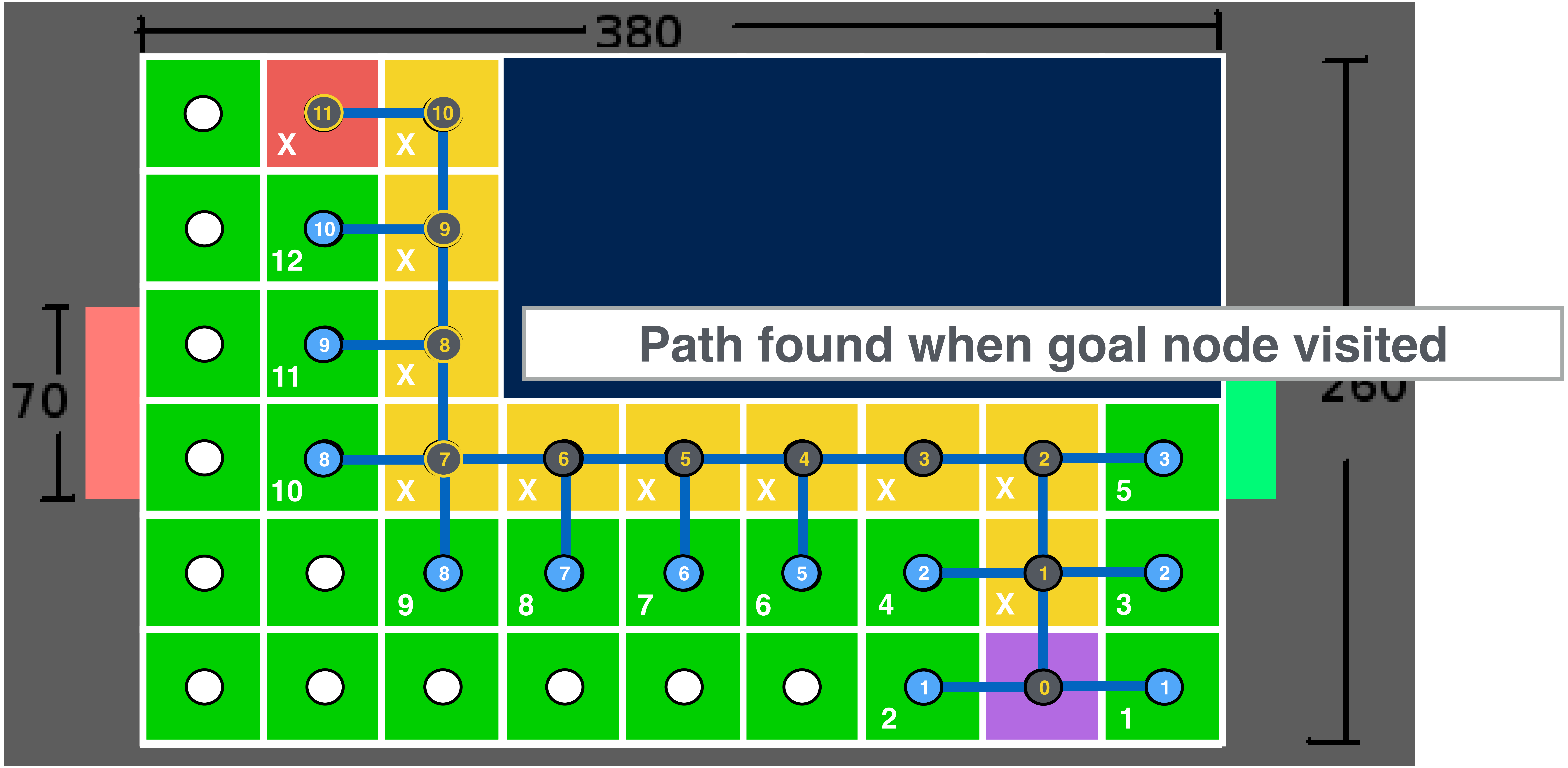


Depth-first search



Depth-first search

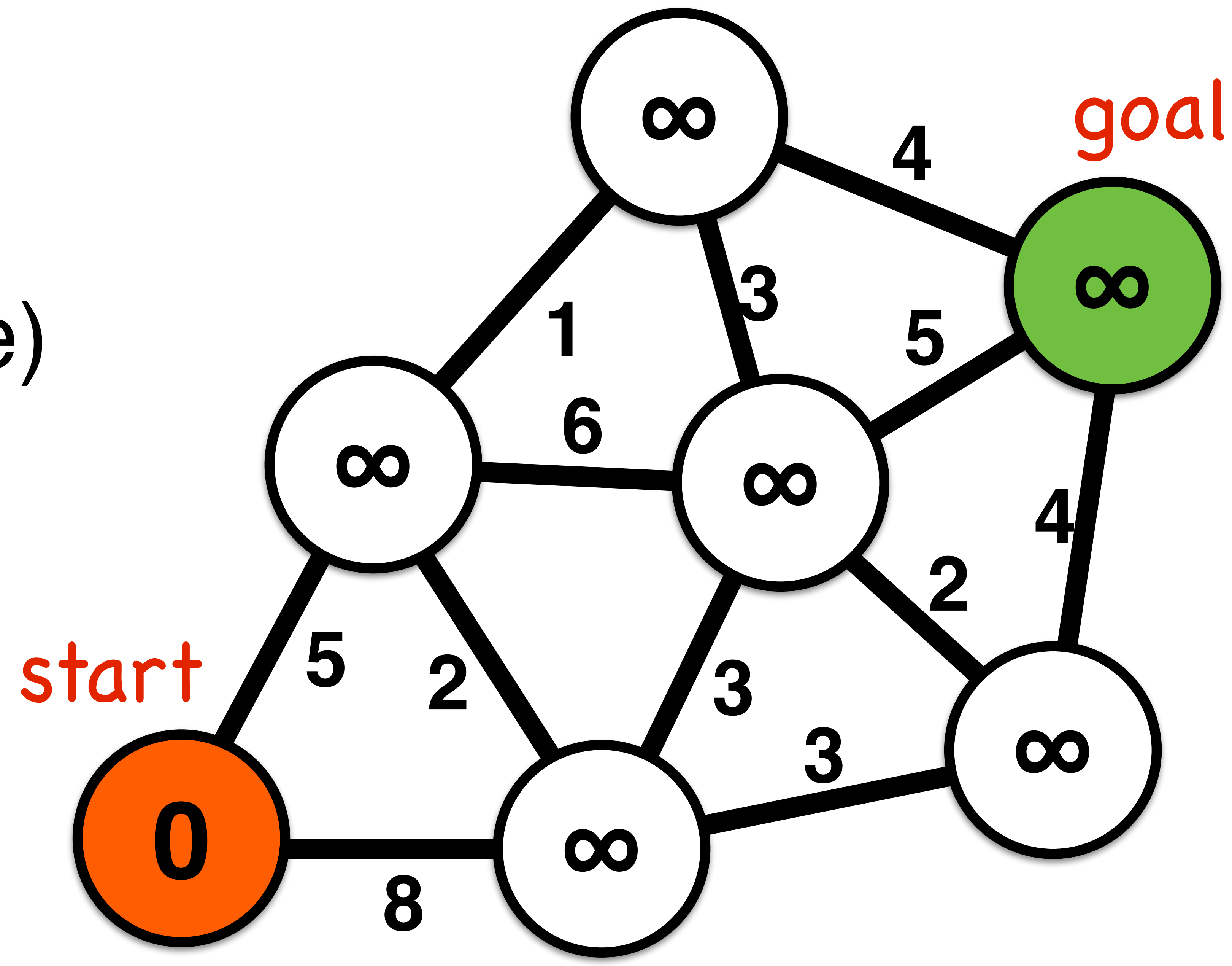
Depth-first search



Let's turn this idea into code

Search algorithm template

```
all nodes ← {dist_start ← infinity, parent_start ← none, visited_start ← false}
start_node ← {dist_start ← 0, parent_start ← none, visited_start ← true}
visit_list ← start_node
    while visit_list != empty && current_node != goal
      cur_node ← highestPriority(visit_list)
      visited_cur_node ← true
      for each nbr in not_visited(adjacent(cur_node))
        add(nbr to visit_list)
        if dist_nbr > dist_cur_node + distStraightLine(nbr,cur_node)
          parent_nbr ← current_node
          dist_nbr ← dist_cur_node + distStraightLine(nbr,cur_node)
        end if
      end for loop
    end while loop
output ← parent, distance
```



Search algorithm template

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_list ← start_node

while visit_list != empty && current_node != goal

Initialization

- each node has a distance and a parent
  - distance: distance along route from start
  - parent: routing from node to start
- visit a chosen start node first
- all other nodes are unvisited and have high distance

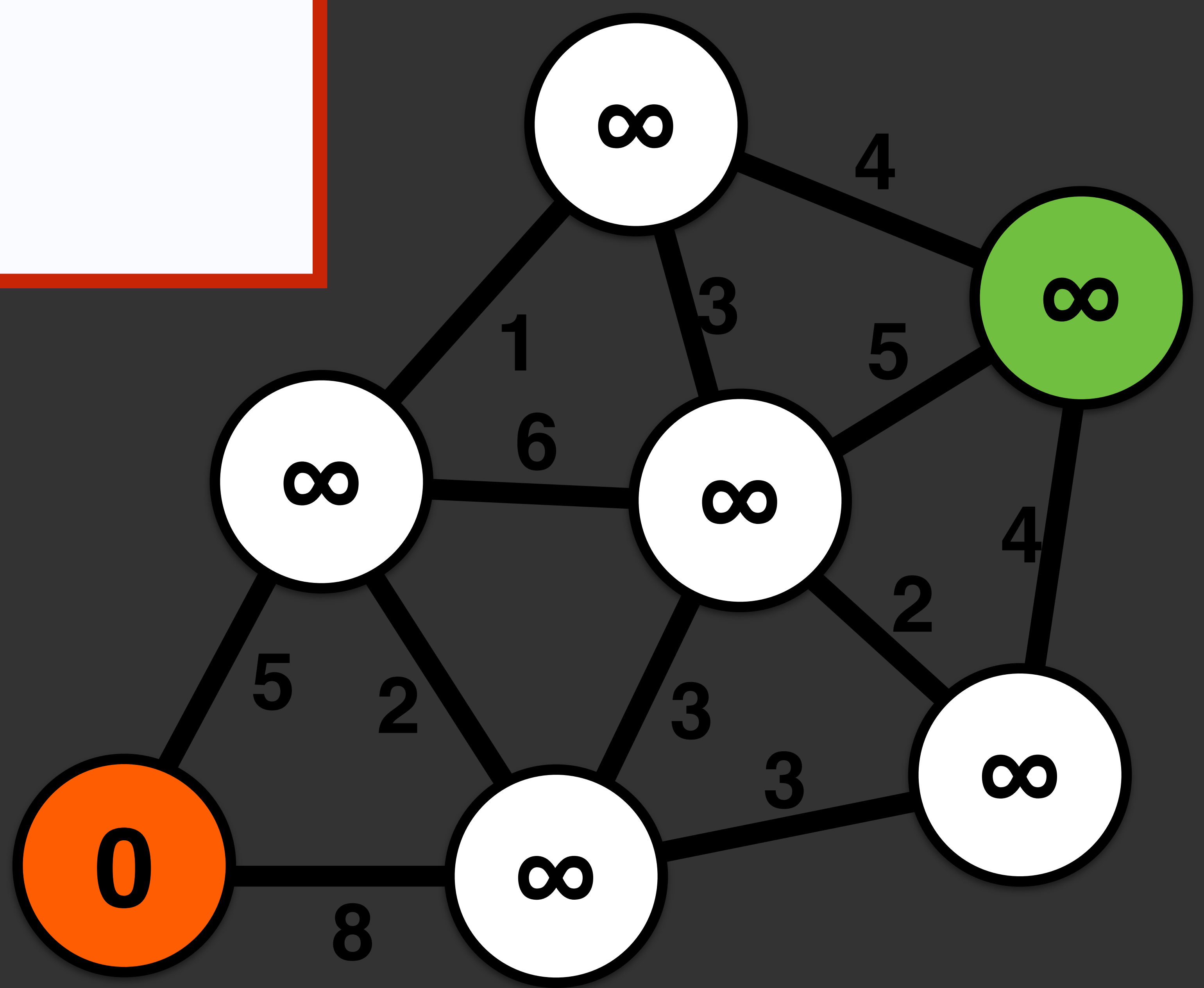
$dist_{nbr}$ ← $dist_{cur\_node}$ + distStraightLine(nbr,cur_node)

end if

end for loop

end while loop

output ← parent, distance



Search algorithm template

all nodes ← {$\text{dist}_{\text{start}}$ ← infinity, $\text{parent}_{\text{start}}$ ← none, $\text{visited}_{\text{start}}$ ← false}

start_node ← {$\text{dist}_{\text{start}}$ ← 0, $\text{parent}_{\text{start}}$ ← none, $\text{visited}_{\text{start}}$ ← true}

visit_list ← start_node

    while visit_list != empty && current_node != goal

        cur_node ← highestPriority(visit_list)

        $\text{visited}_{\text{cur\_node}}$ ← true

        for each nbr in not_visited(adjacent(cur_node))

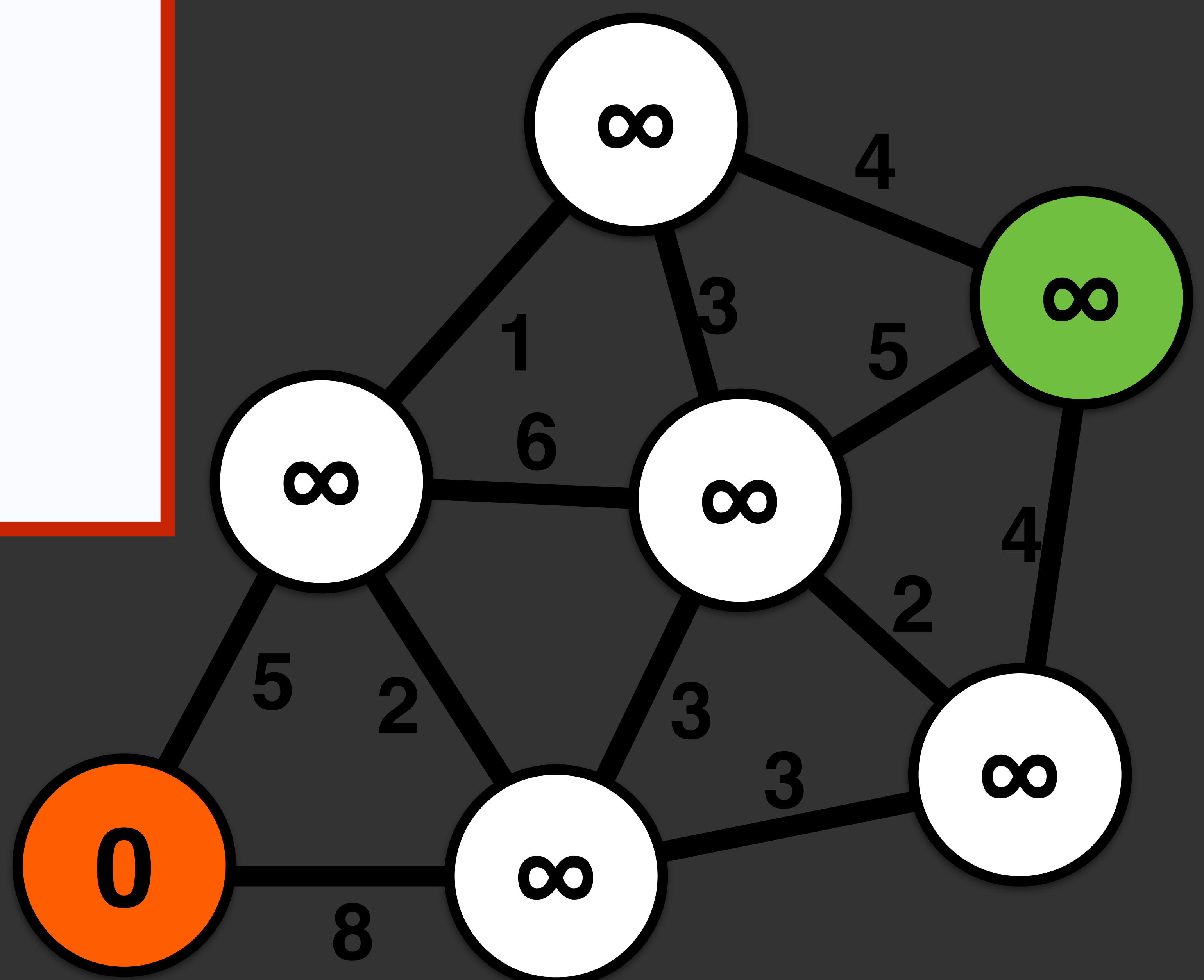
Main Loop

- visits every node to compute its distance and parent
- at each iteration:
  - select the node to visit based on its priority
  - remove current node from visit_list

        end for loop

    end while loop

output ← parent, distance



Search algorithm template

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_list ← start_node

while visit_list != empty && current_node != goal

cur_node ← highestPriority(visit_list)

$visited_{cur\_node}$ ← true

for each nbr in not_visited(adjacent(cur_node))

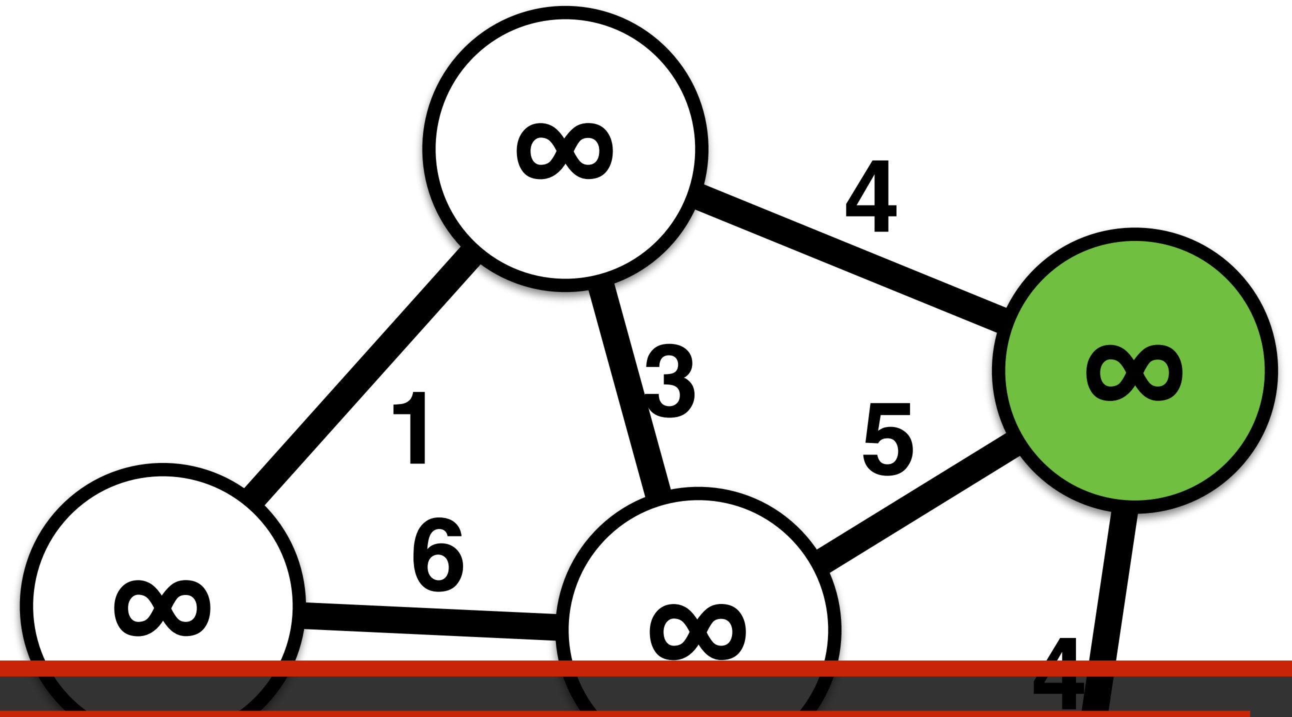
add(nbr to visit_list)

if $dist_{nbr}$ > $dist_{cur\_node}$ + distStraightLine(nbr,cur_node)

$parent_{nbr}$ ← current_node

$dist_{nbr}$ ← $dist_{cur\_node}$ + distStraightLine(nbr,cur_node)

end if



For each iteration on a single node

- add all unvisited neighbors of the node to the visit list
- assign node as a parent to a neighbor, if it creates a shorter route

Search algorithm template

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

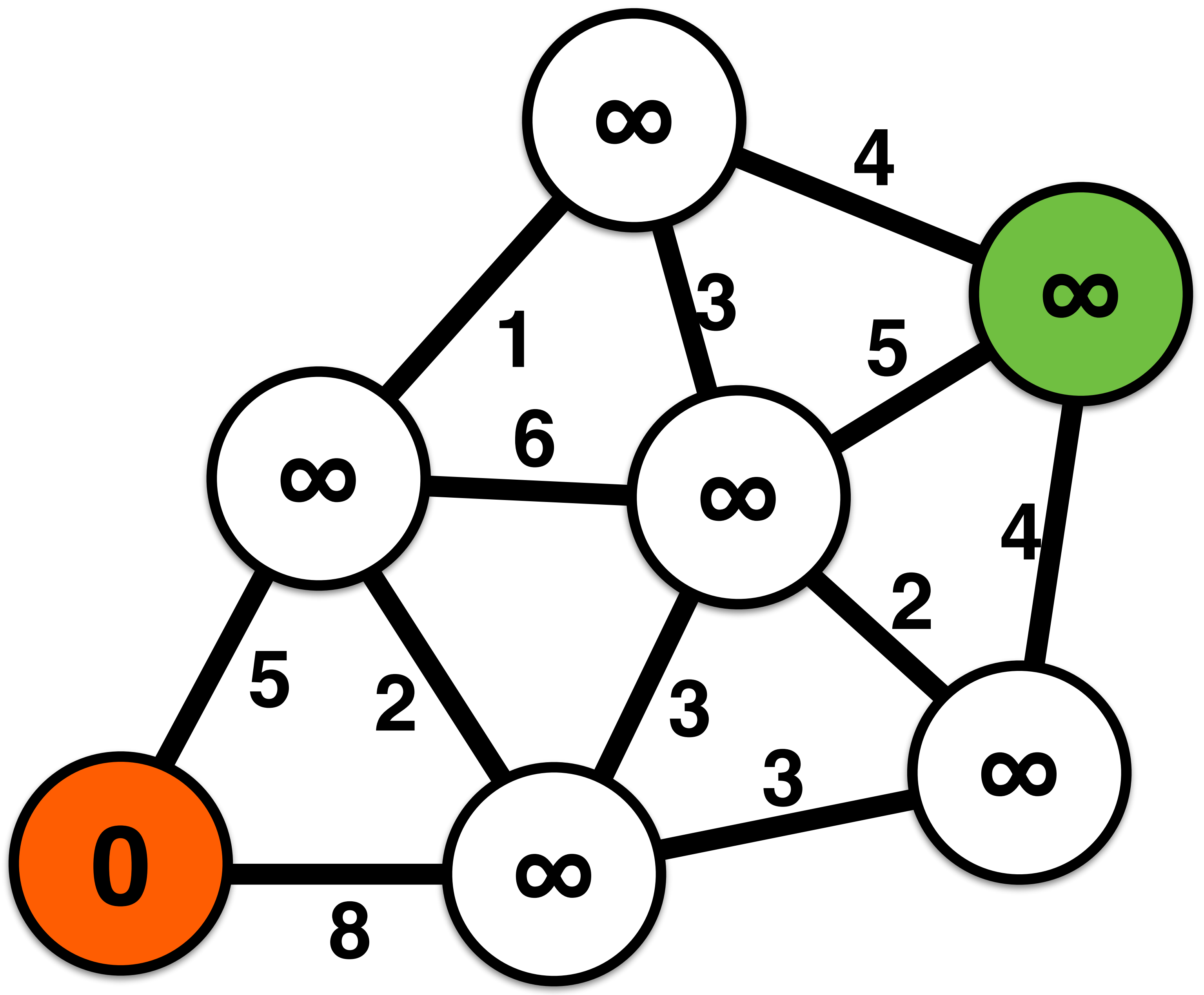
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_list ← start_node

```
    while visit_list != empty && current_node != goal
        cur_node ← highestPriority(visit_list)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
                parent_nbr ← current_node
                dist_nbr ← dist_cur_node + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
```

output ← parent, distance

Output the resulting routing and path distance at each node



Depth-first search

Search algorithm template

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_list ← start_node

while visit_list != empty && current_node != goal

cur_node ← **highestPriority**(visit_list)

$visited_{cur\_node}$ ← true

for each nbr in not_visited(adjacent(cur_node))

add(nbr to visit_list)

if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)

$parent_{nbr}$ ← current_node

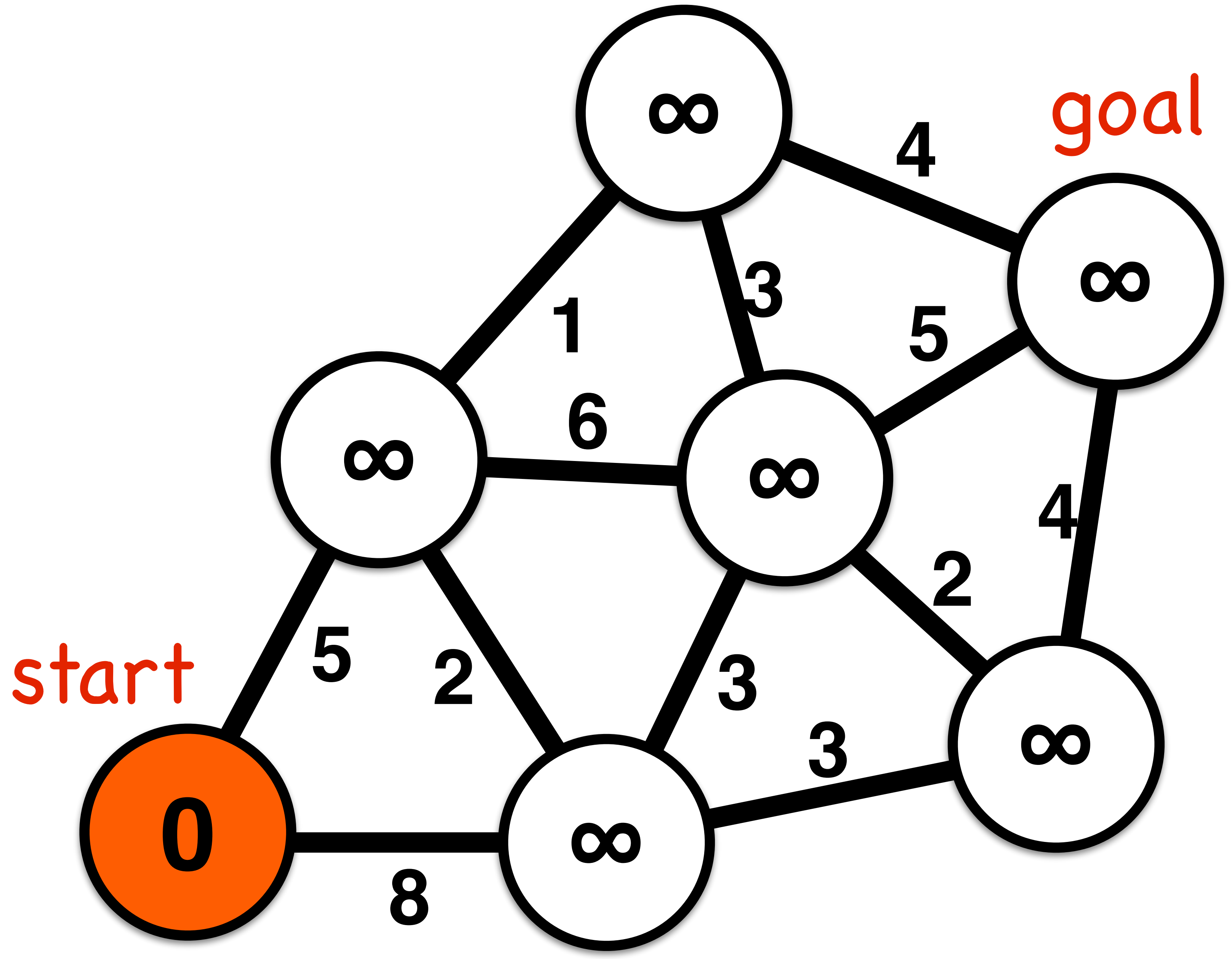
$dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

end if

end for loop

end while loop

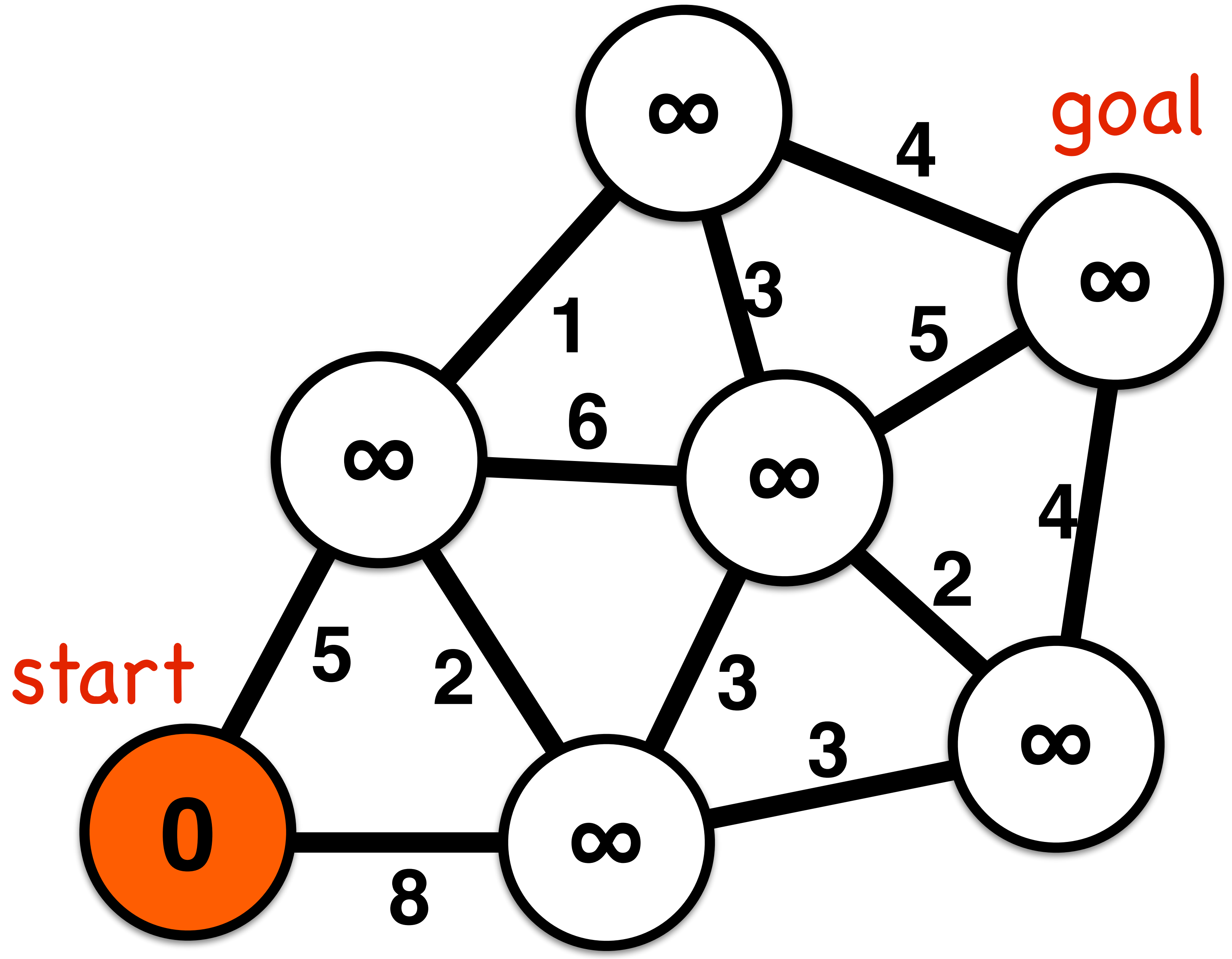
output ← parent, distance



Depth-first search

```
all nodes ← {dist_start ← infinity, parent_start ← none, visited_start ← false}
start_node ← {dist_start ← 0, parent_start ← none, visited_start ← true}
visit_stack ← start_node
    while visit_stack != empty && current_node != goal
        cur_node ← pop(visit_stack)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            push(nbr to visit_stack)
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
                parent_nbr ← current_node
                dist_nbr ← dist_cur_node + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance
```

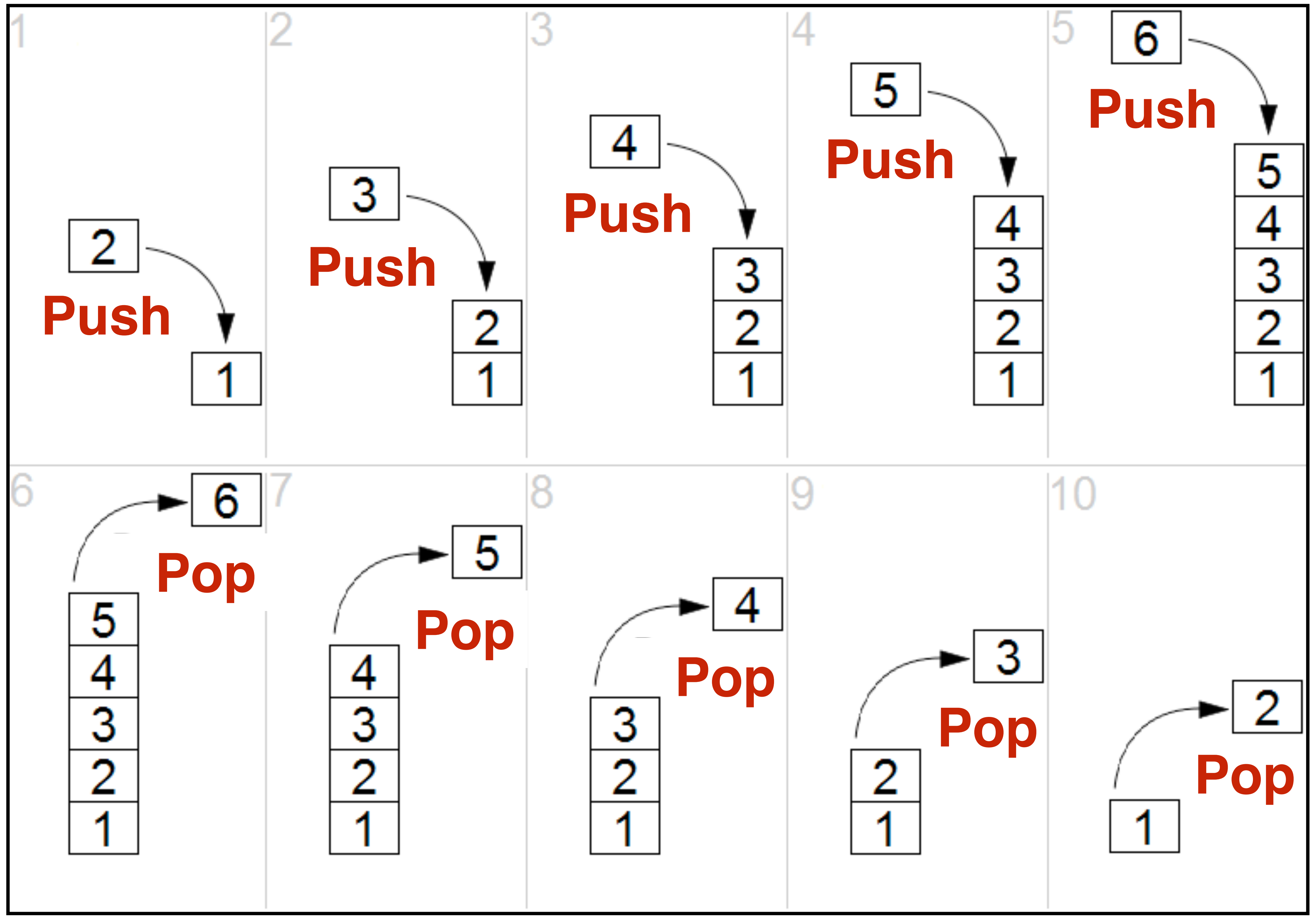
Priority:
Most recent

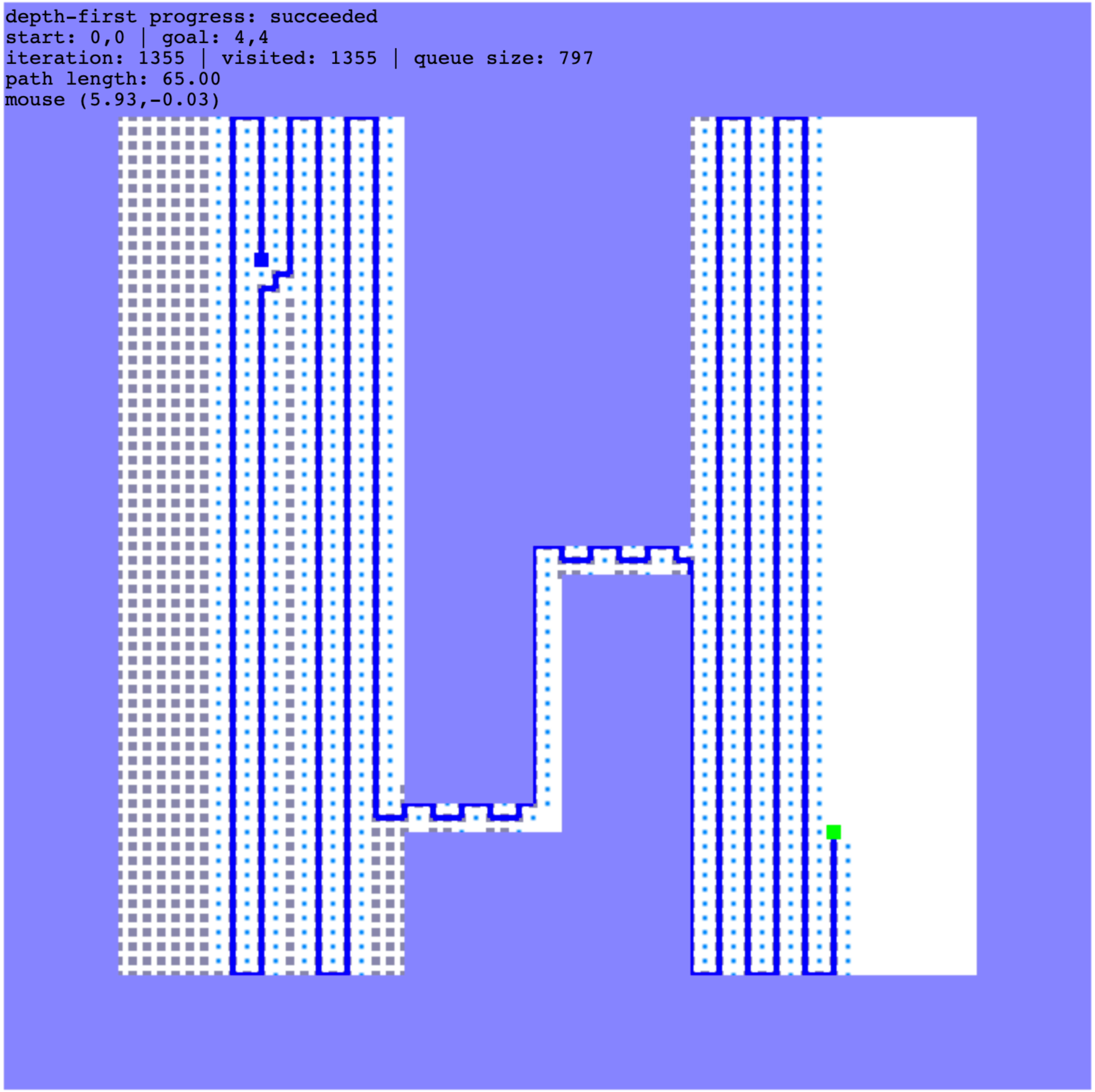


Stack data structure

A stack is a "last in, first out" (or LIFO) structure, with two operations:
push: to add an element to the top of the stack
pop: to remove and element from the top of the stack

Stack example for reversing the order of six elements

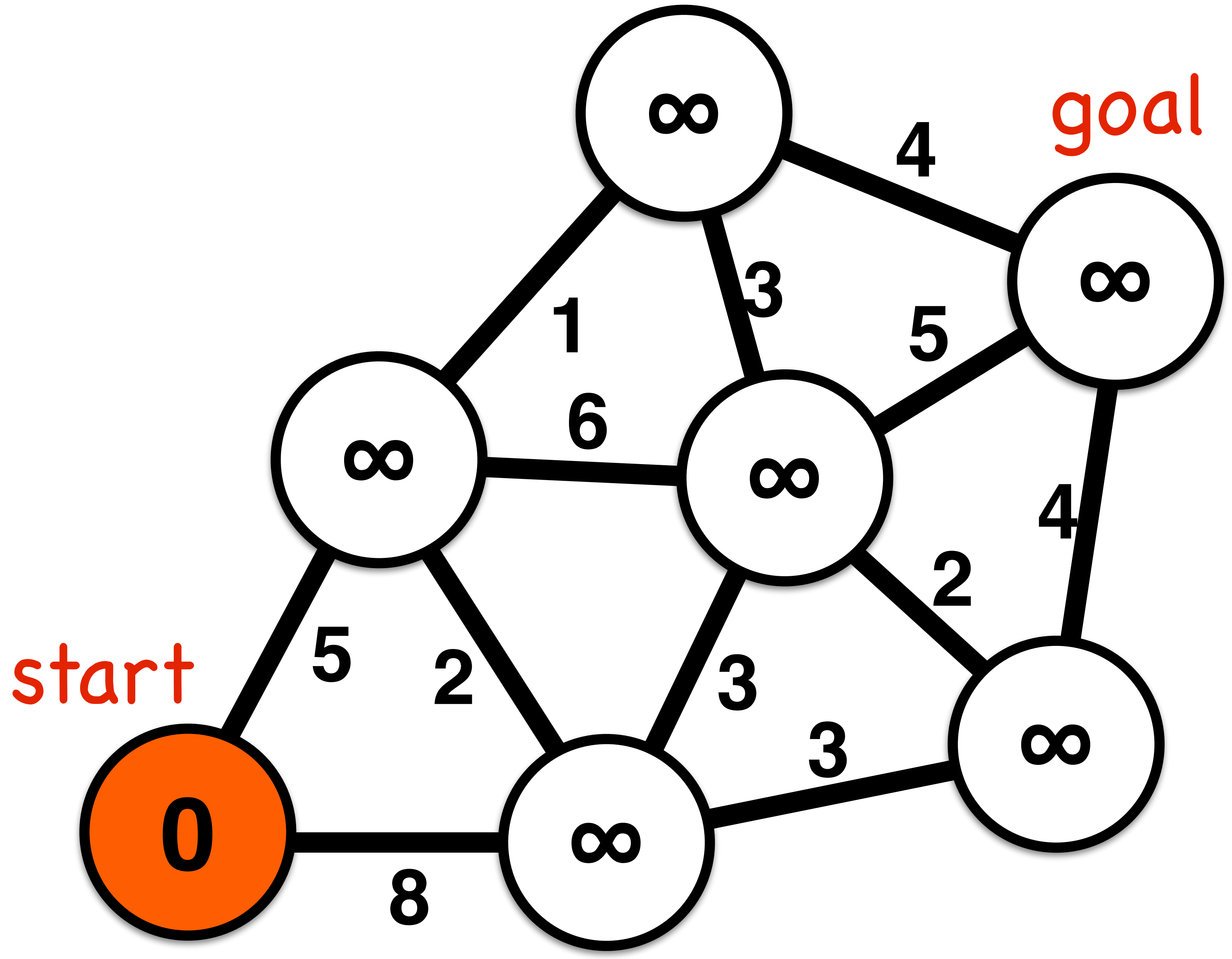



depth-first progress: succeeded
start: 0,0 | goal: 4,4
iteration: 1355 | visited: 1355 | queue size: 797
path length: 65.00
mouse (5.93,-0.03)

Breadth-first search

Search algorithm template

```
all nodes ← {dist_start ← infinity, parent_start ← none, visited_start ← false}
start_node ← {dist_start ← 0, parent_start ← none, visited_start ← true}
visit_list ← start_node
    while visit_list != empty && current_node != goal
        cur_node ← highestPriority(visit_list)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
                parent_nbr ← current_node
                dist_nbr ← dist_cur_node + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance
```



Breadth-first search

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

    while visit_queue != empty && current_node != goal

        cur_node ← **dequeue**(**visit_queue**) ← Priority: Least recent

        $visited_{cur\_node}$ ← true

        for each nbr in not_visited(adjacent(cur_node))

            enqueue(nbr to **visit_queue**)

            if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)

                $parent_{nbr}$ ← current_node

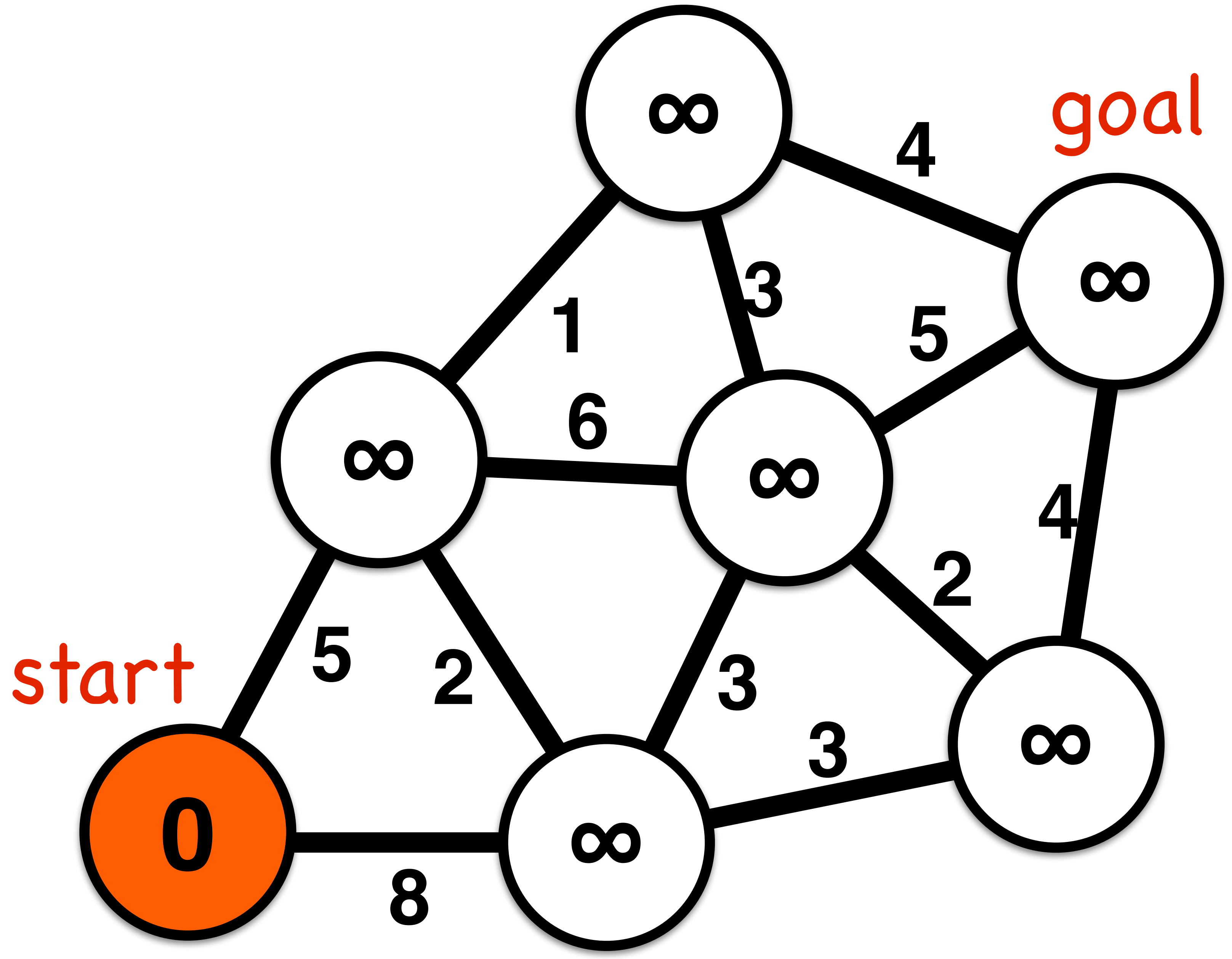
                $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

            end if

        end for loop

    end while loop

output ← parent, distance

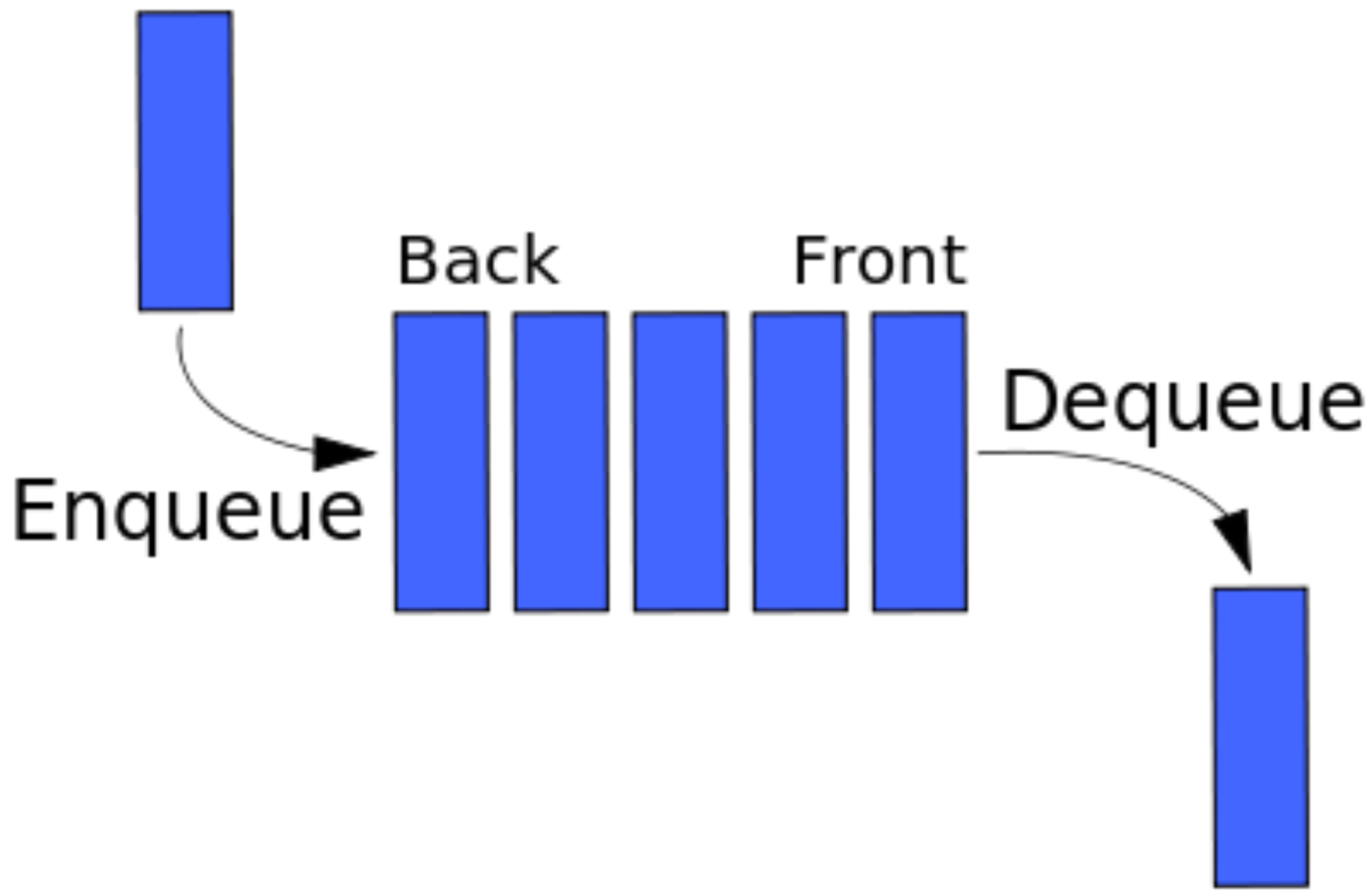


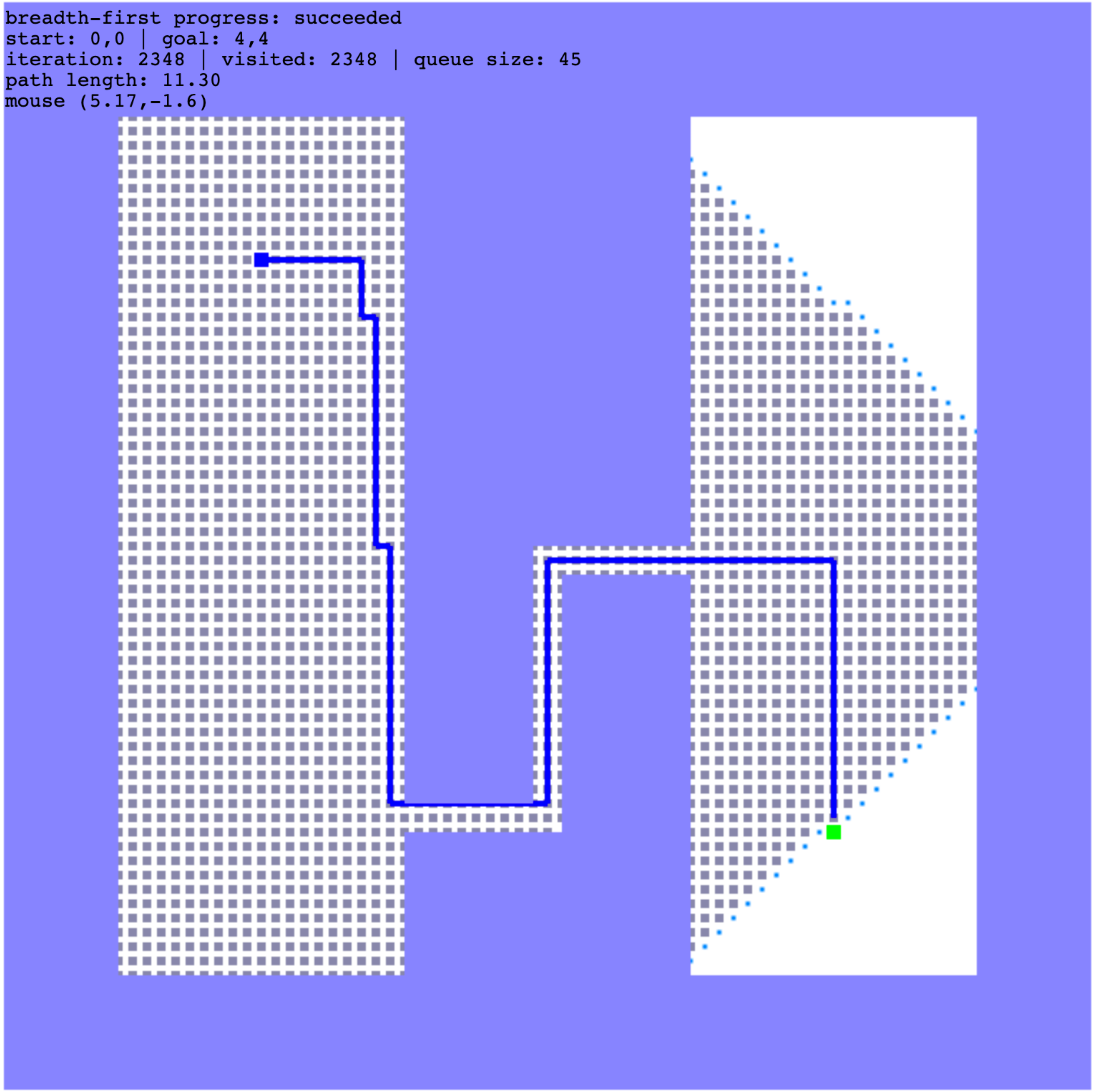
Queue data structure

A queue is a "first in, first out" (or FIFO) structure, with two operations

enqueue: to add an element to the back of the stack

dequeue: to remove an element from the front of the stack

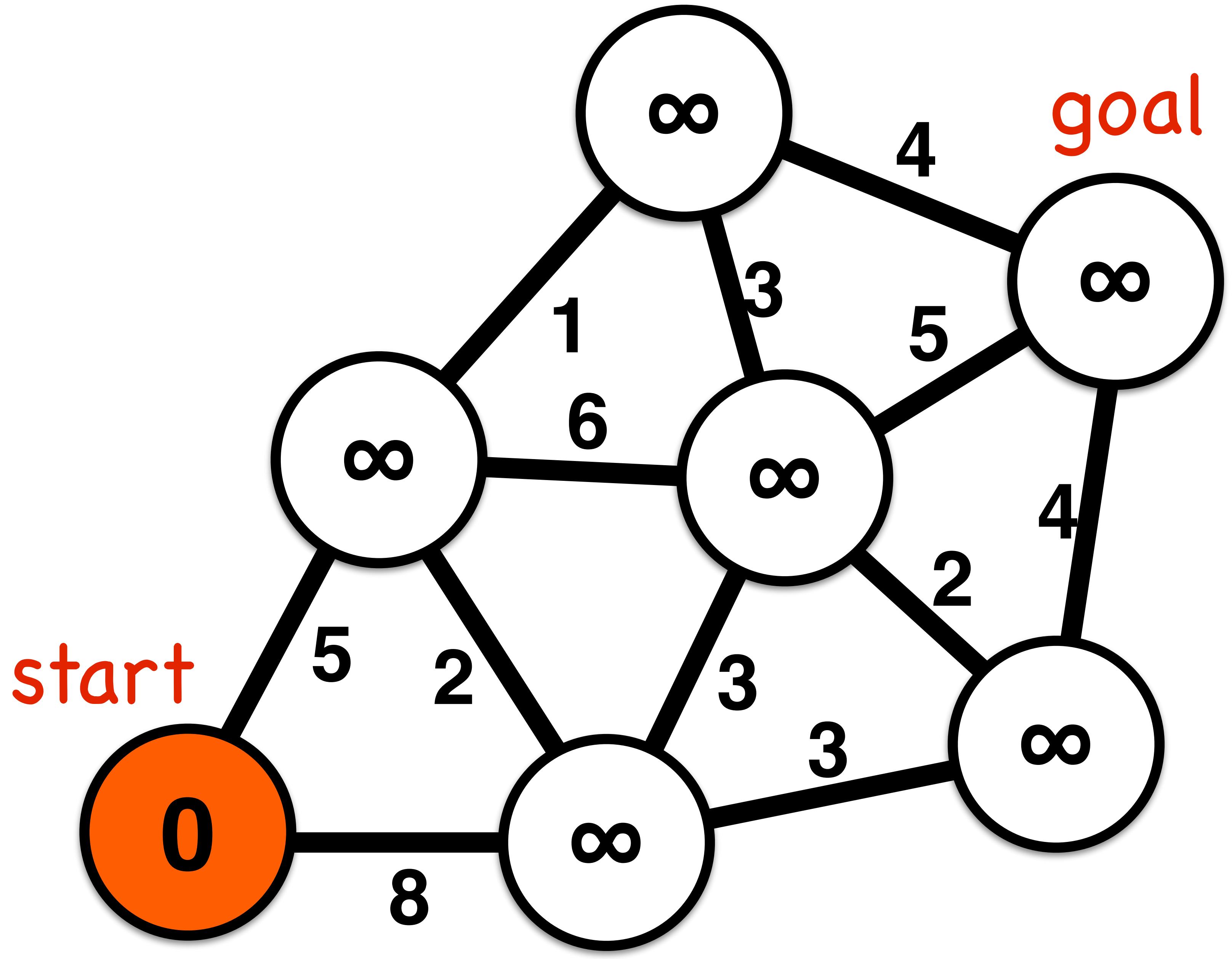



breadth-first progress: succeeded
start: 0,0 | goal: 4,4
iteration: 2348 | visited: 2348 | queue size: 45
path length: 11.30
mouse (5.17,-1.6)

Dijkstra’s algorithm

Search algorithm template

```
all nodes ← {dist_start ← infinity, parent_start ← none, visited_start ← false}
start_node ← {dist_start ← 0, parent_start ← none, visited_start ← true}
visit_list ← start_node
    while visit_list != empty && current_node != goal
        cur_node ← highestPriority(visit_list)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            add(nbr to visit_list)
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
                parent_nbr ← current_node
                dist_nbr ← dist_cur_node + distance(nbr,cur_node)
            end if
        end for loop
    end while loop
output ← parent, distance
```



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

    while visit_queue != empty ~~**&& current_node != goal**~~

      cur_node ← **min_distance**(**visit_queue**)

      $visited_{cur\_node}$ ← true

      for each nbr in not_visited(adjacent(cur_node))

        if $dist_{nbr} > dist_{cur\_node}$ + distance(nbr,cur_node)

          $parent_{nbr}$ ← current_node

          $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

          enqueue(nbr to **visit_queue**)

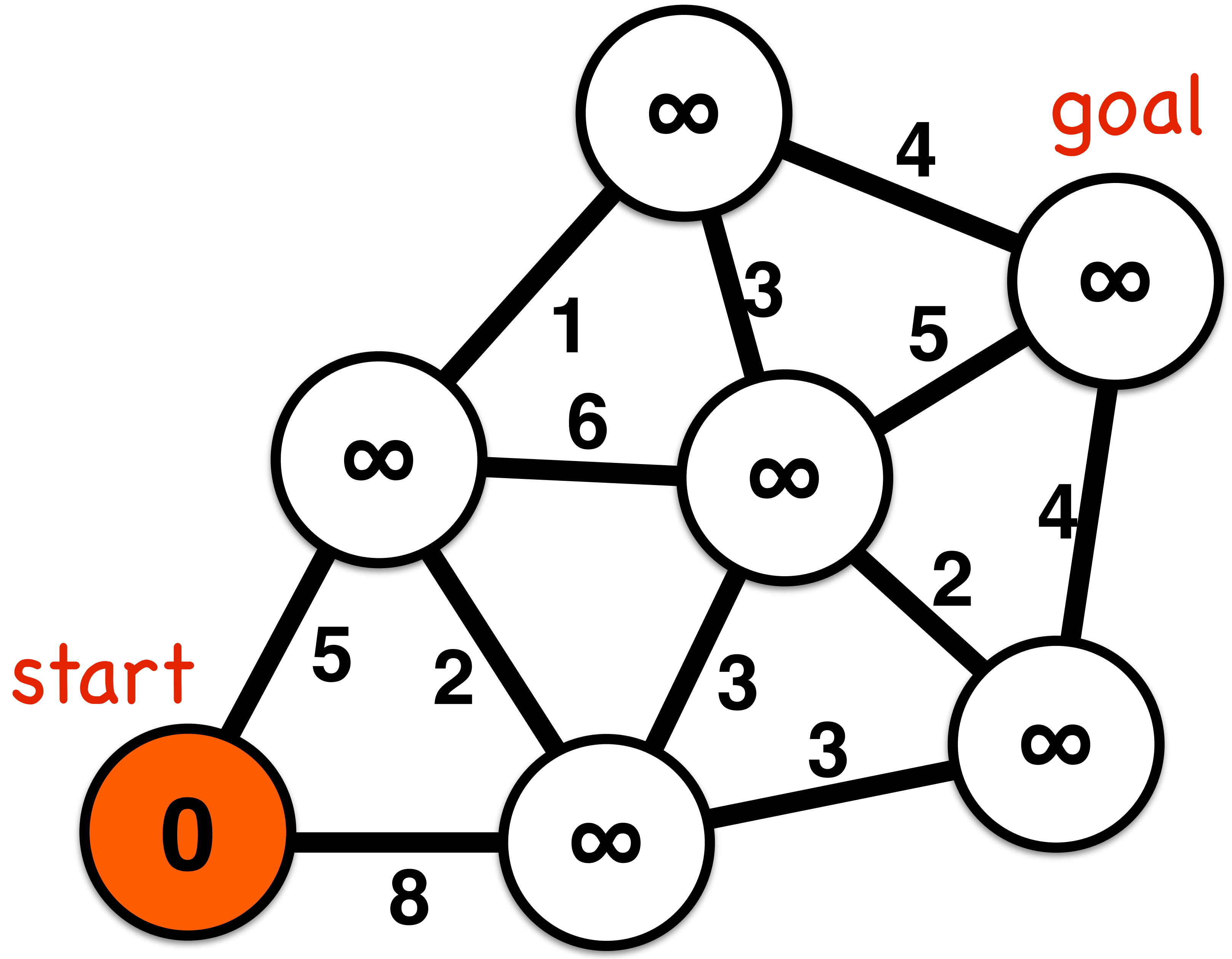
        end if

      end for loop

    end while loop

output ← parent, distance

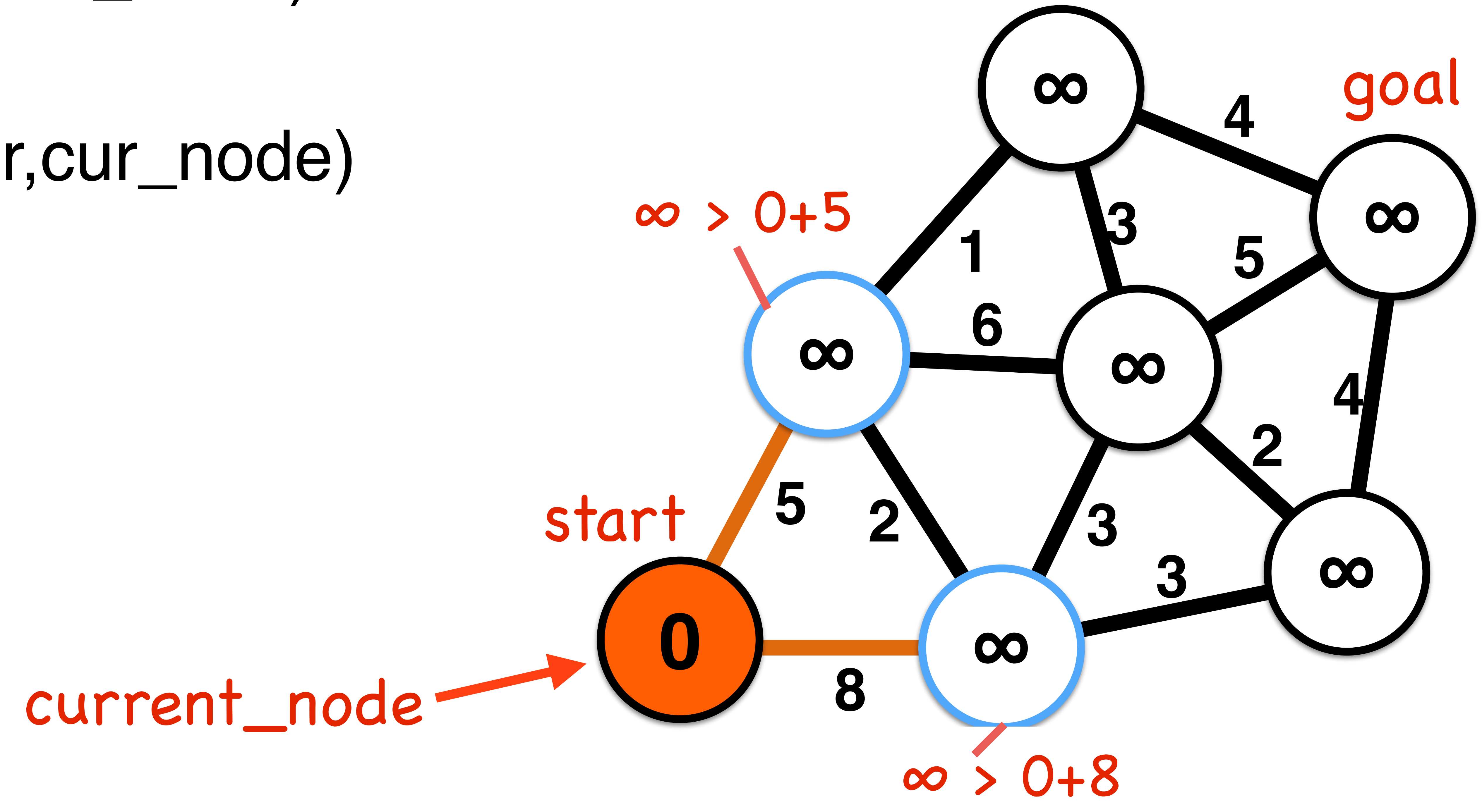
Priority:
Minimum route distance from start



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
while visit_queue != empty ~~&& current_node != goal~~
cur_node ← min_distance(visit_queue)
$visited_{cur\_node}$ ← true
for each nbr in not_visited(adjacent(cur_node))
if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
$parent_{nbr}$ ← current_node
$dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
enqueue(nbr to visit_queue)
end if
end for loop
end while loop
output ← parent, distance

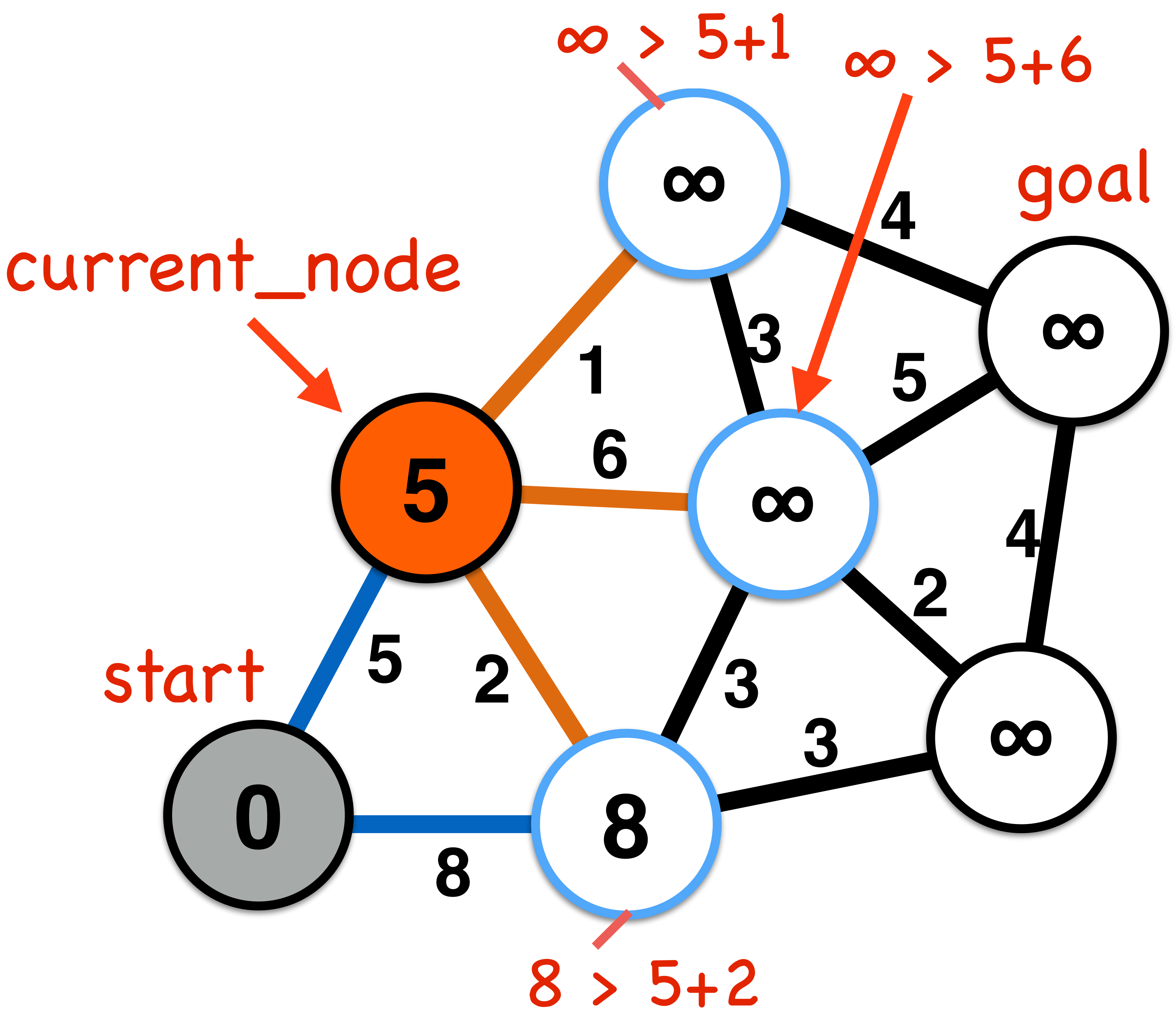
Dijkstra walkthrough



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}
start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}
visit_queue ← start_node
while visit_queue != empty ~~&& current_node != goal~~
cur_node ← min_distance(visit_queue)
$visited_{cur\_node}$ ← true
for each nbr in not_visited(adjacent(cur_node))
if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)
$parent_{nbr}$ ← current_node
$dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)
enqueue(nbr to visit_queue)
end if
end for loop
end while loop
output ← parent, distance

Dijkstra walkthrough



Dijkstra shortest path algorithm

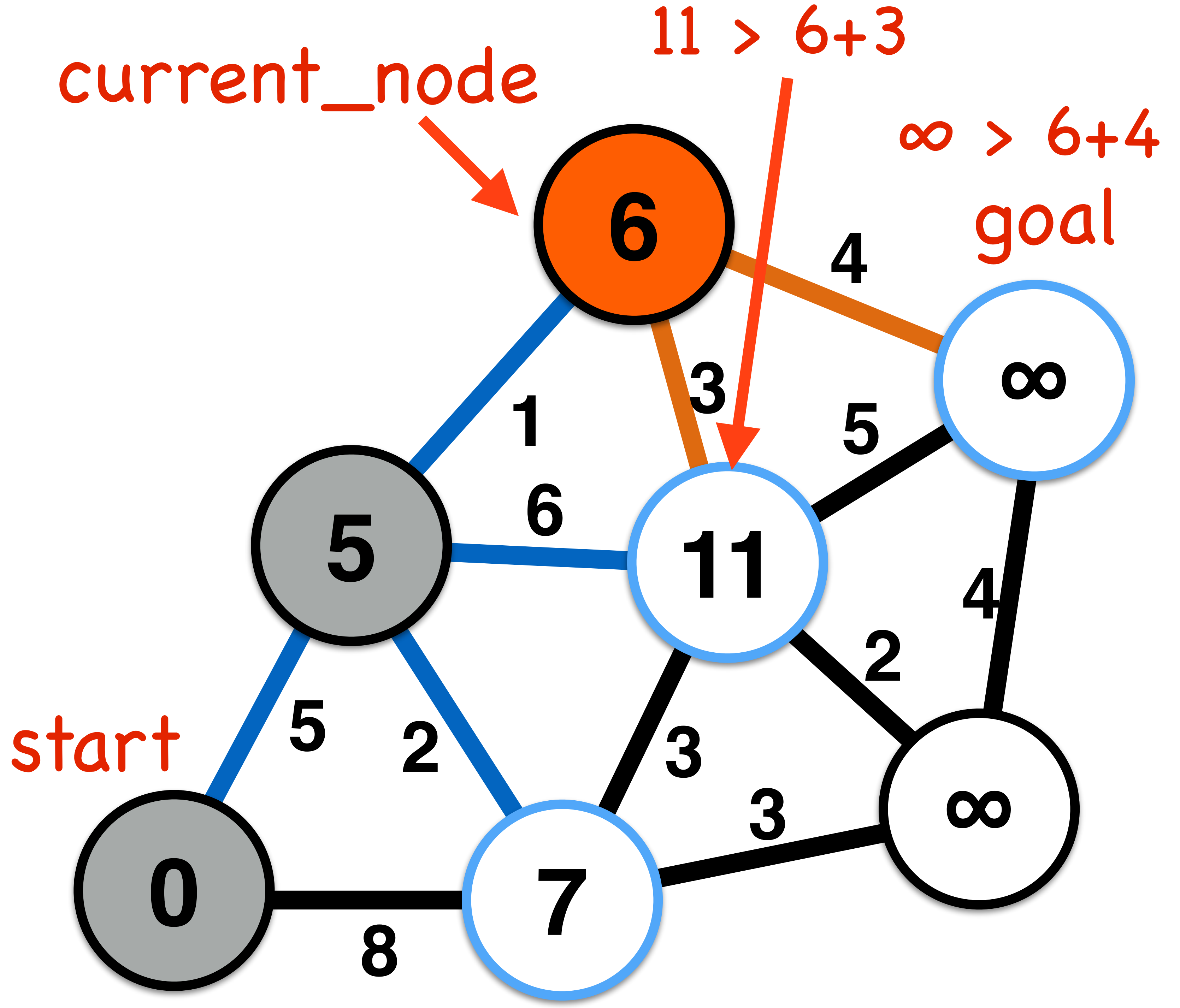
all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

```
    while visit_queue != empty ~~&& current_node != goal~~
      cur_node ← min_distance(visit_queue)
      visited[cur_node] ← true
      for each nbr in not_visited(adjacent(cur_node))
        if dist[nbr] > dist[cur_node] + distance(nbr,cur_node)
          parent[nbr] ← current_node
          dist[nbr] ← dist[cur_node] + distance(nbr,cur_node)
          enqueue(nbr to visit_queue)
        end if
      end for loop
    end while loop
output ← parent, distance
```

Dijkstra walkthrough



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

while visit_queue != empty ~~&& current_node != goal~~

cur_node ← min_distance(visit_queue)

$visited_{cur\_node}$ ← true

for each nbr in not_visited(adjacent(cur_node))

if $dist_{nbr} > dist_{cur\_node}$ + distance(nbr,cur_node)

$parent_{nbr}$ ← current_node

$dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

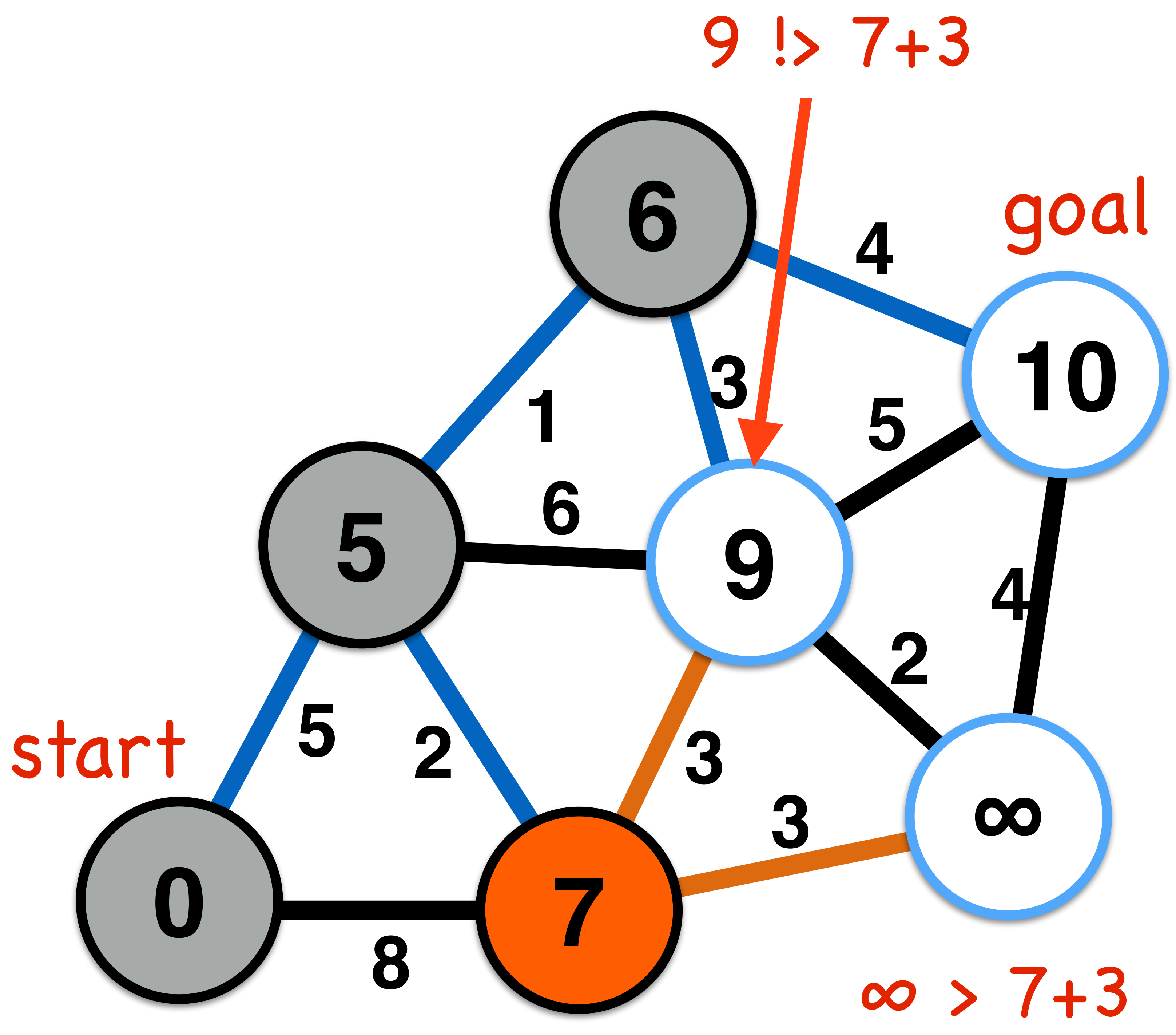
enqueue(nbr to visit_queue)

end if

end for loop

end while loop

output ← parent, distance



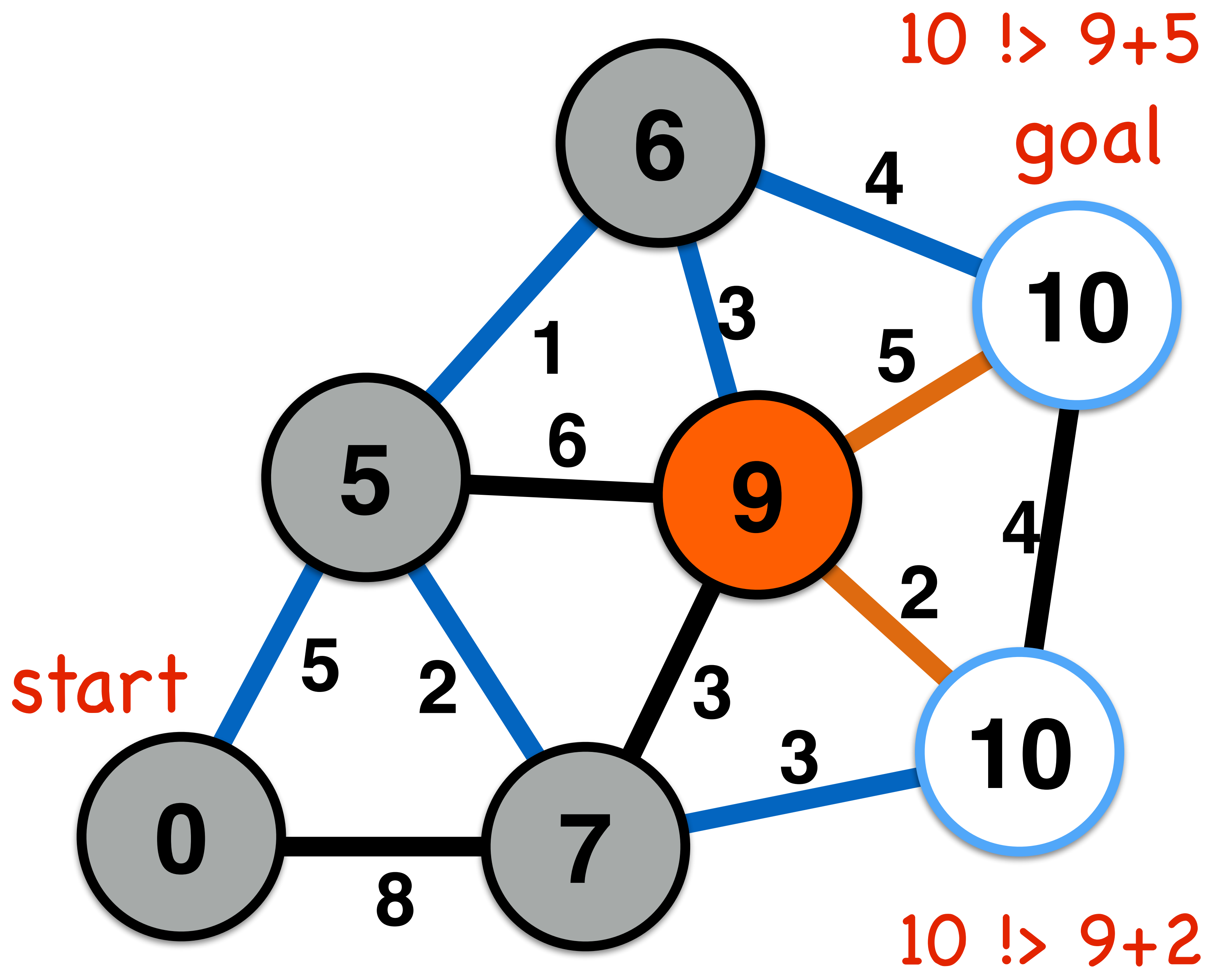
Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

```
    while visit_queue != empty && current_node != goal
        cur_node ← min_distance(visit_queue)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
                parent_nbr ← current_node
                dist_nbr ← dist_cur_node + distance(nbr,cur_node)
                enqueue(nbr to visit_queue)
            end if
        end for loop
    end while loop
output ← parent, distance
```



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

while visit_queue != empty ~~&& current_node != goal~~

cur_node ← min_distance(visit_queue)

$visited_{cur\_node}$ ← true

for each nbr in not_visited(adjacent(cur_node))

if $dist_{nbr} > dist_{cur\_node}$ + distance(nbr,cur_node)

$parent_{nbr}$ ← current_node

$dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

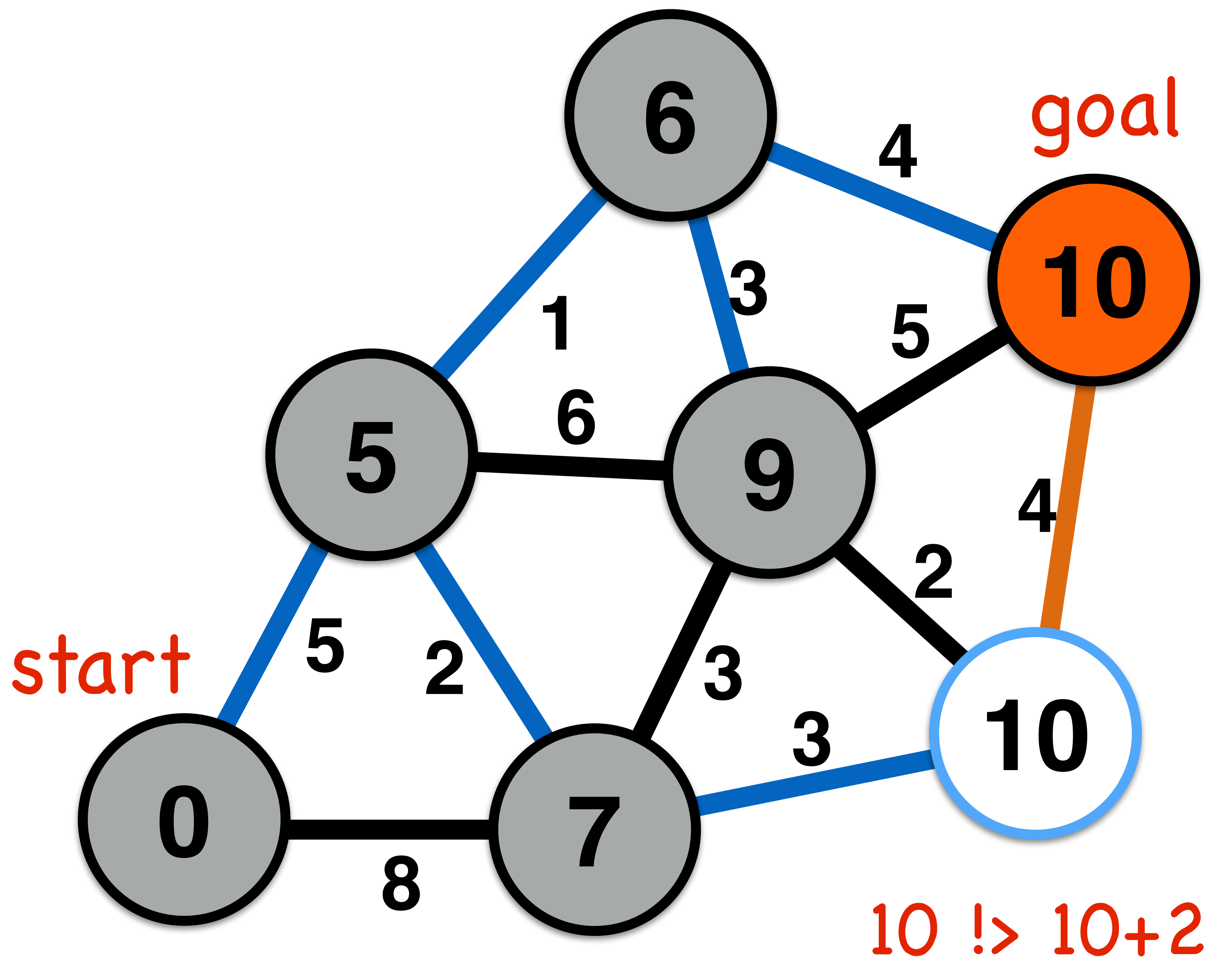
enqueue(nbr to visit_queue)

end if

end for loop

end while loop

output ← parent, distance



Dijkstra shortest path algorithm

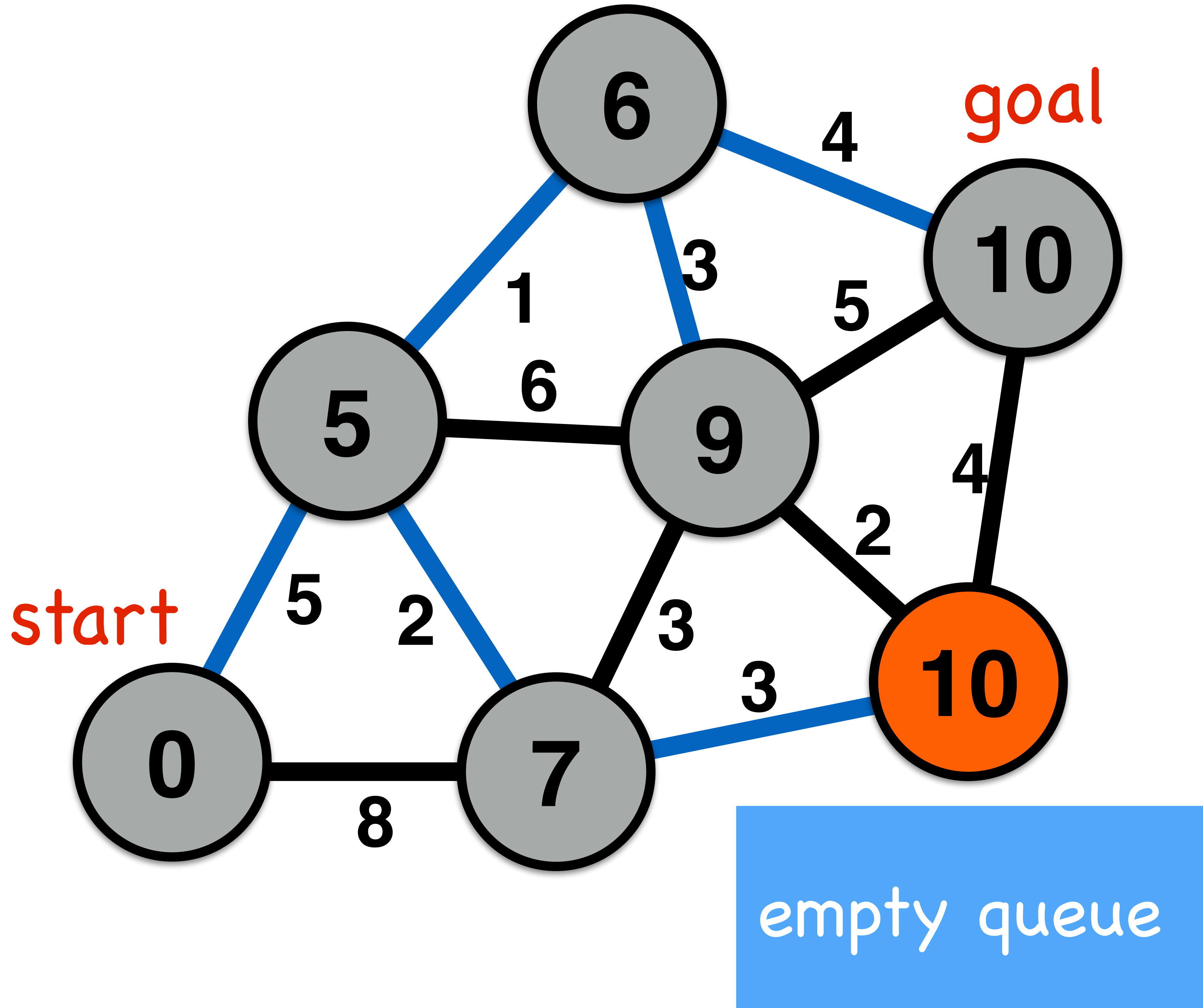
all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

```
while visit_queue != empty ~~&& current_node != goal~~
    cur_node ← min_distance(visit_queue)
    visited_cur_node ← true
    for each nbr in not_visited(adjacent(cur_node))
        if dist_nbr > dist_cur_node + distance(nbr,cur_node)
            parent_nbr ← current_node
            dist_nbr ← dist_cur_node + distance(nbr,cur_node)
            enqueue(nbr to visit_queue)
        end if
    end for loop
end while loop
```

output ← parent, distance



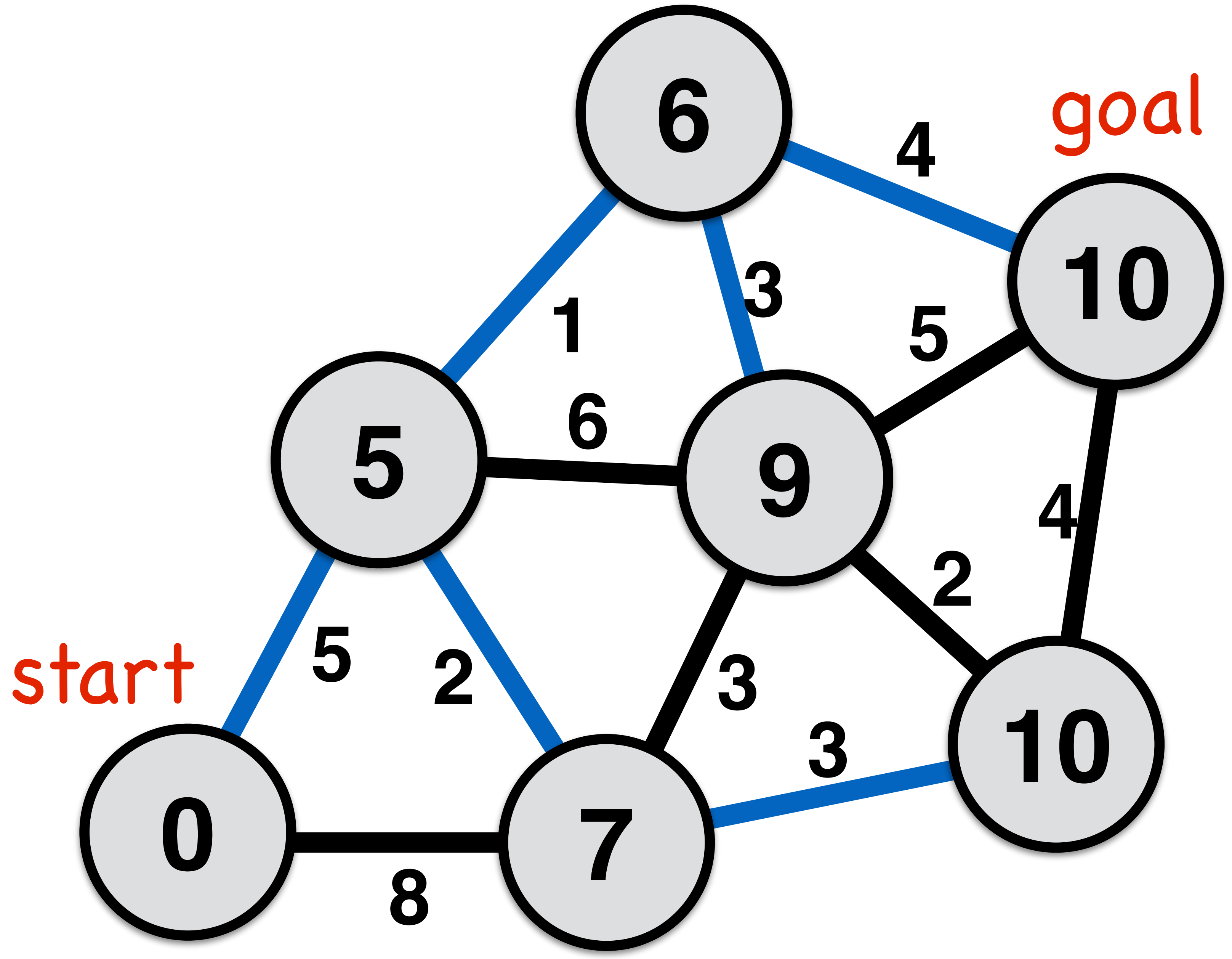
Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

```
    while visit_queue != empty ~~&& current_node != goal~~
        cur_node ← min_distance(visit_queue)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
                parent_nbr ← current_node
                dist_nbr ← dist_cur_node + distance(nbr,cur_node)
                enqueue(nbr to visit_queue)
            end if
        end for loop
    end while loop
output ← parent, distance
```



Dijkstra shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

    while visit_queue != empty ~~&& current_node != goal~~

      cur_node ← min_distance(visit_queue)

      $visited_{cur\_node}$ ← true

      for each nbr in not_visited(adjacent(cur_node))

        if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)

          $parent_{nbr}$ ← current_node

          $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

          enqueue(nbr to visit_queue)

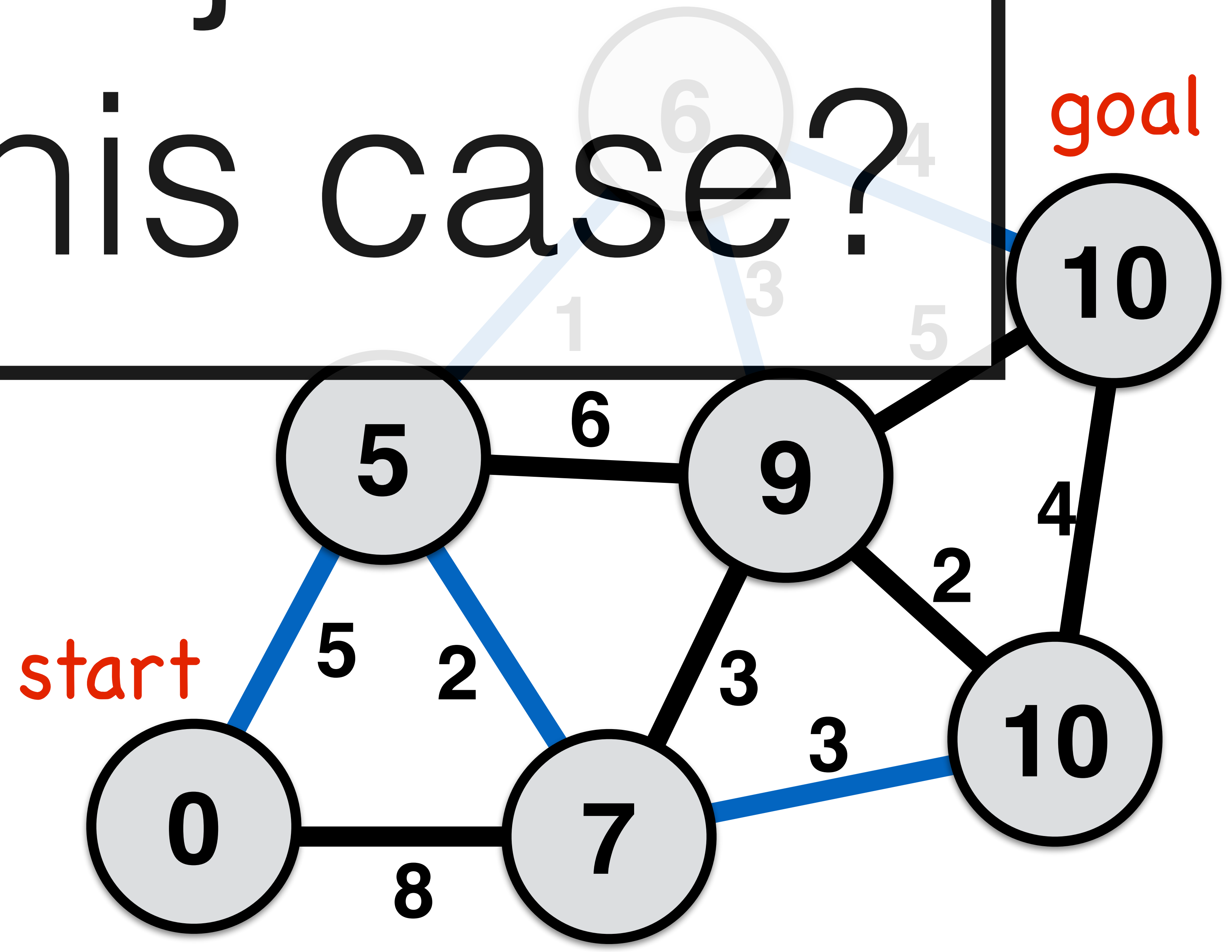
        end if

      end for loop

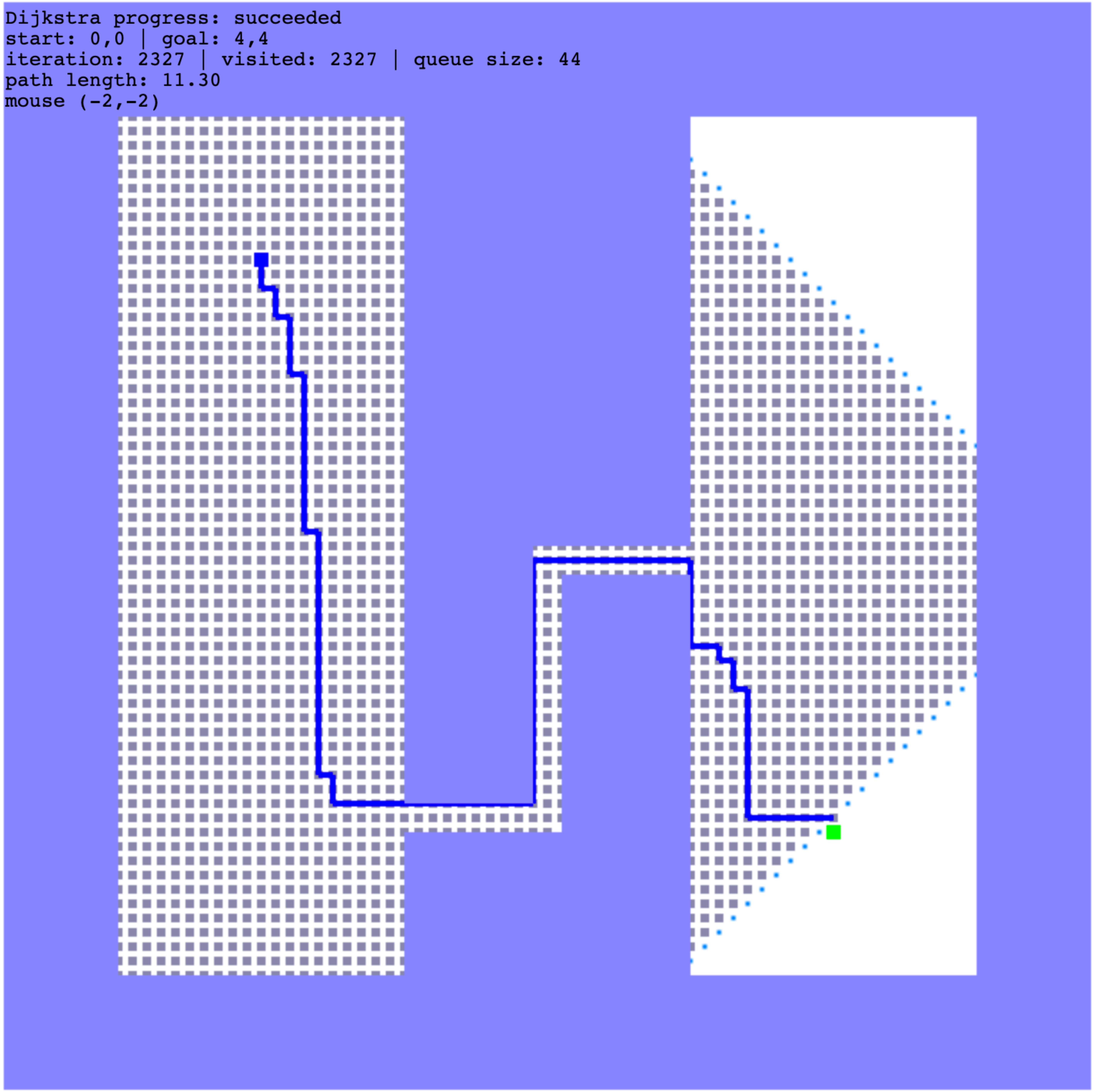
    end while loop

output ← parent, distance

What will search with Dijkstra's algorithm look like in this case?

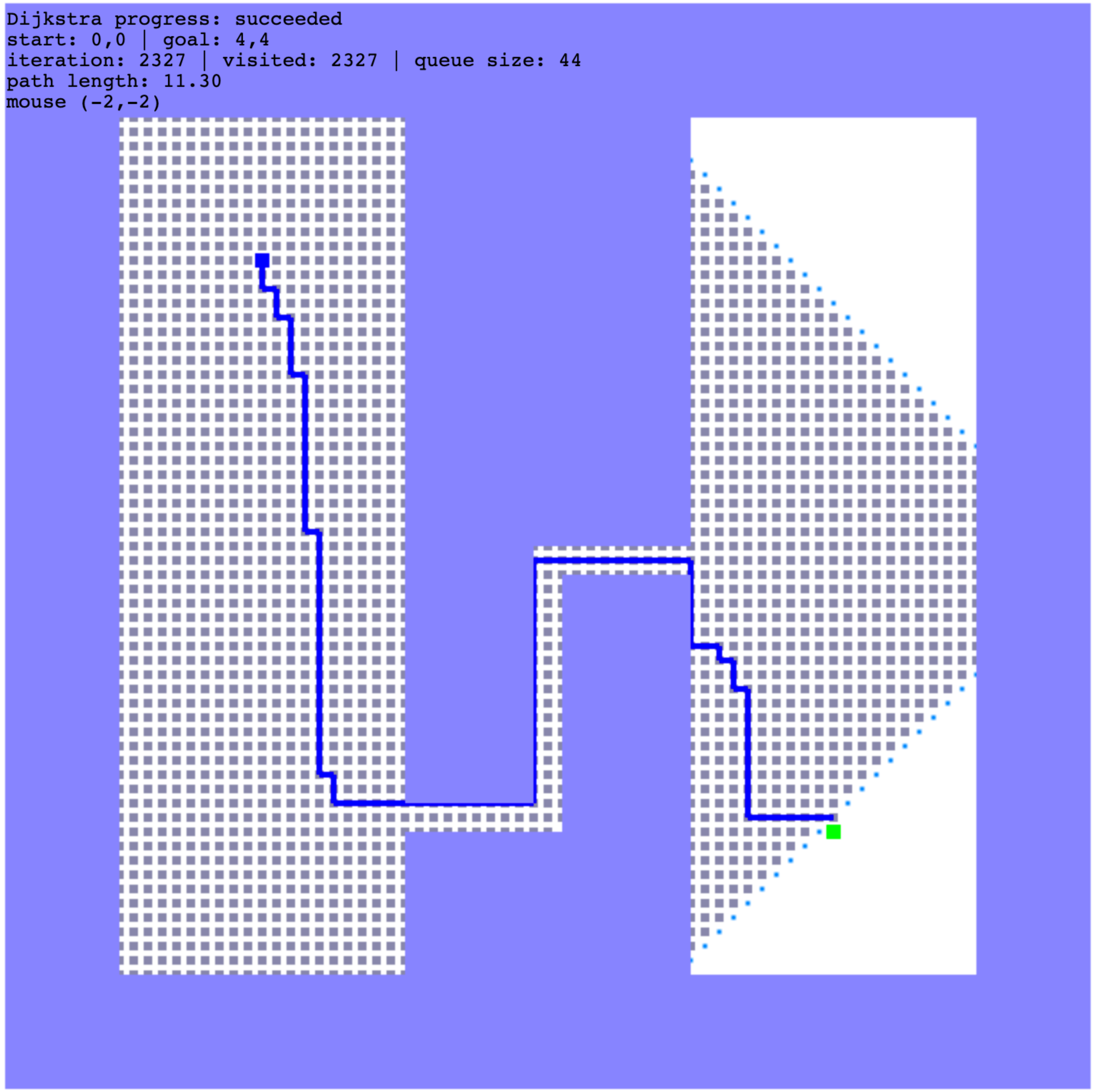


What will search with Dijkstra's algorithm look like in this case?

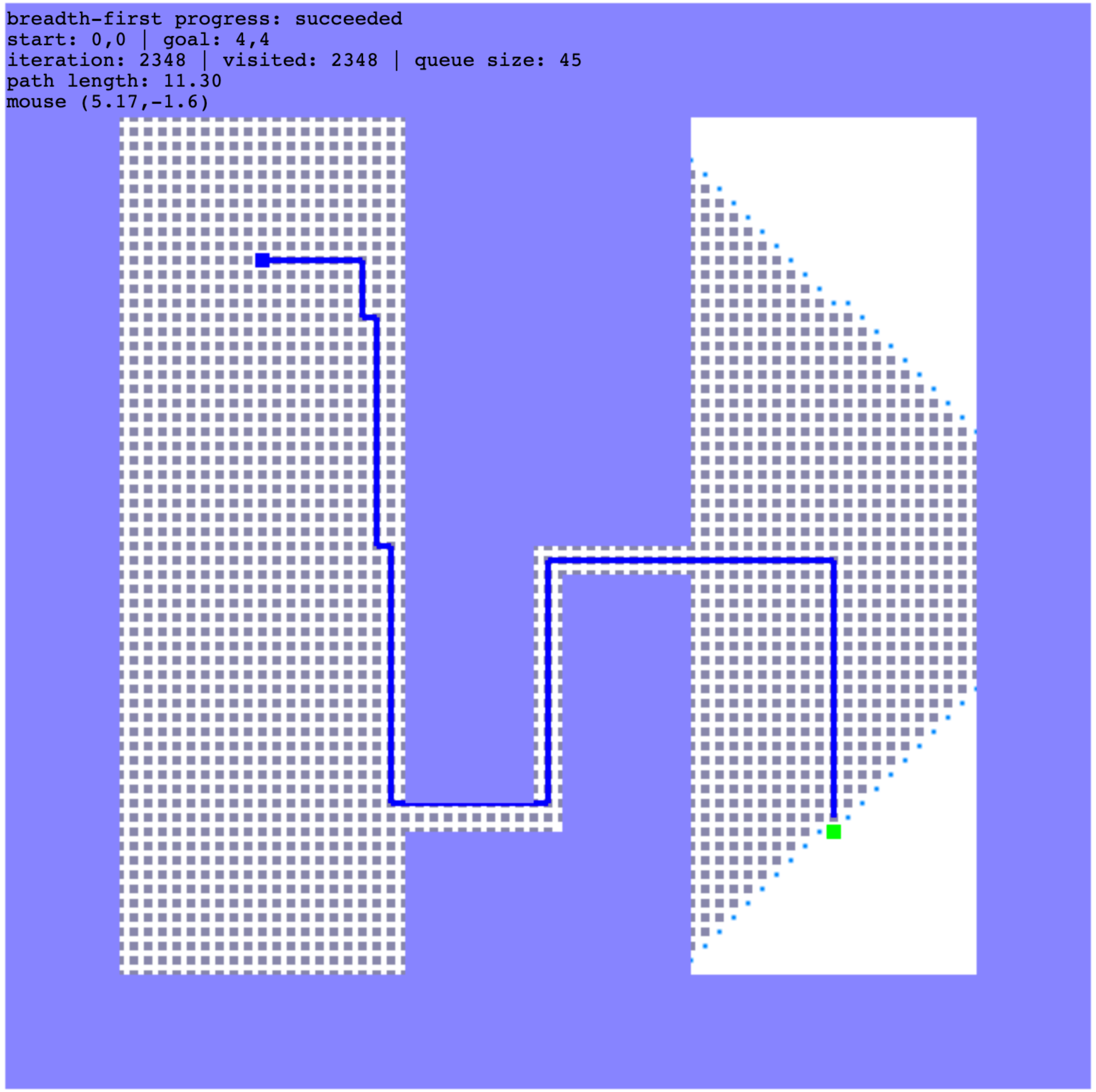


Dijkstra progress: succeeded
start: 0,0 | goal: 4,4
iteration: 2327 | visited: 2327 | queue size: 44
path length: 11.30
mouse (-2,-2)

Dijkstra



BFS



Why does their visit pattern look similar?

A-star Algorithm

A Formal Basis for the Heuristic Determination of Minimum Cost Paths

PETER E. HART, MEMBER, IEEE, NILS J. NILSSON, MEMBER, IEEE, AND BERTRAM RAPHAEL

***Abstract*—Although the problem of determining the minimum cost path through a graph arises naturally in a number of interesting applications, there has been no underlying theory to guide the development of efficient search procedures. Moreover, there is no adequate conceptual framework within which the various ad hoc search strategies proposed to date can be compared. This paper describes how heuristic information from the problem domain can be incorporated into a formal mathematical theory of graph searching and demonstrates an optimality property of a class of search strategies.**

I. Introduction

A. *The Problem of Finding Paths Through Graphs*

MANY PROBLEMS of engineering and scientific importance can be related to the general problem of finding a path through a graph. Examples of such problems include routing of telephone traffic, navigation through a maze, layout of printed circuit boards, and mechanical theorem-proving and problem-solving. These problems have usually been approached in one of two ways, which we shall call the *mathematical approach* and the *heuristic approach.*

1) The mathematical approach typically deals with the properties of abstract graphs and with algorithms that prescribe an orderly examination of nodes of a graph to establish a minimum cost path. For example, Pollock and Wiebenson[1] review several algorithms which are guaranteed to find such a path for any graph. Busacker and Saaty[2] also discuss several algorithms, one of which uses the concept of dynamic programming.[3] The mathematical approach is generally more concerned with the ultimate achievement of solutions than it is with the computational feasibility of the algorithms developed.

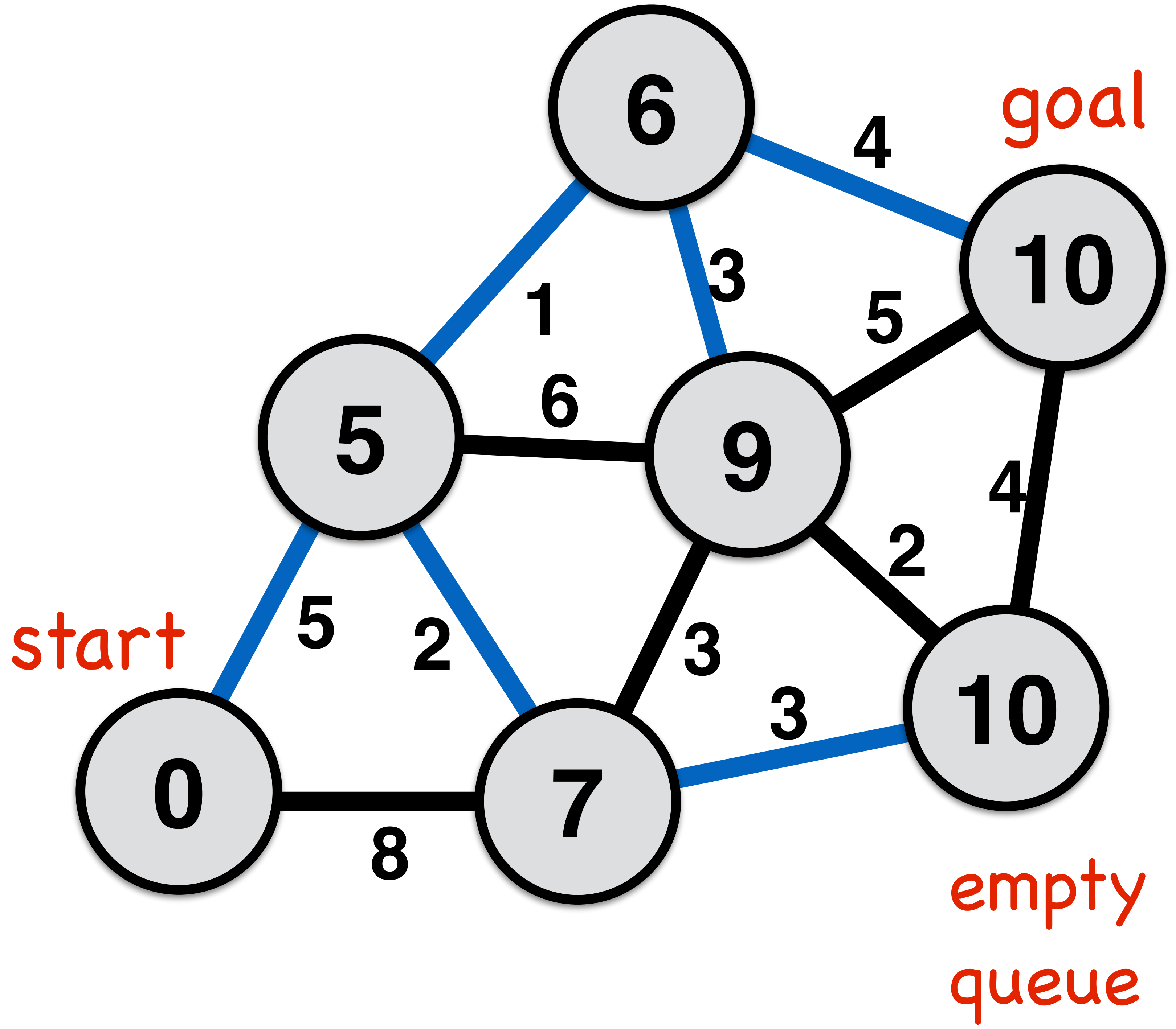
2) The heuristic approach typically uses special knowledge about the domain of the problem being represented by a graph to improve the computational efficiency of solutions to particular graph-searching problems. For example, Gelernter's[4] program used Euclidean diagrams to direct the search for geometric proofs. Samuel[5] and others have used ad hoc characteristics of particular games to reduce

Manuscript received November 24, 1967.
The authors are with the Artificial Intelligence Group of the Applied Physics Laboratory, Stanford Research Institute, Menlo Park, Calif.

Hart, Nilsson, and Raphael
IEEE Transactions of System Science and Cybernetics, 4(2):100-107, 1968

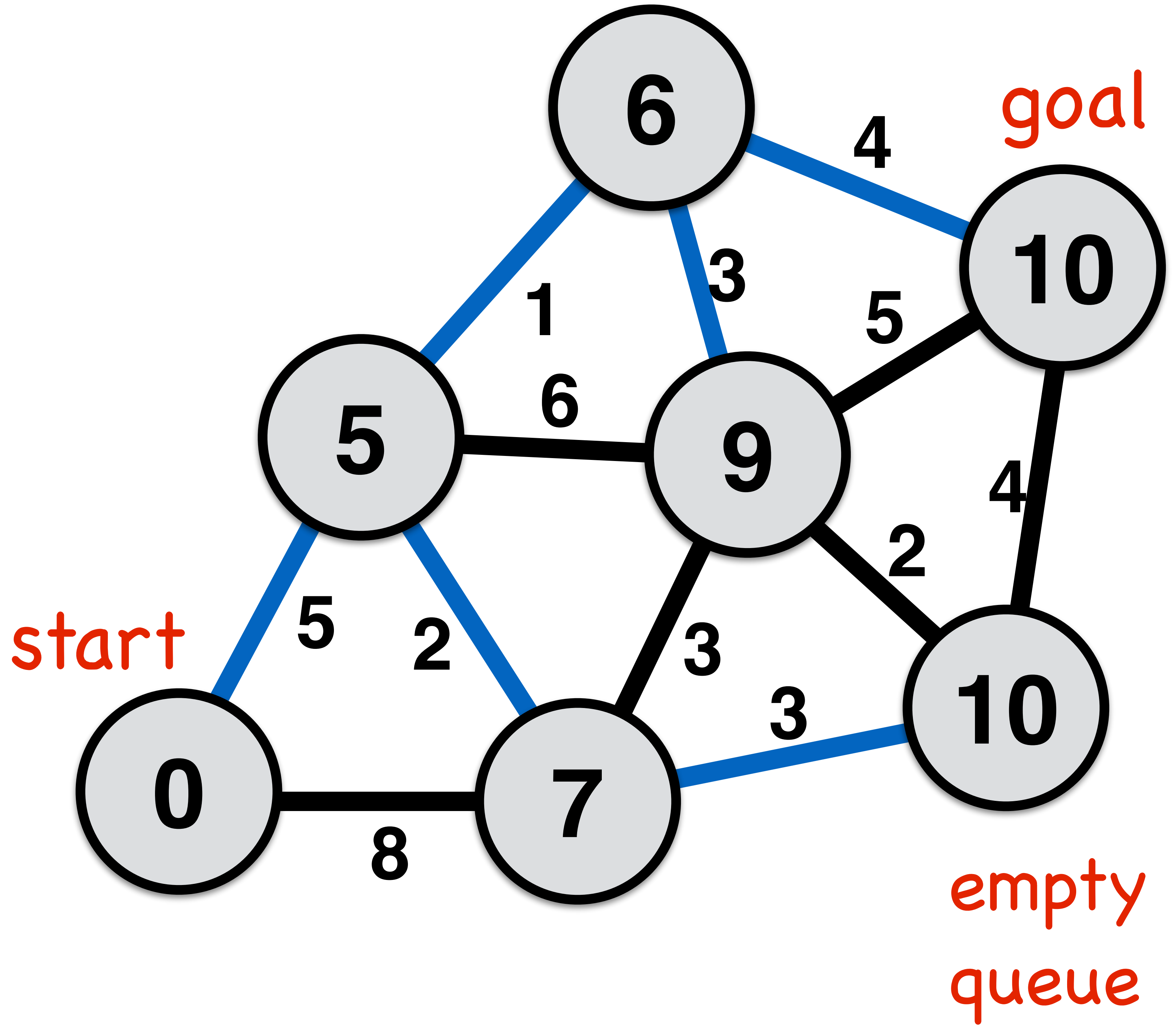
<u>Dijkstra shortest path algorithm</u>

```
all nodes ← {dist_start ← infinity, parent_start ← none, visited_start ← false}
start_node ← {dist_start ← 0, parent_start ← none, visited_start ← true}
visit_queue ← start_node
      while visit_queue != empty && current_node != goal
         cur_node ← min_distance(visit_queue)
         visited_cur_node ← true
         for each nbr in not_visited(adjacent(cur_node))
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
               parent_nbr ← current_node
               dist_nbr ← dist_cur_node + distance(nbr,cur_node)
               enqueue(nbr to visit_queue)
            end if
         end for loop
      end while loop
output ← parent, distance
```



A-star shortest path algorithm

```
all nodes ← {dist_start ← infinity, parent_start ← none, visited_start ← false}
start_node ← {dist_start ← 0, parent_start ← none, visited_start ← true}
visit_queue ← start_node
    while (visit_queue != empty) && current_node != goal
        cur_node ← dequeue(visit_queue, f_score)
        visited_cur_node ← true
        for each nbr in not_visited(adjacent(cur_node))
            if dist_nbr > dist_cur_node + distance(nbr,cur_node)
                parent_nbr ← current_node
                dist_nbr ← dist_cur_node + distance(nbr,cur_node)
                f_score ← distance_nbr + line_distance_nbr,goal
                enqueue(nbr to visit_queue)
            end if
        end for loop
    end while loop
output ← parent, distance
```



A-star shortest path algorithm

all nodes ← {$\text{dist}_{\text{start}}$ ← infinity, $\text{parent}_{\text{start}}$ ← none, $\text{visited}_{\text{start}}$ ← false}

start_node ← {$\text{dist}_{\text{start}}$ ← 0, $\text{parent}_{\text{start}}$ ← none, $\text{visited}_{\text{start}}$ ← true}

visit_queue ← start_node

  while (visit_queue != empty) && current_node != goal

    cur_node ← **dequeue(visit_queue, f_score)**

    $\text{visited}_{\text{cur\_node}}$ ← true

    for each nbr in not_visited(adjacent(cur_node))

      if dist_{nbr} > $\text{dist}_{\text{cur\_node}}$ + distance(nbr,cur_node)

        $\text{parent}_{\text{nbr}}$ ← current_node

        dist_{nbr} ← $\text{dist}_{\text{cur\_node}}$ + distance(nbr,cur_node)

        f_score ← $\text{distance}_{\text{nbr}}$ + $\text{line\_distance}_{\text{nbr,goal}}$

        enqueue(nbr to visit_queue)

      end if

    end for loop

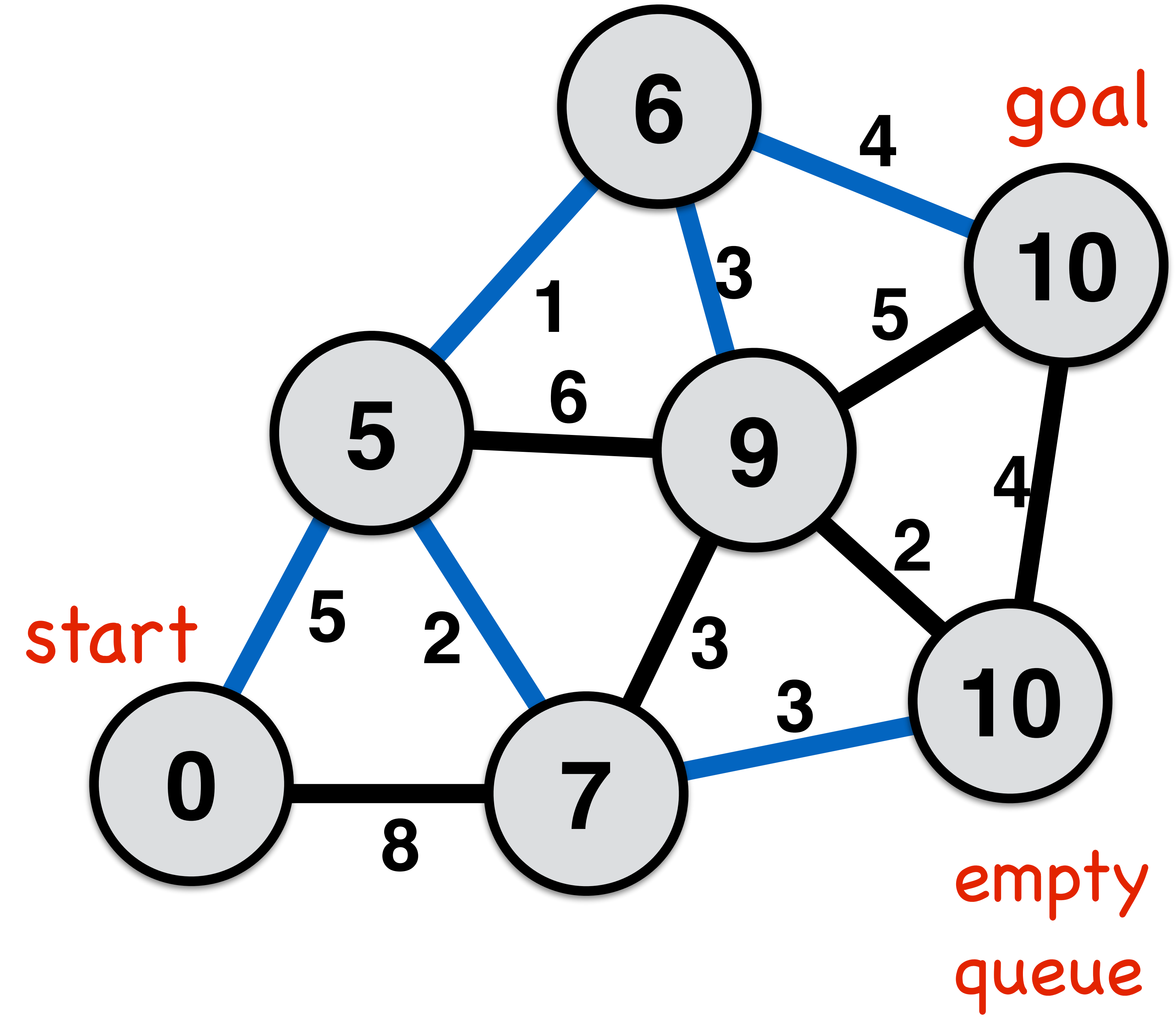
  end while loop

output ← parent, distance

priority queue wrt. f_score
(implement min binary heap)

g_score: distance along current path back to start

h_score: best possible distance to goal



A-star shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

while (visit_queue != empty) && current_node != goal

  cur_node ← **dequeue(visit_queue, f_score)**

priority queue wrt. f_score
(implement min binary heap)

  $visited_{cur\_node}$ ← true

  for each nbr in not_visited(adjacent(cur_node))

   if $dist_{nbr}$ > $dist_{cur\_node}$ + distance(nbr,cur_node)

    $parent_{nbr}$ ← current_node

    $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

    f_score ← $distance_{nbr}$ + $line\_distance_{nbr,goal}$

    enqueue(nbr to visit_queue)

   end if

  end for loop

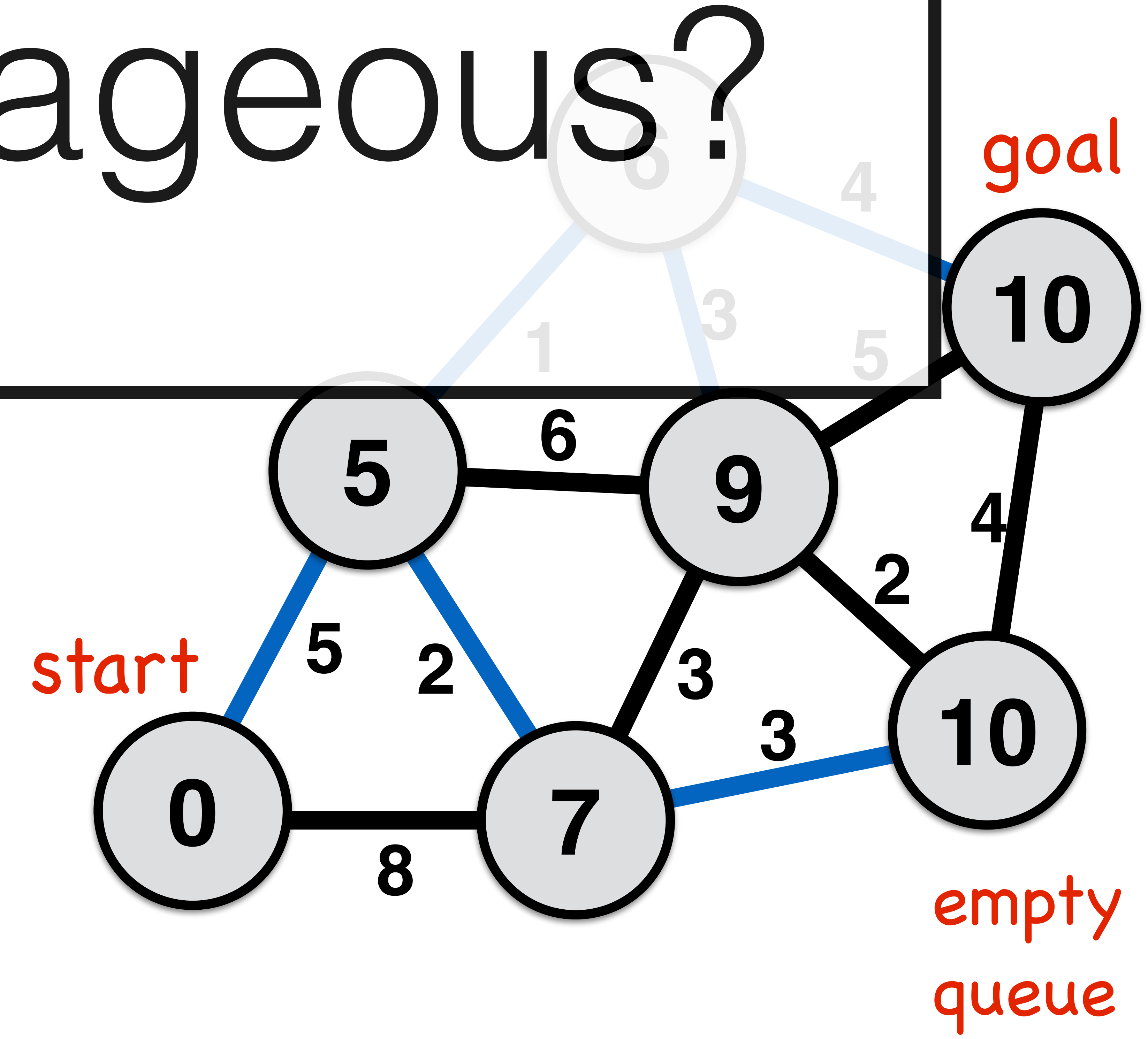
end while loop

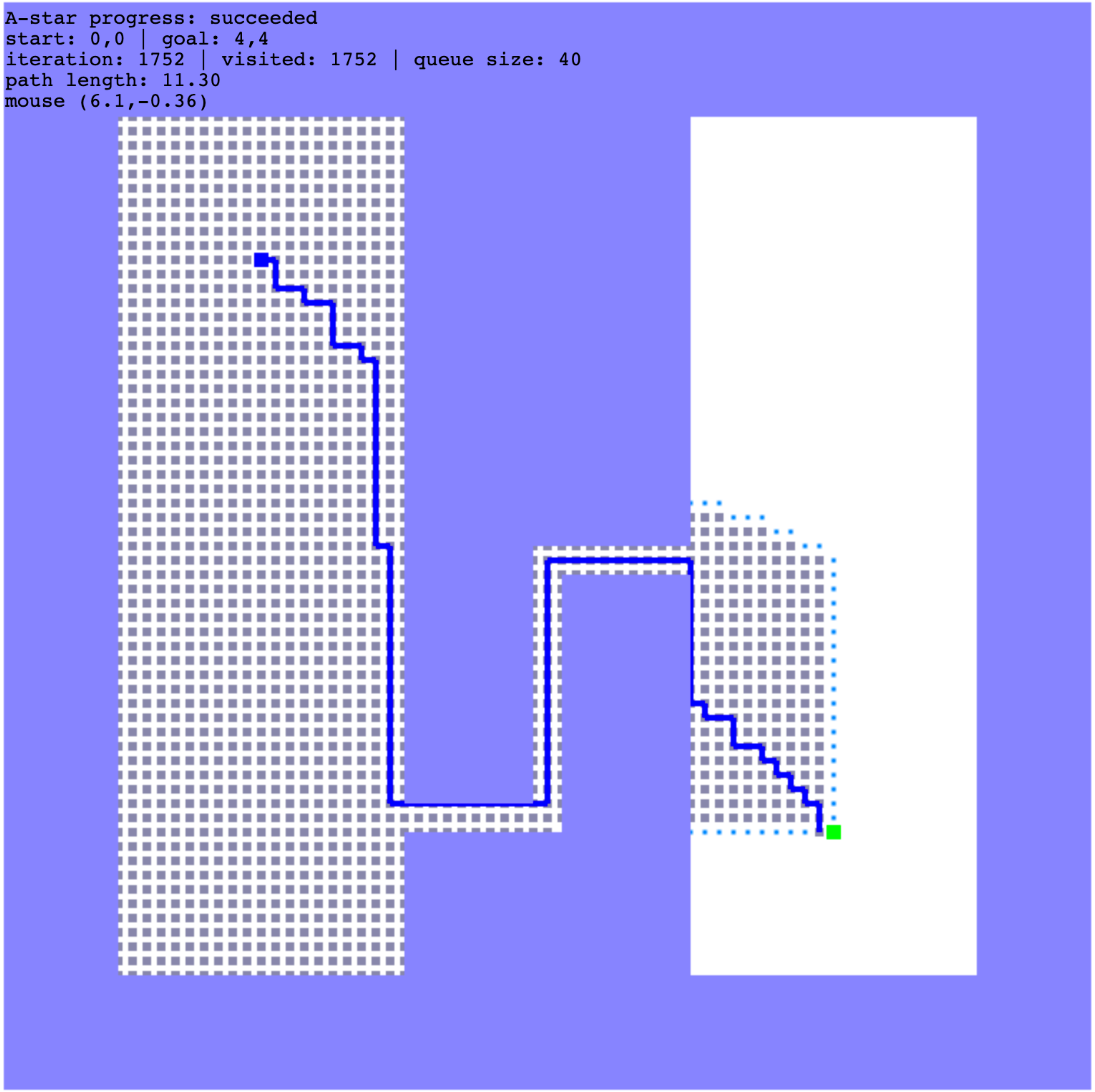
output ← parent, distance

Why is A-star advantageous?

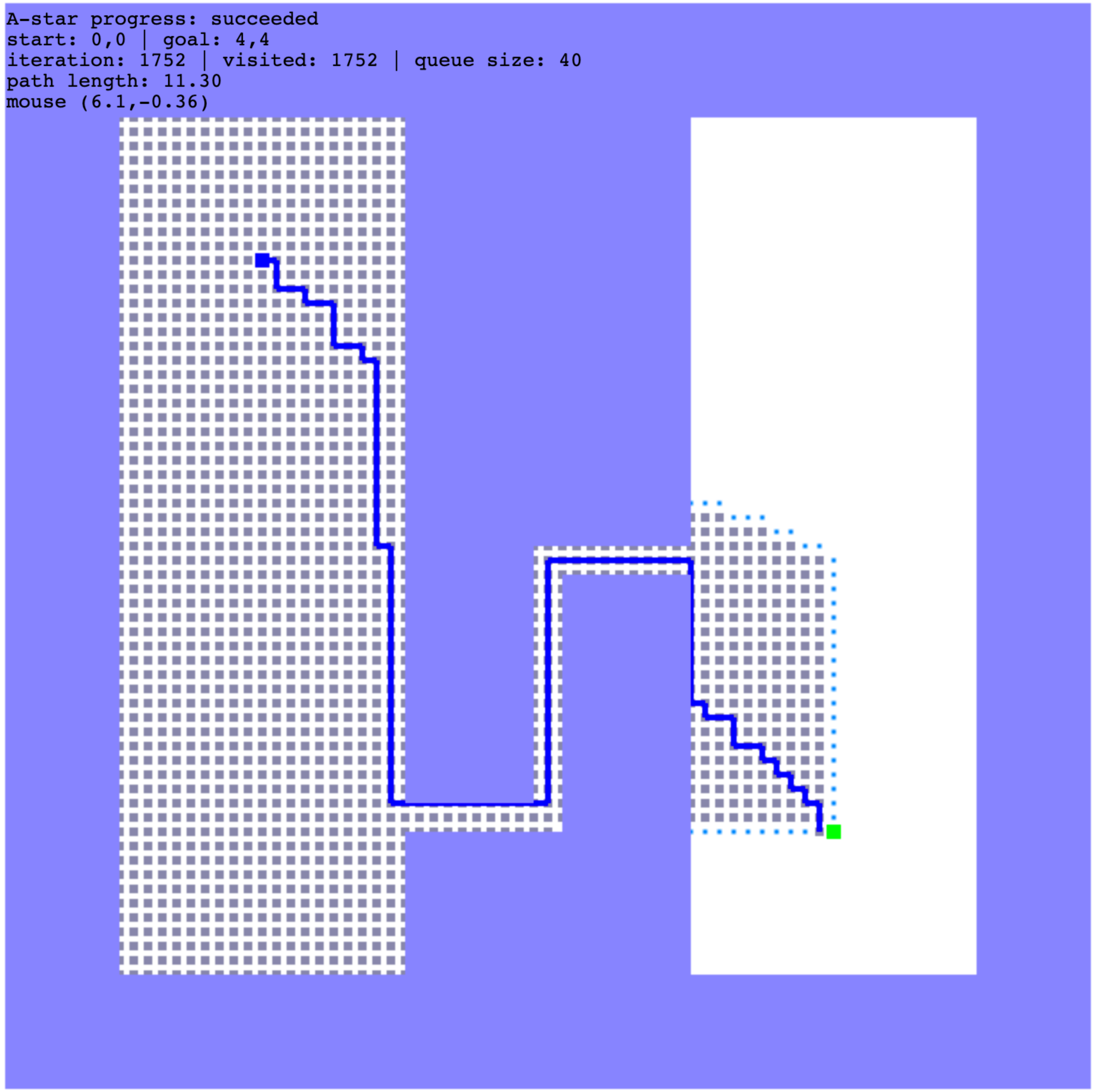
g_score: distance along current path back to start

h_score: best possible distance to goal

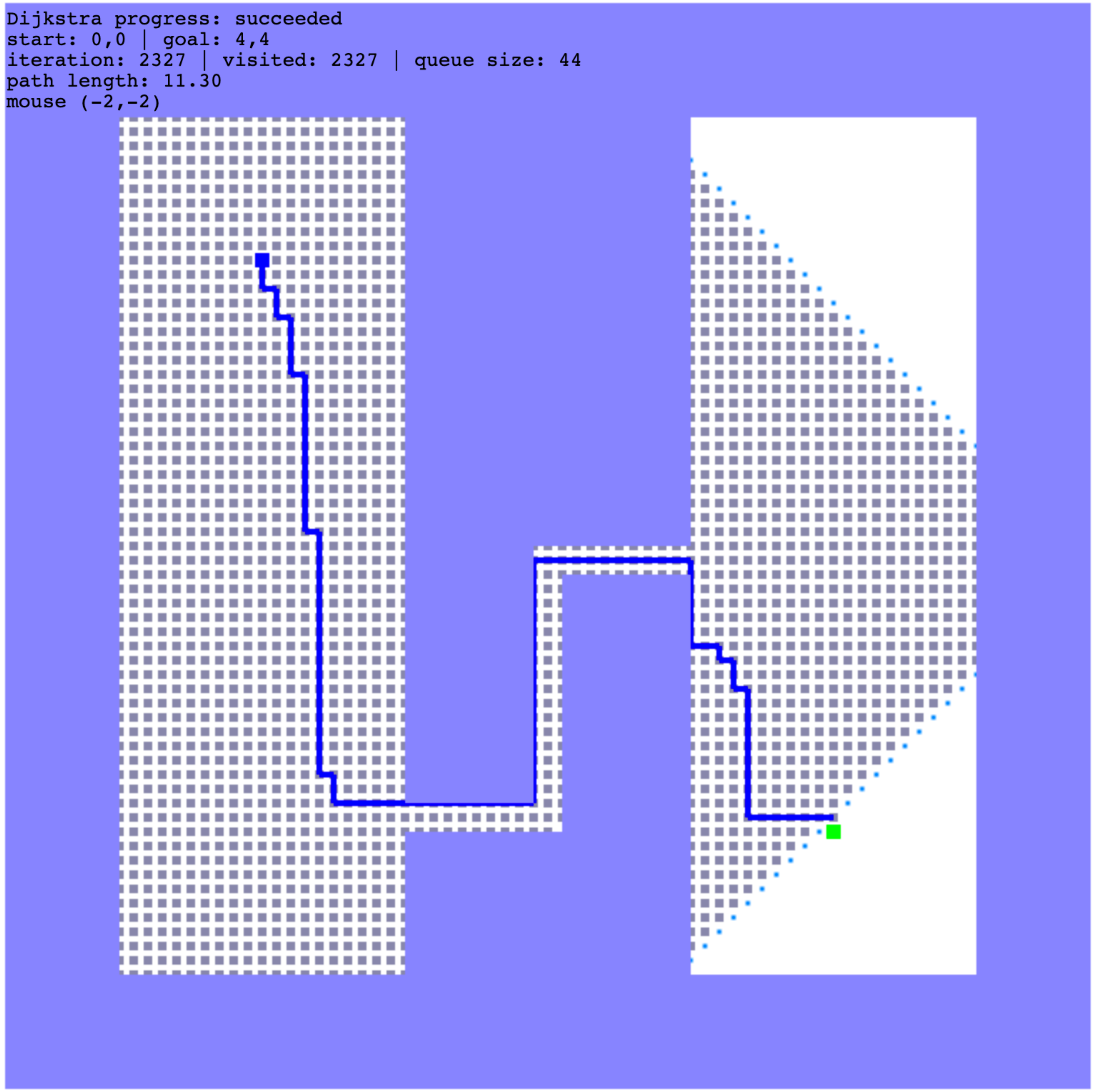



A-star progress: succeeded
start: 0,0 | goal: 4,4
iteration: 1752 | visited: 1752 | queue size: 40
path length: 11.30
mouse (6.1,-0.36)

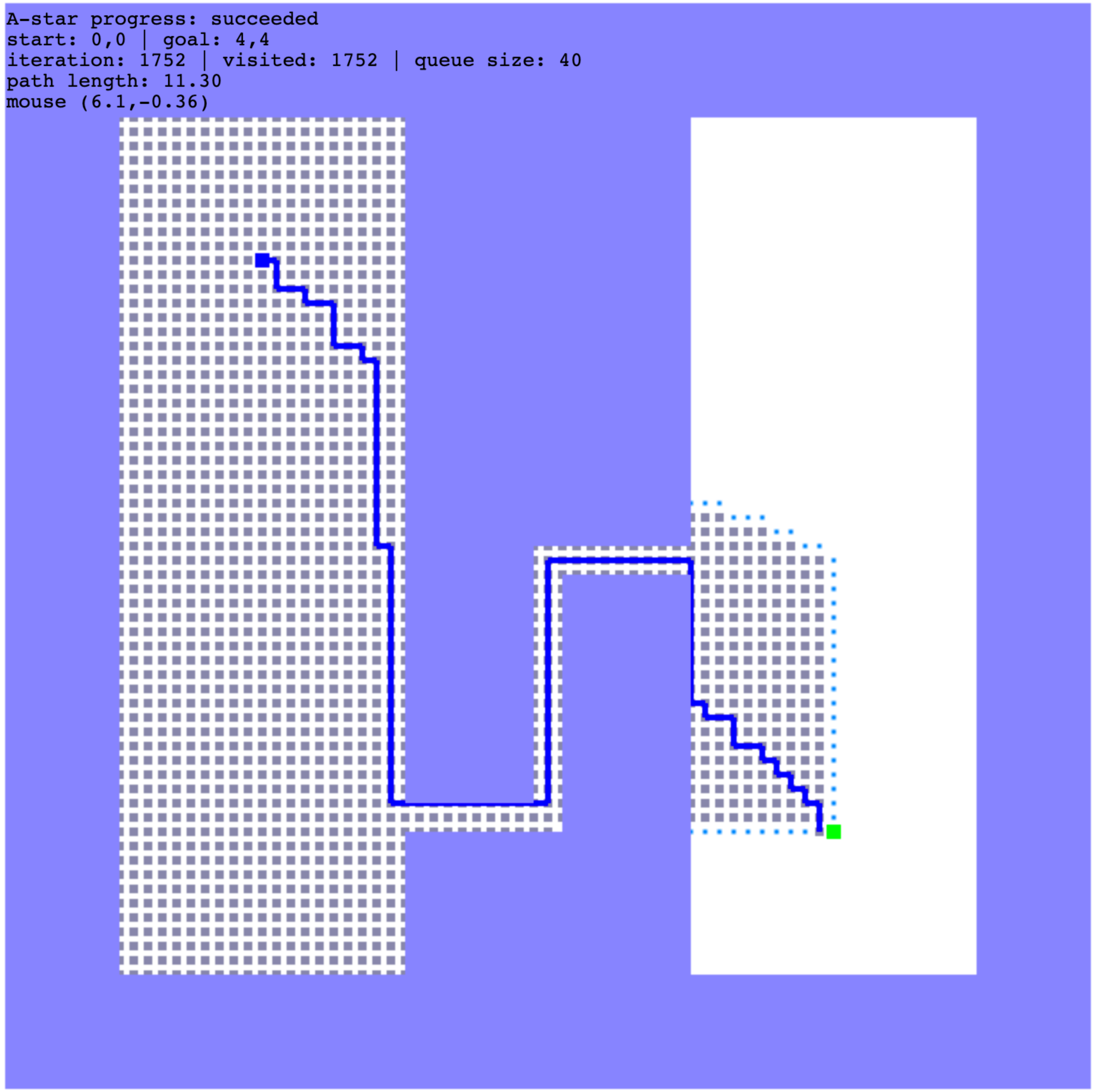
A-Star



Dijkstra

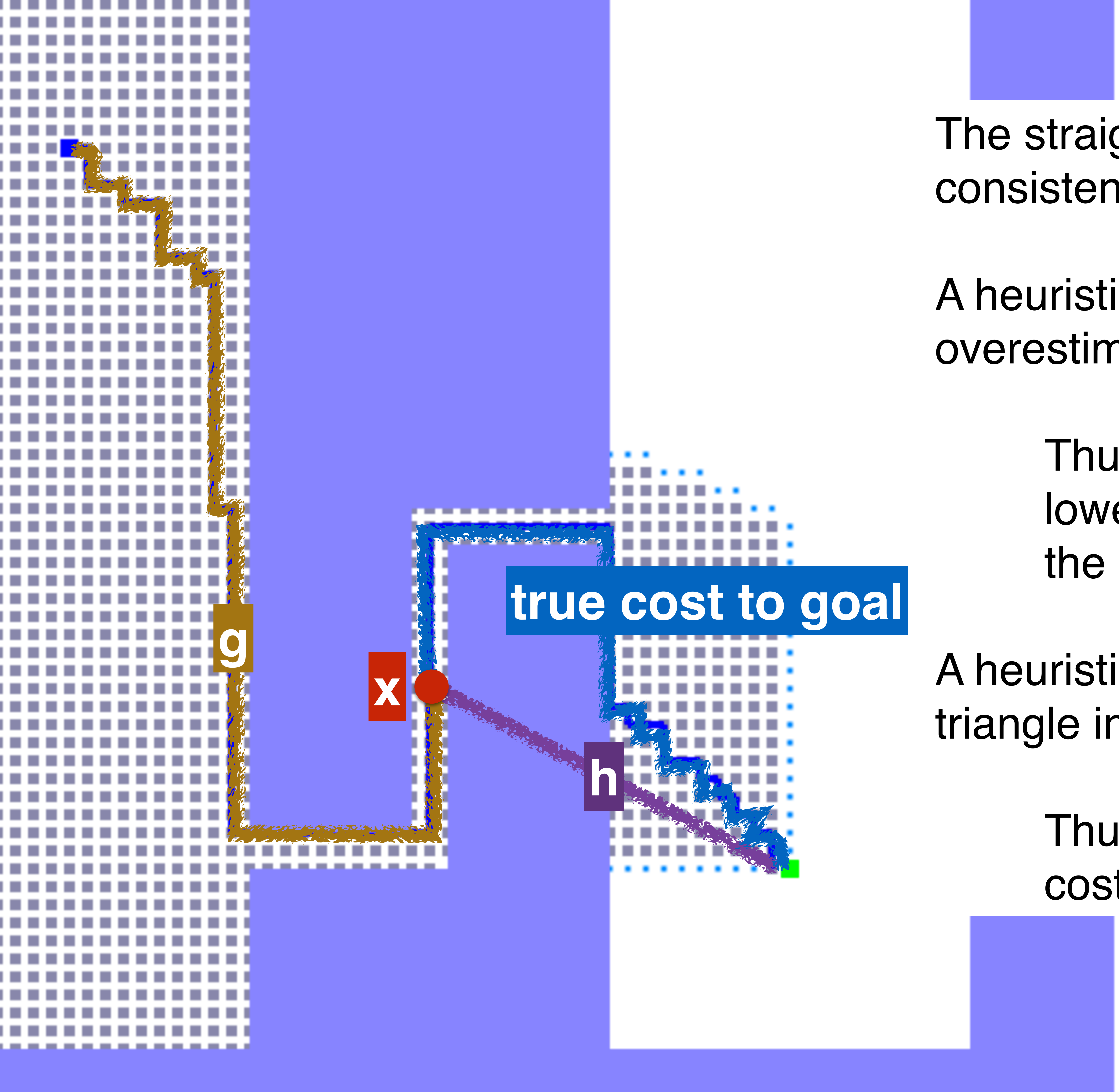


How can A-star visit few nodes?



How can A-star visit few nodes?

A-Star uses an admissible heuristic to estimate the cost to goal from a node



The straight line h_score is an admissible and consistent heuristic function.

A heuristic function is **admissible** if it never overestimates the cost of reaching the goal.

Thus, h_score(x) is less than or equal to the lowest possible cost from current location to the goal.

A heuristic function is **consistent** if obeys the triangle inequality

Thus, h_score(x) is less than or equal to cost(x,action,x') + h_score(x') https://www.cs.cmu.edu/~./awm/tutorials/astar08.pdf

Proof: A* with Admissible Heuristic Guarantees Optimal Path

- Suppose it finds a suboptimal path, ending in goal state G_1 where $f(G_1) > f^*$ where $f^* = h^*(start)$ = cost of optimal path.
- There must exist a node n which is
  - Unexpanded
  - The path from start to n (stored in the BackPointers(n) values) is the start of a true optimal path

Why must such a node exist? Consider any optimal path *s*,*n1*,*n2*...goal. If all along it were expanded, the goal would've been reached along the shortest path.

- $f(n) >= f(G_1)$ (else search wouldn't have ended)
- Also $f(n) = g(n) + h(n)$

$= g^*(n) + h(n)$ because it's on optimal path

$<= g^*(n) + h^*(n)$ By the admissibility assumption

$= f^*$ Because n is on the optimal path

So $f^* >= f(n) >= f(G_1)$ contradicting top of slide

Slide 21

Heaps and Priority Queues

A-star shortest path algorithm

all nodes ← {$dist_{start}$ ← infinity, $parent_{start}$ ← none, $visited_{start}$ ← false}

start_node ← {$dist_{start}$ ← 0, $parent_{start}$ ← none, $visited_{start}$ ← true}

visit_queue ← start_node

  while (visit_queue != empty) && current_node != goal

    cur_node ← **dequeue(visit_queue, f_score)**

    $visited_{cur\_node}$ ← true

    for each nbr in not_visited(adjacent(cur_node))

      if $dist_{nbr} > dist_{cur\_node}$ + distance(nbr,cur_node)

        $parent_{nbr}$ ← current_node

        $dist_{nbr}$ ← $dist_{cur\_node}$ + distance(nbr,cur_node)

        f_score ← $distance_{nbr}$ + $line\_distance_{nbr,goal}$

        enqueue(nbr to visit_queue)

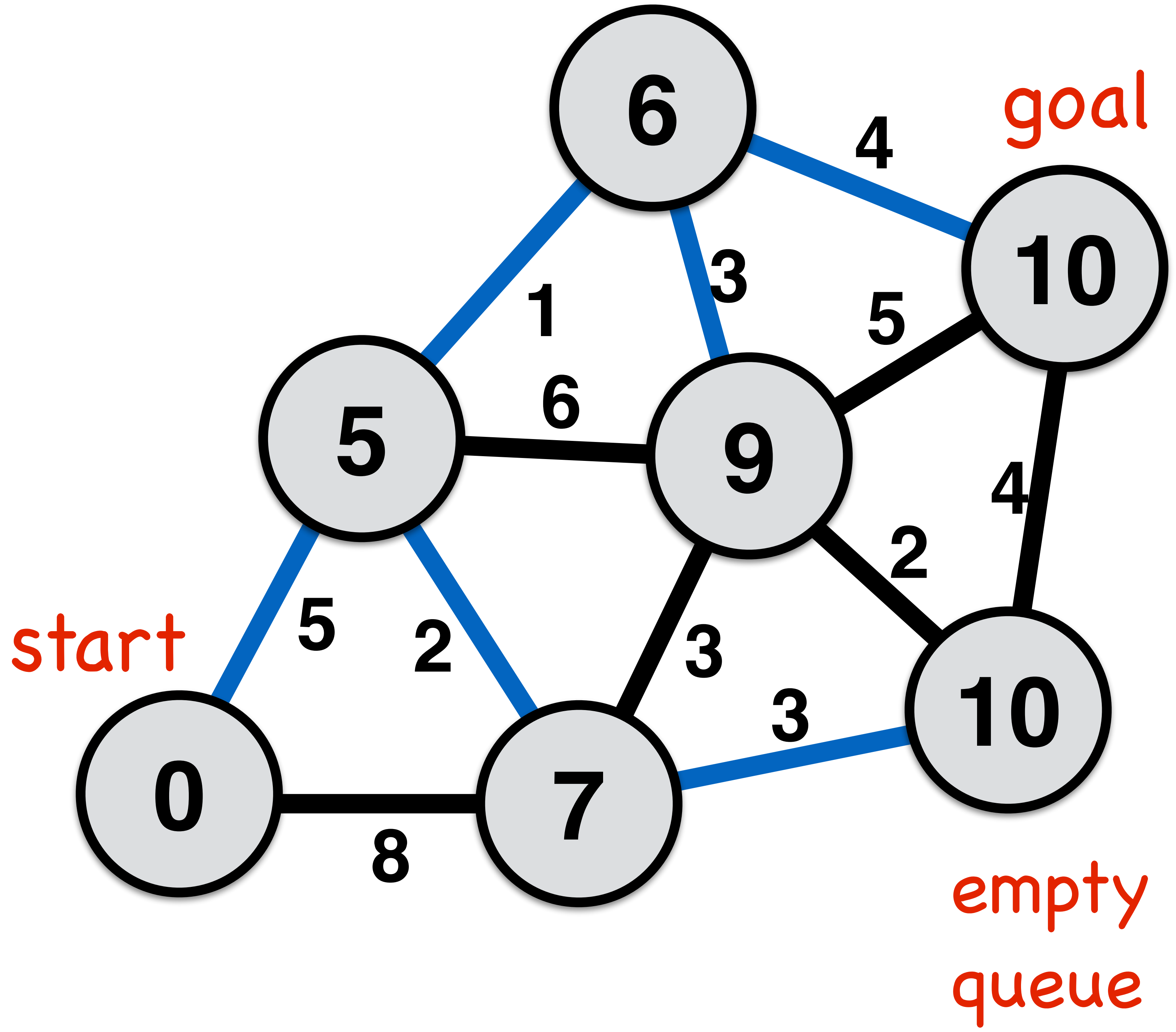
      end if

    end for loop

  end while loop

output ← parent, distance

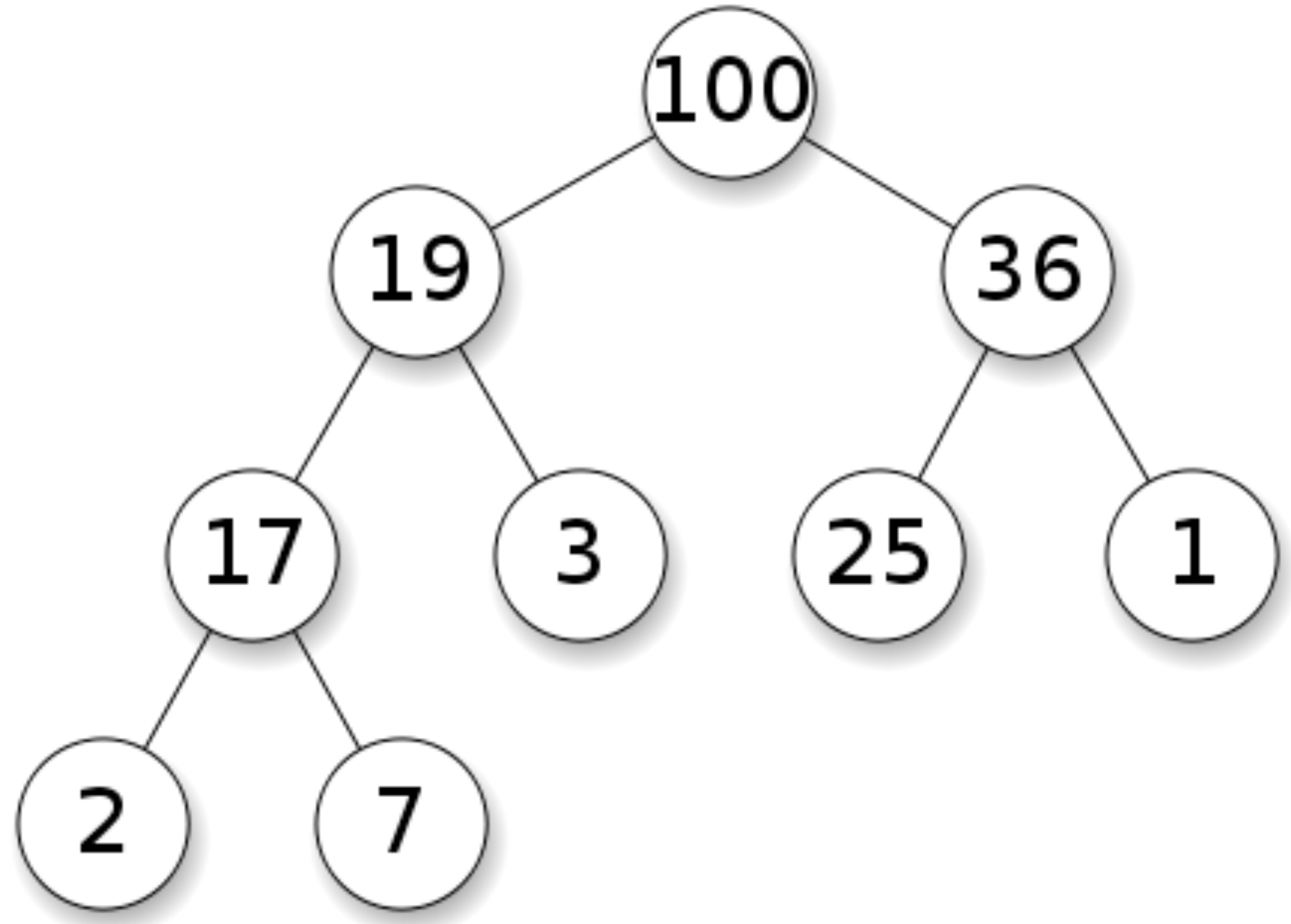
min binary heap for priority queue



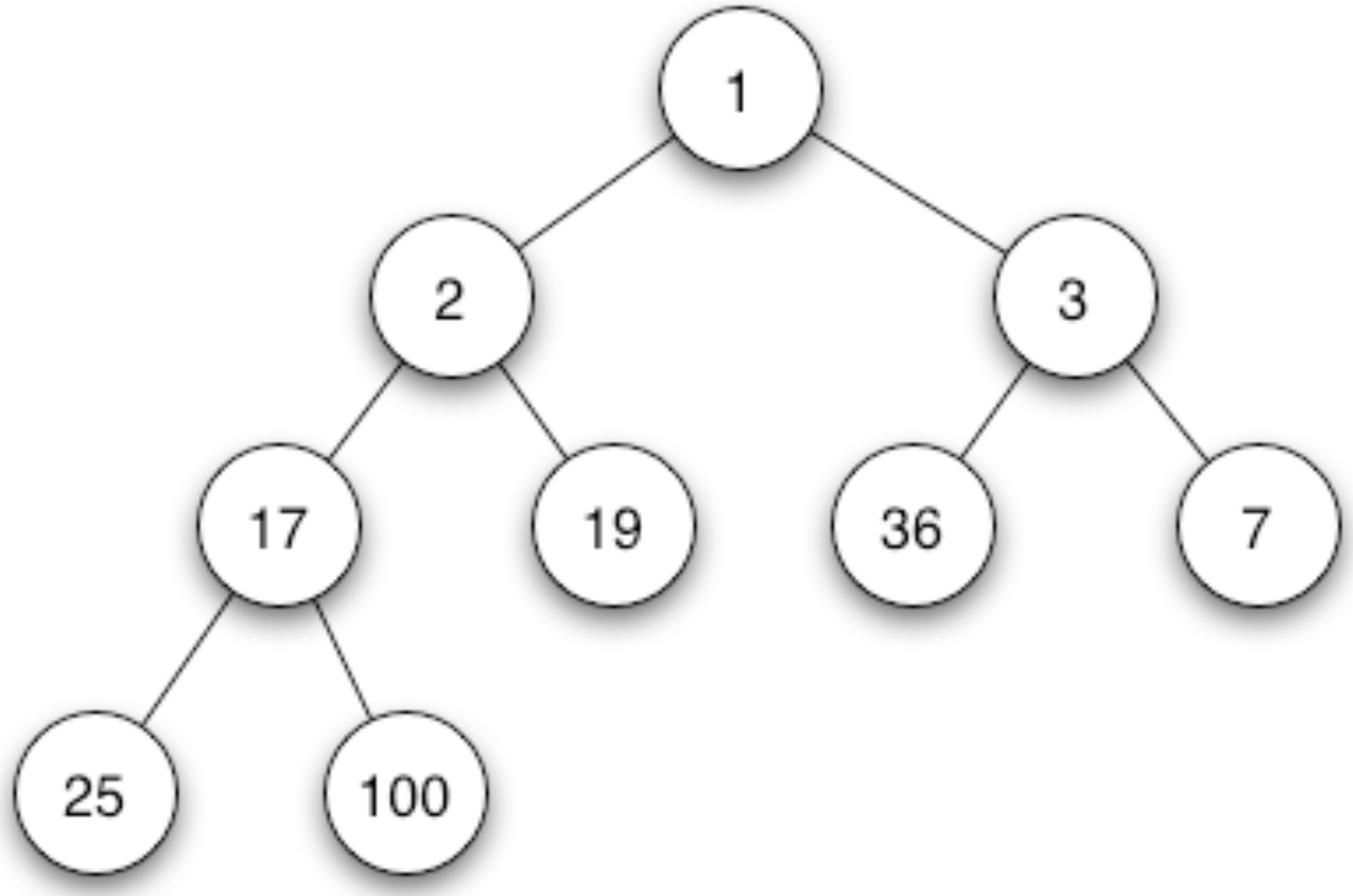
Binary Heaps

A heap is a tree-based data structure satisfying the heap property:
every element is greater (or less) than its children

Binary heaps allow nodes to have up to 2 children

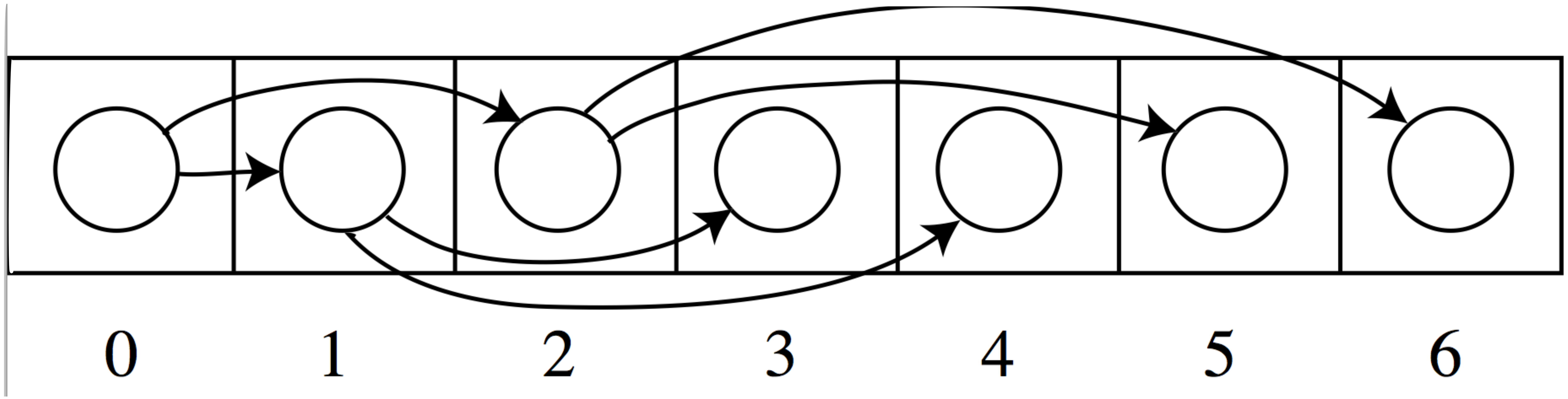


max heap



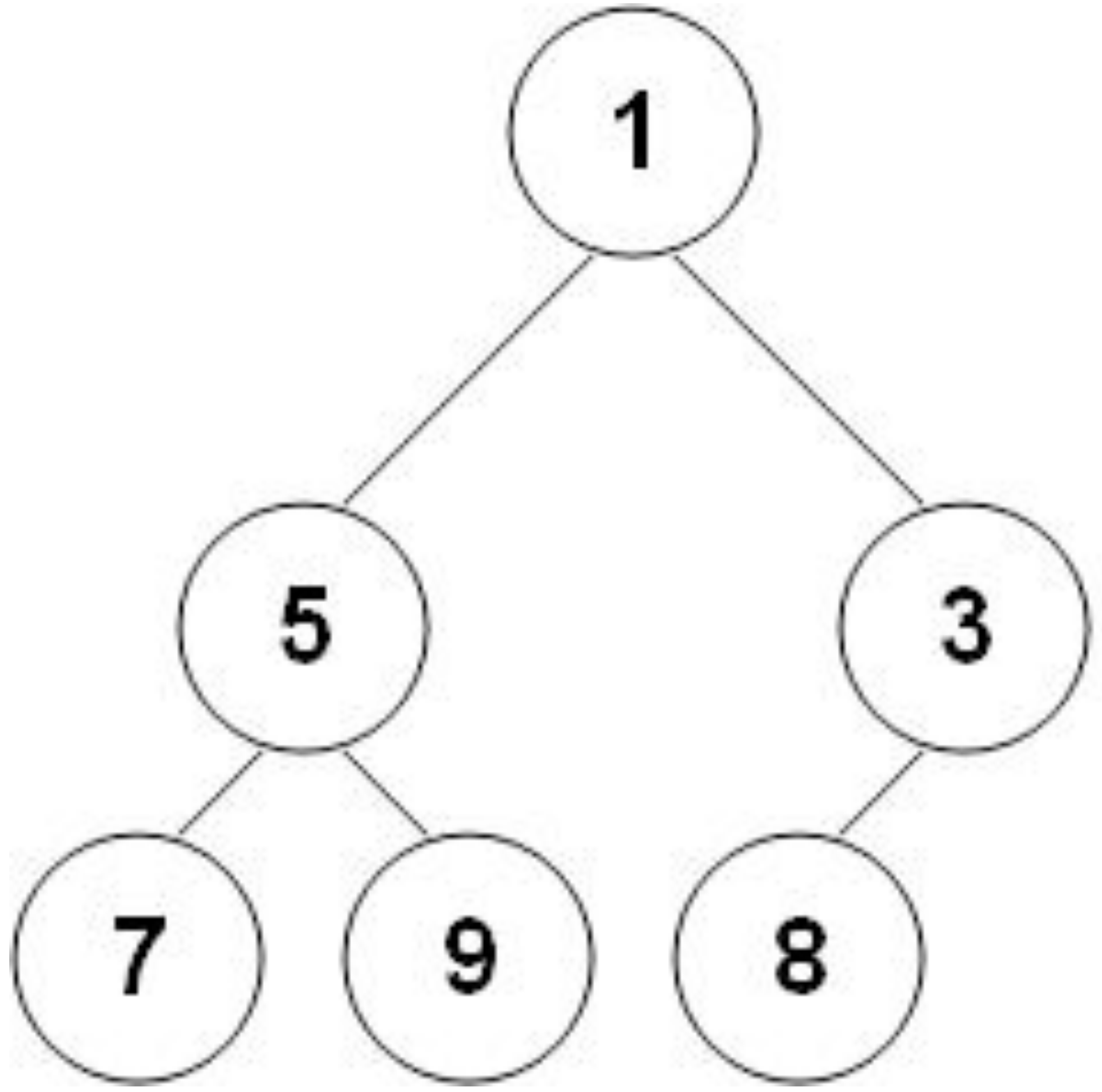
min heap

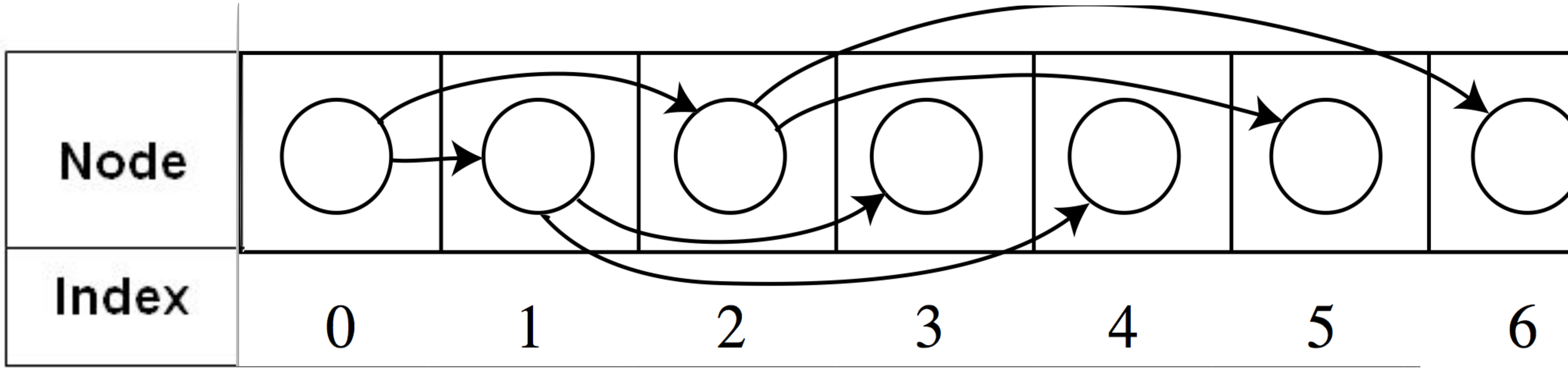
Heaps as arrays



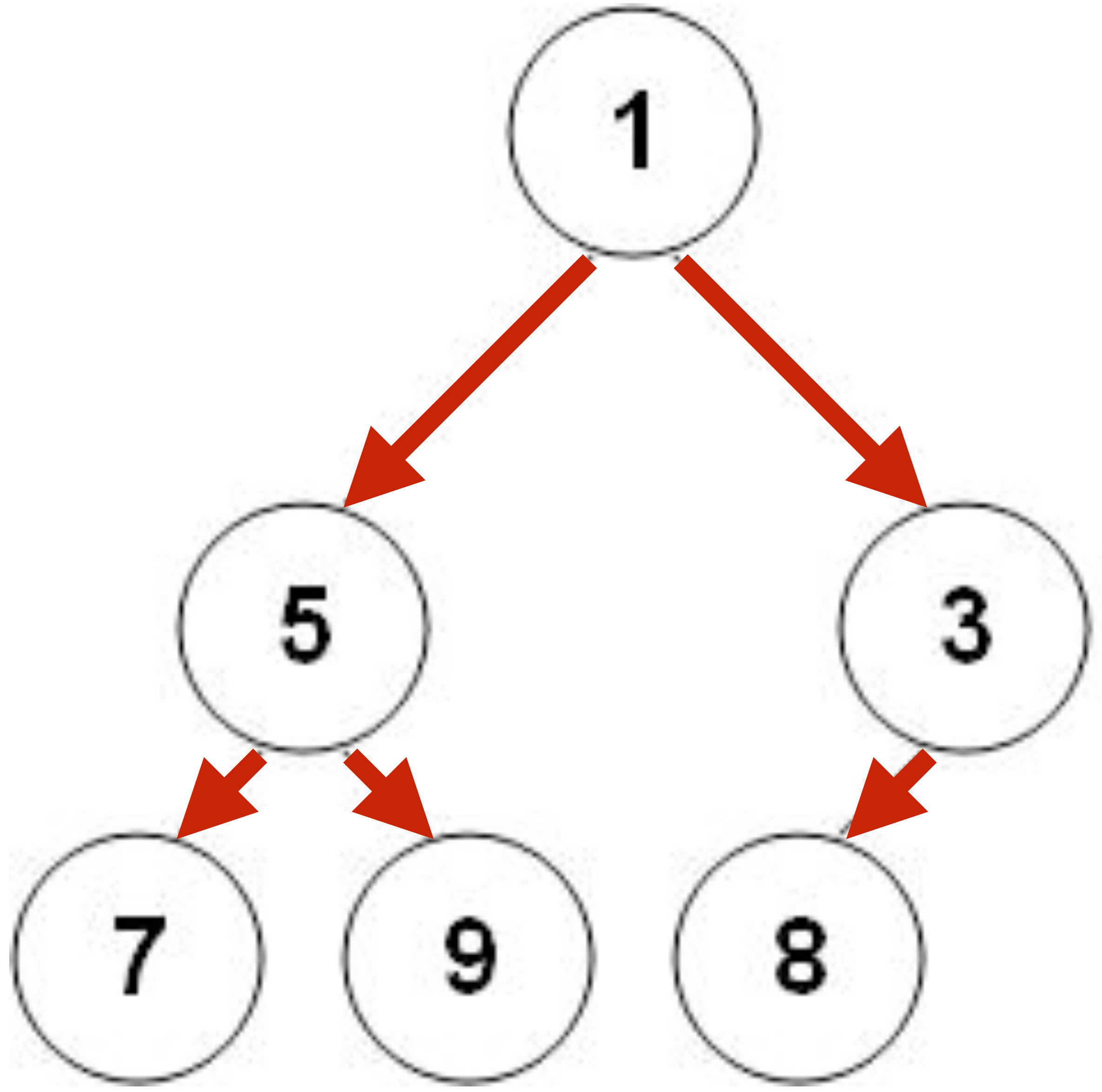
- Heap element at array location *i* has
  - children at array locations *2i+1* and *2i+2*
  - parent at *floor((i-1)/2)*

Heap array example





Heap array example



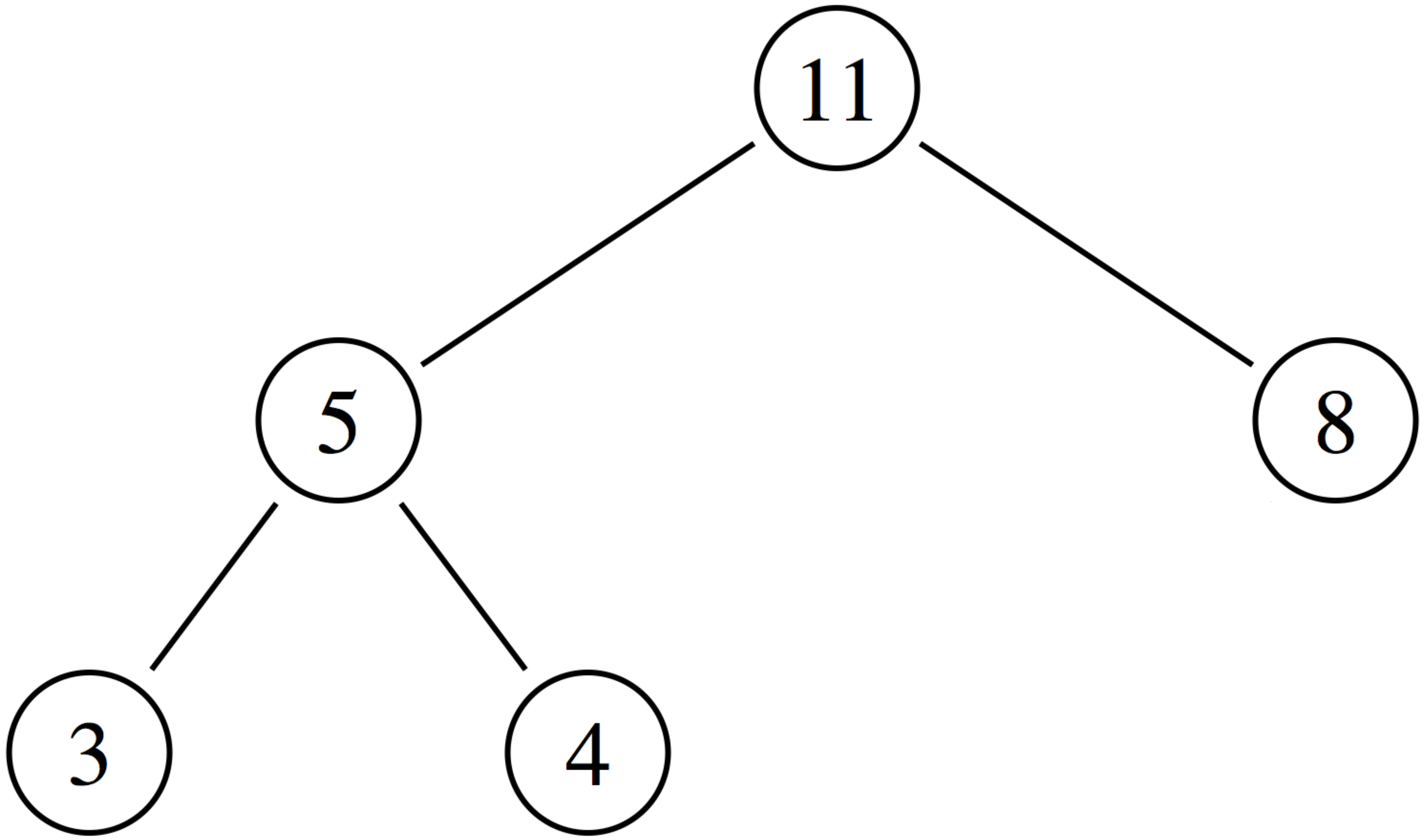
Node	1	5	3	7	9	8
Index	0	1	2	3	4	5

How do we insert a new heap element?

Heap operations: Insert

New element

Current heap

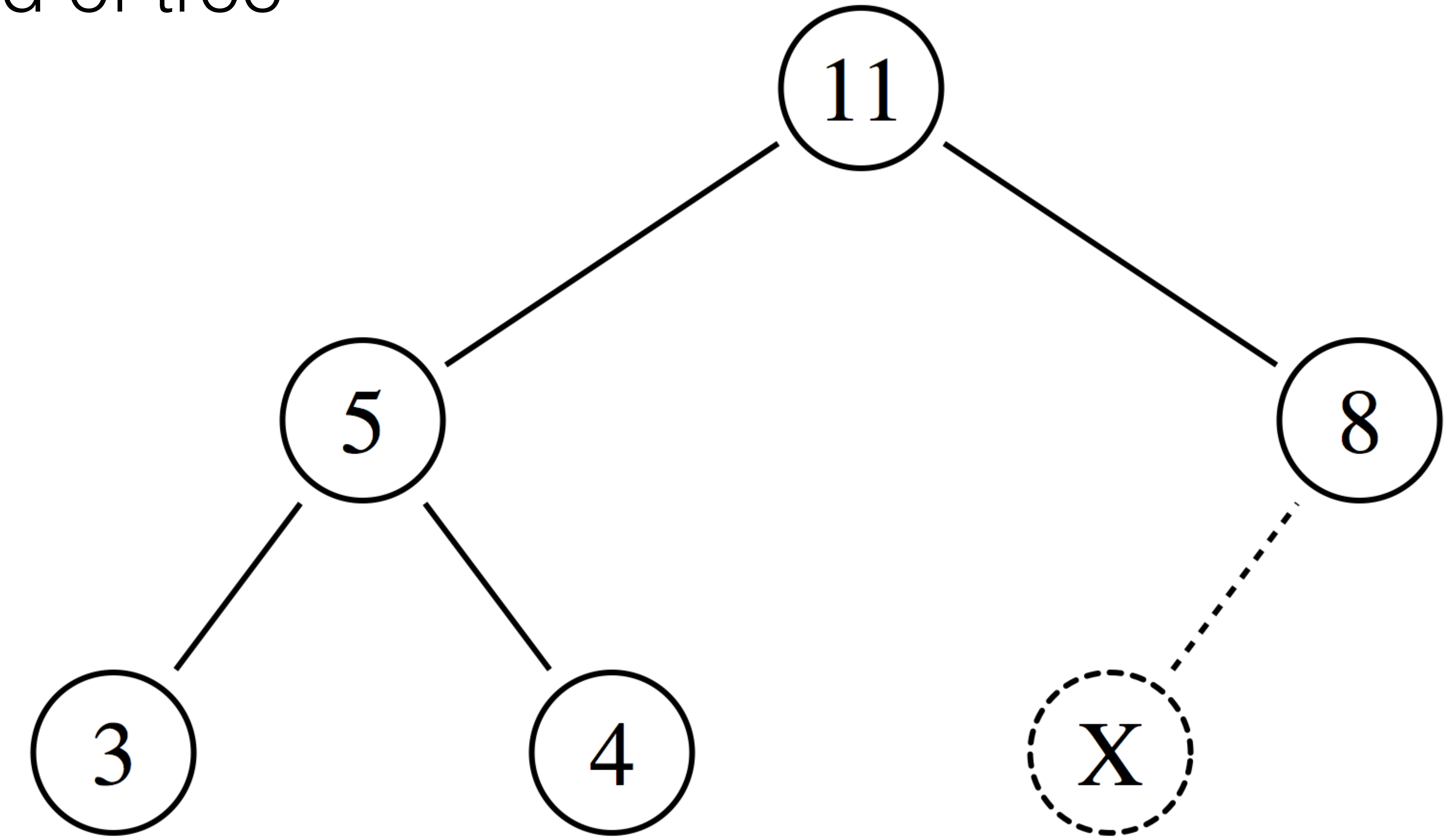


Heap operations: Insert

Step 1) add new element to end of tree

New element

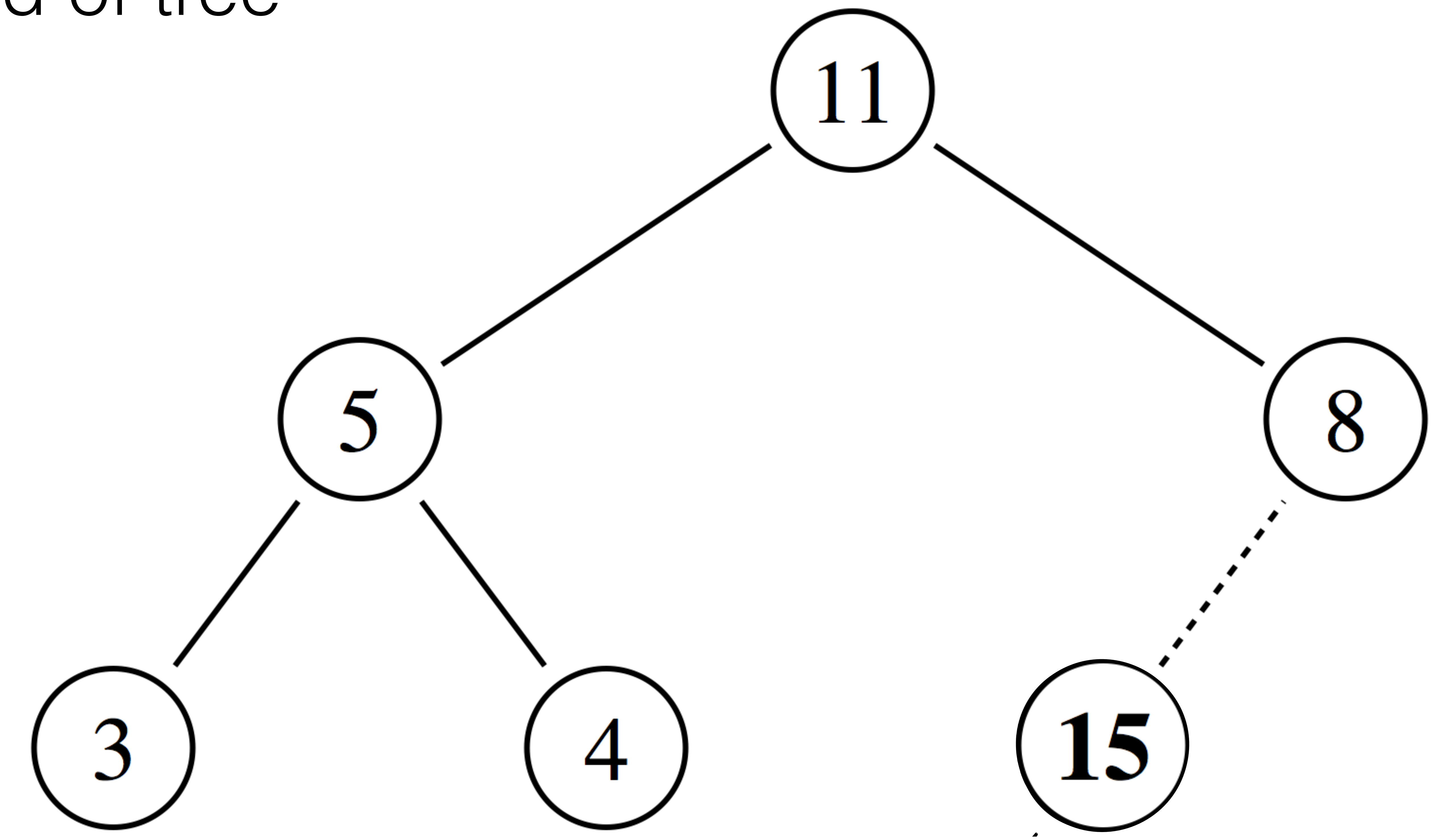
Current heap



Heap operations: Insert

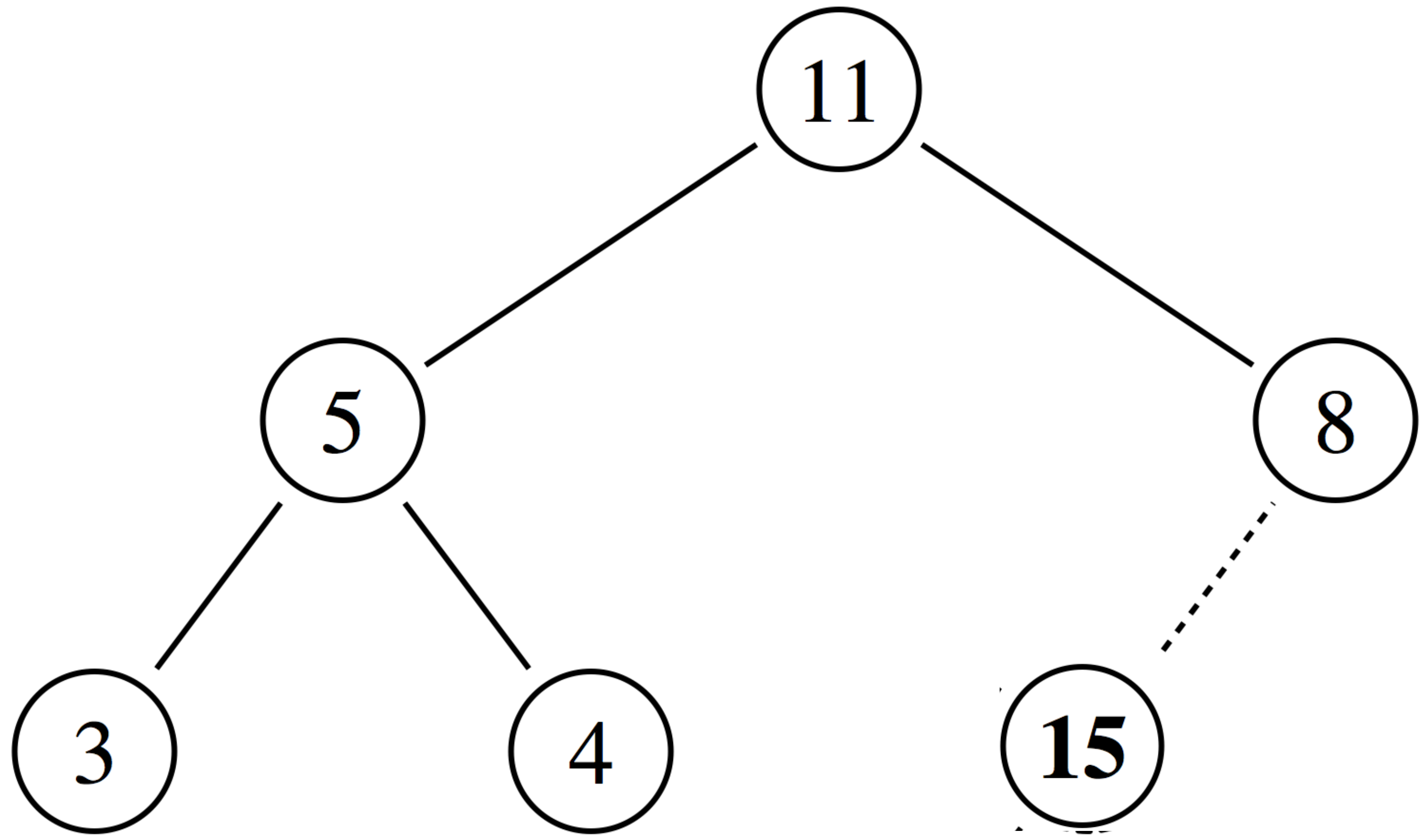
Step 1) add new element to end of tree

Current heap

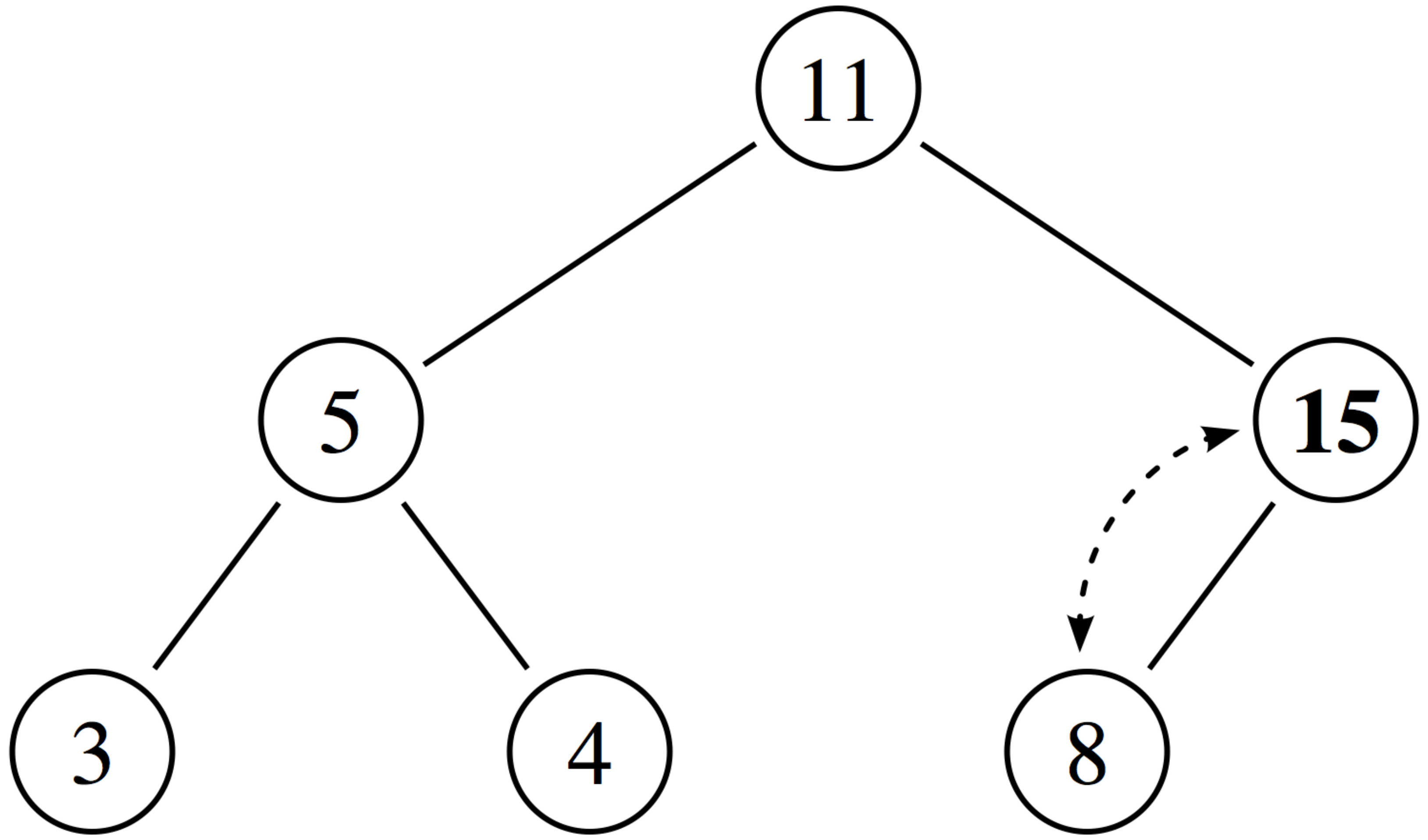


Heap operations: Insert

1) add new element to end of tree

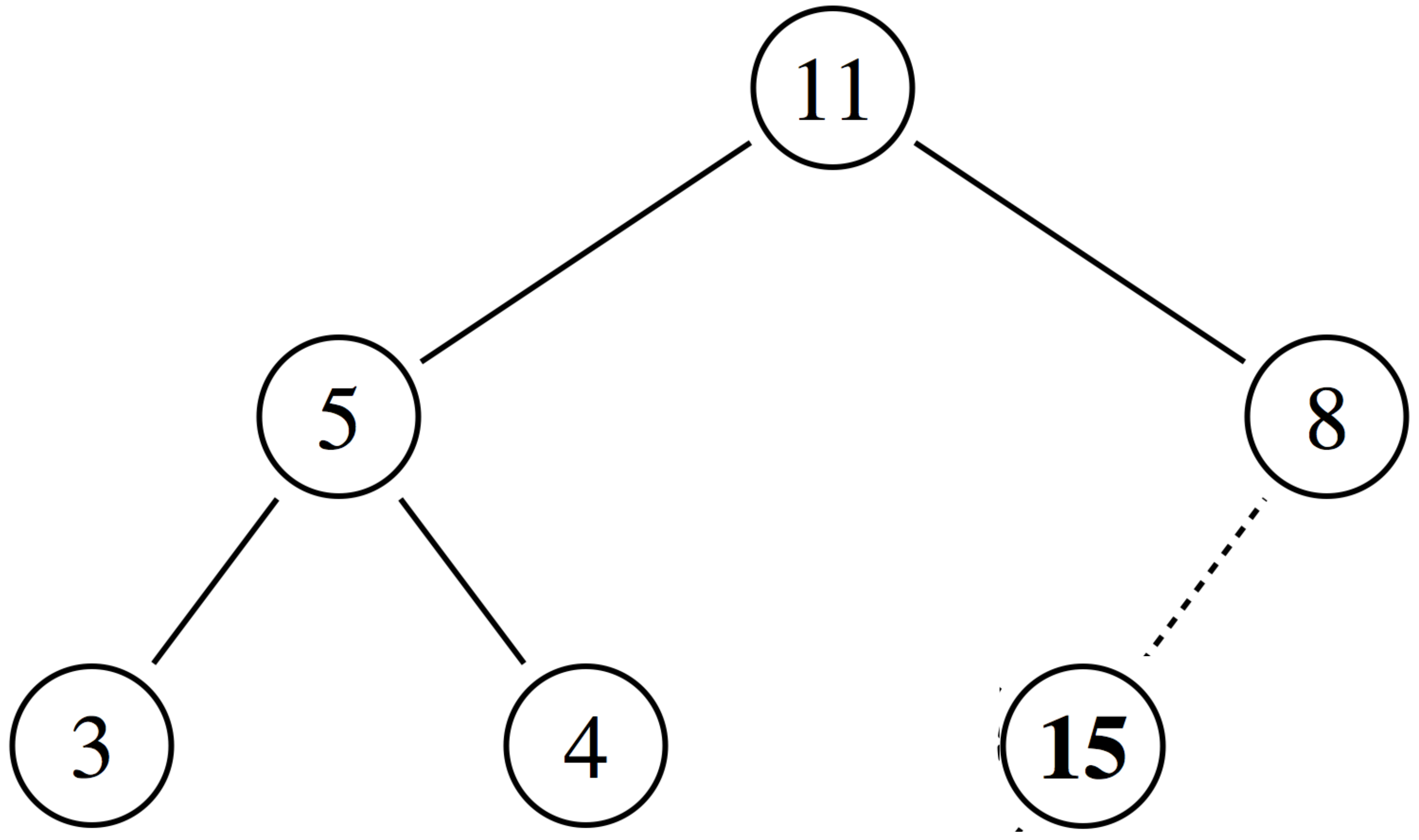


2) if heap condition not satisfied, swap inserted node with parent

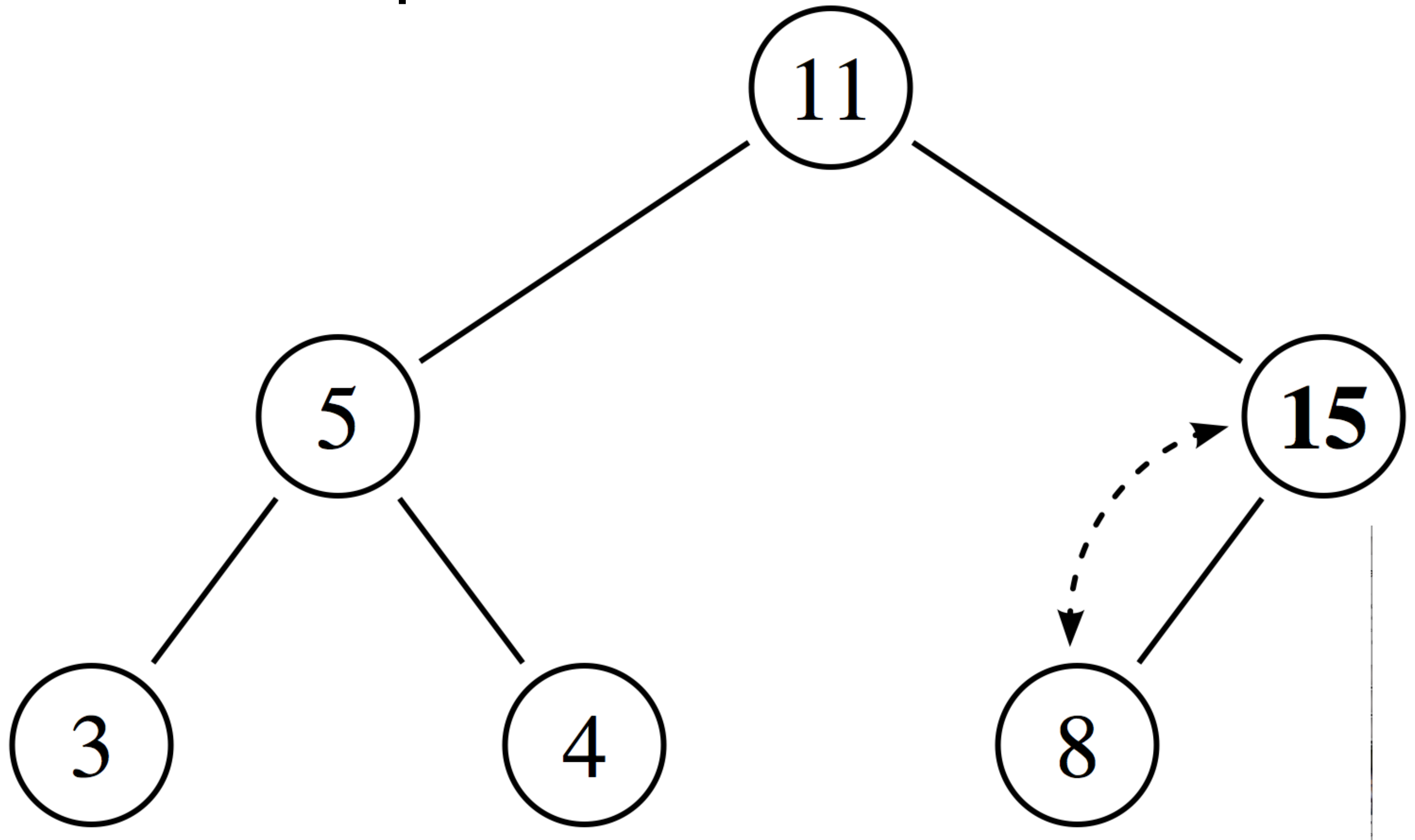


Heap operations: Insert

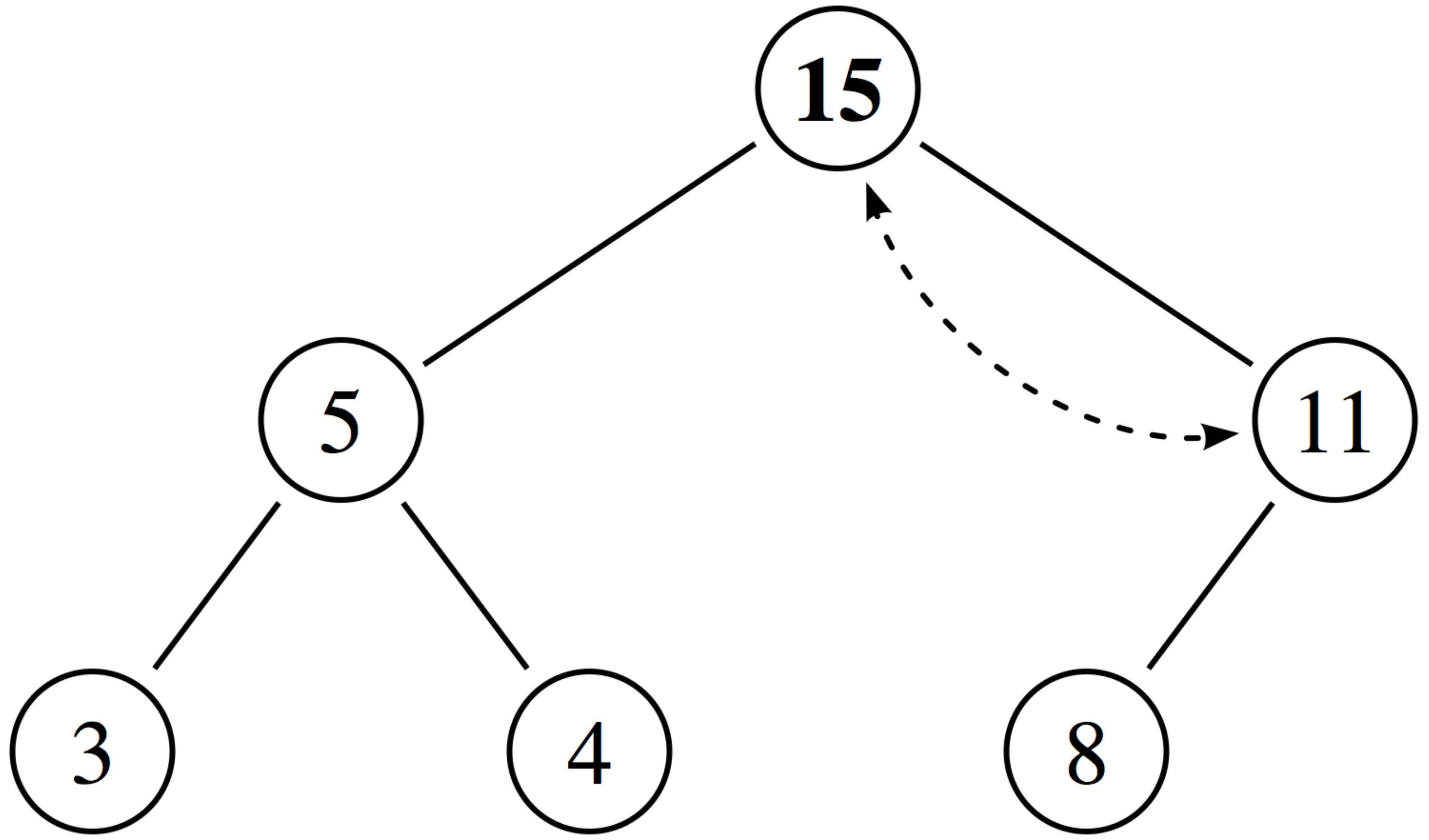
1) add new element to end of tree



2) if heap condition not satisfied, swap inserted node with parent



3) until heap condition satisfied, repeat (2)

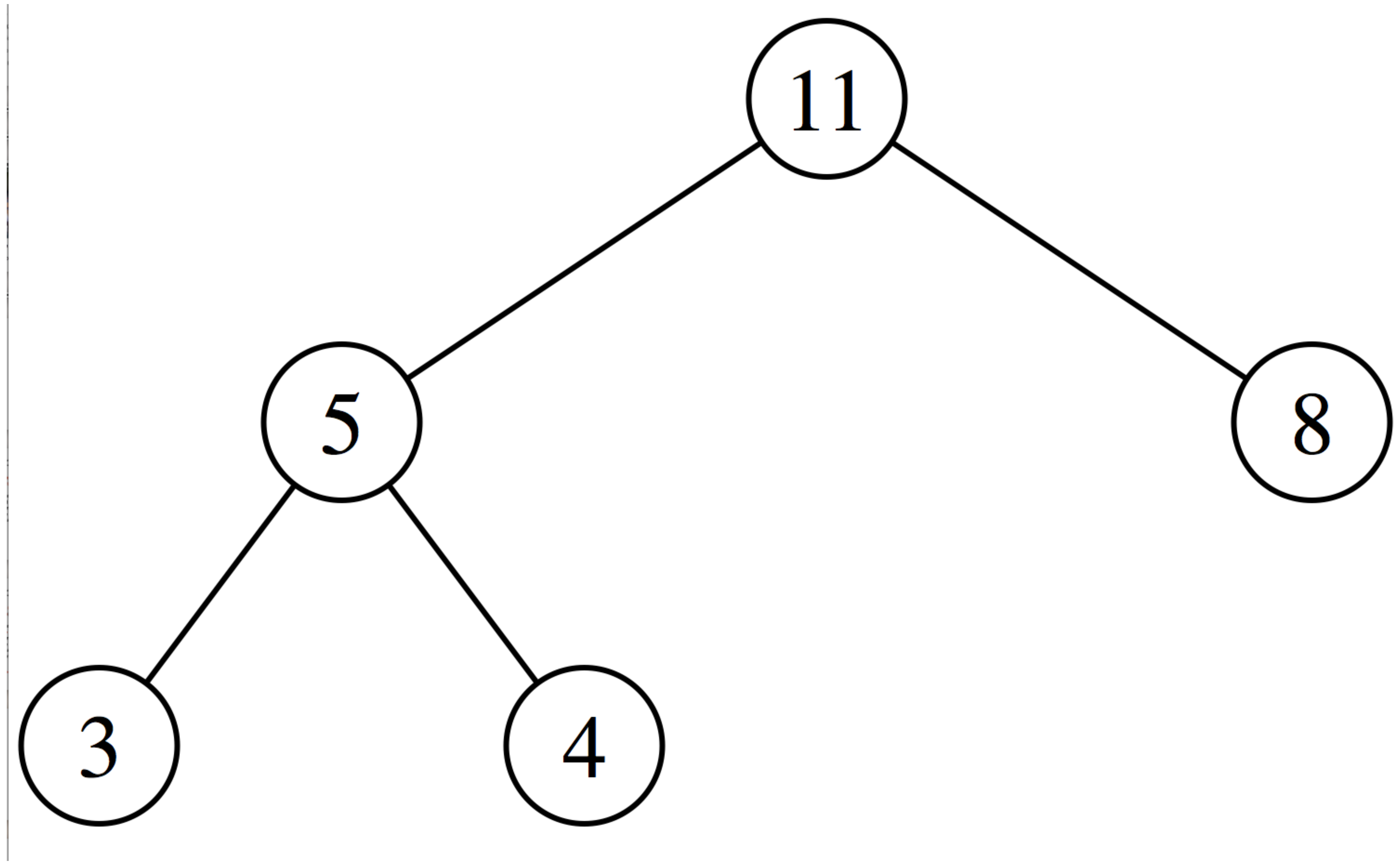


For priority queue, previously non-queued locations will be inserted with f_score priority

What happens when we extract a heap element?

Heap operations: Extract

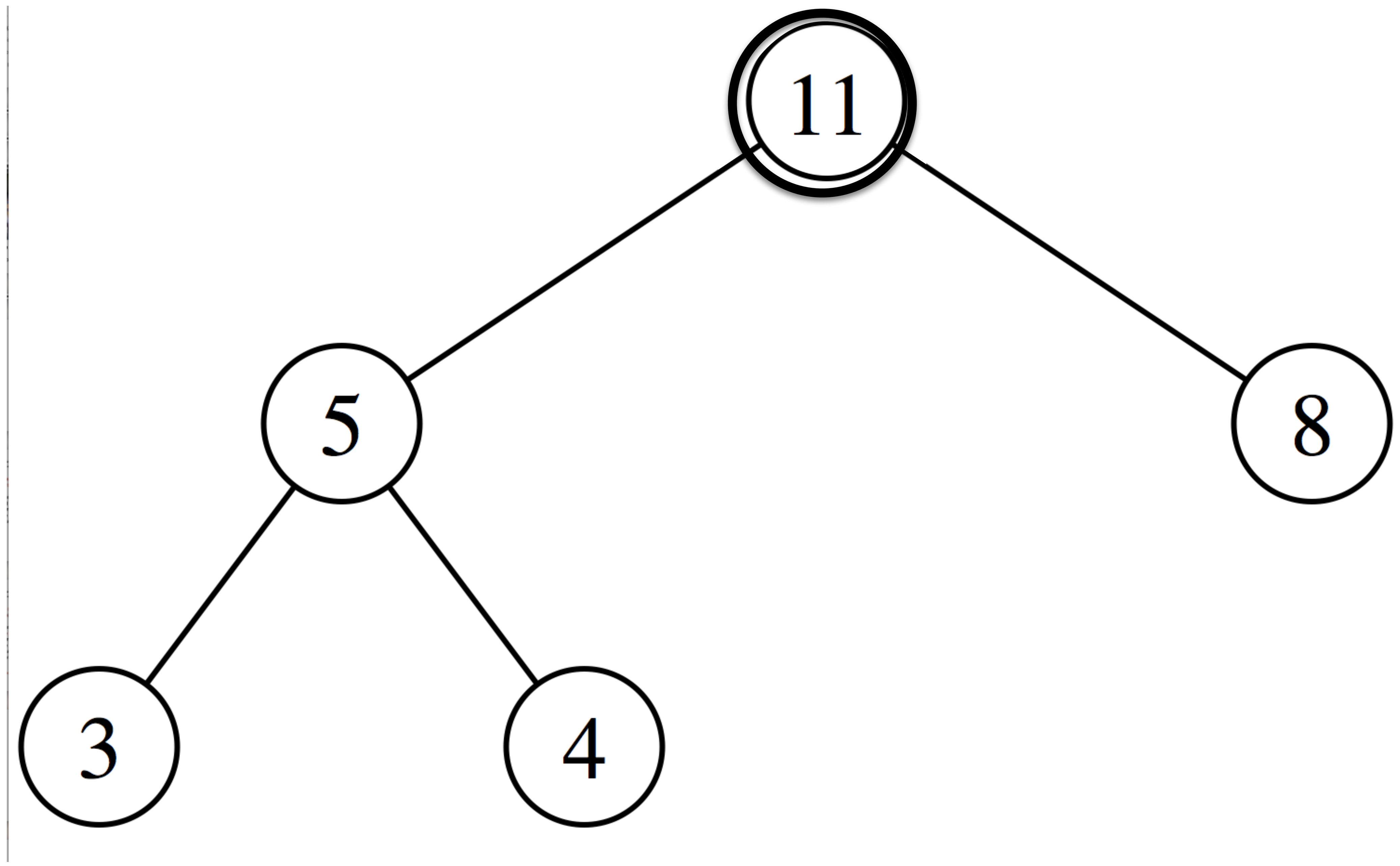
1) extract root element



For priority queue, the root of the heap will be the next node to visit

Heap operations: Extract

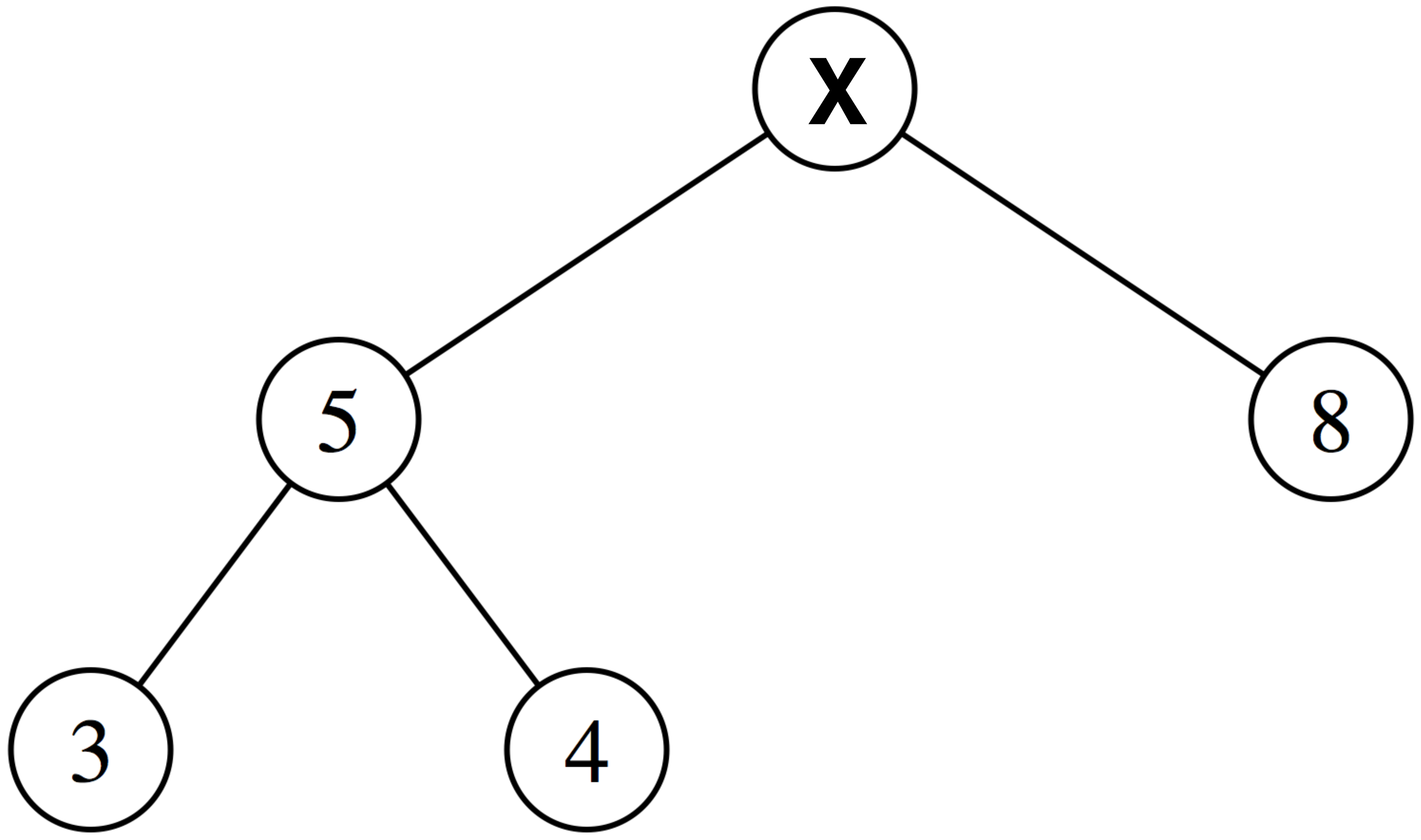
1) extract root element



For priority queue, the root of the heap will be the next node to visit

Heap operations: Extract

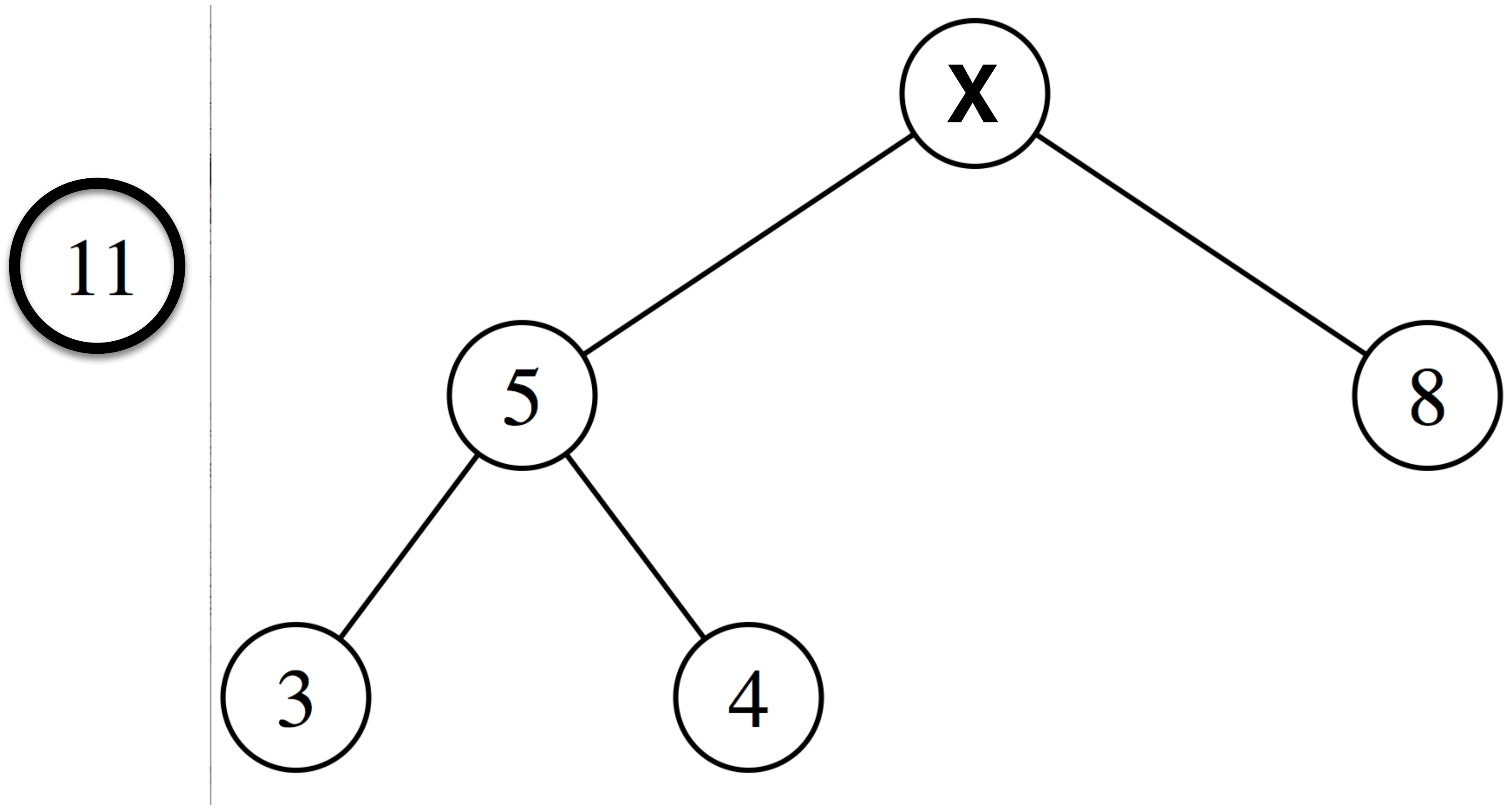
1) extract root element



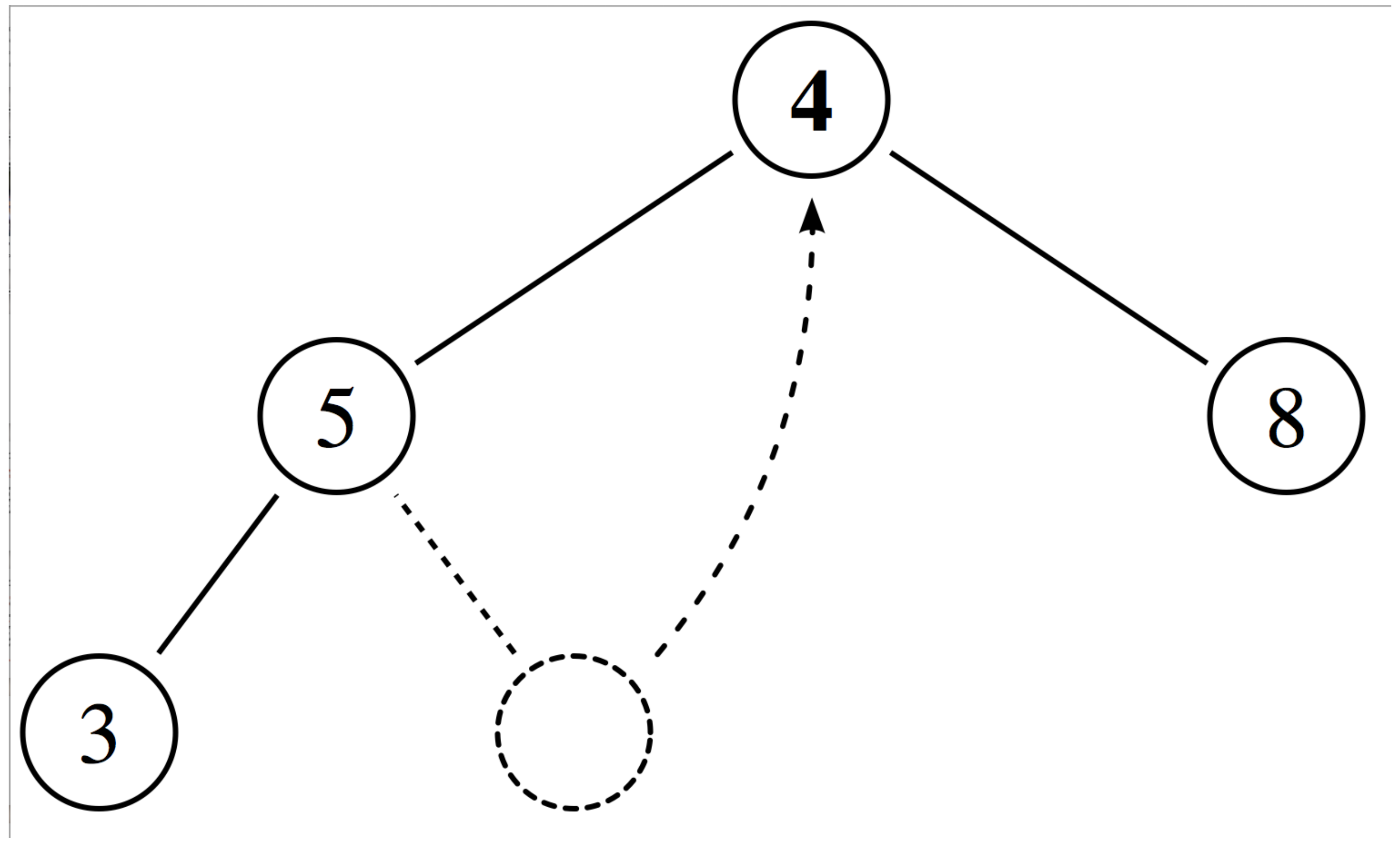
For priority queue, the root of the heap
will be the next node to visit

Heap operations: Extract

1) extract root element

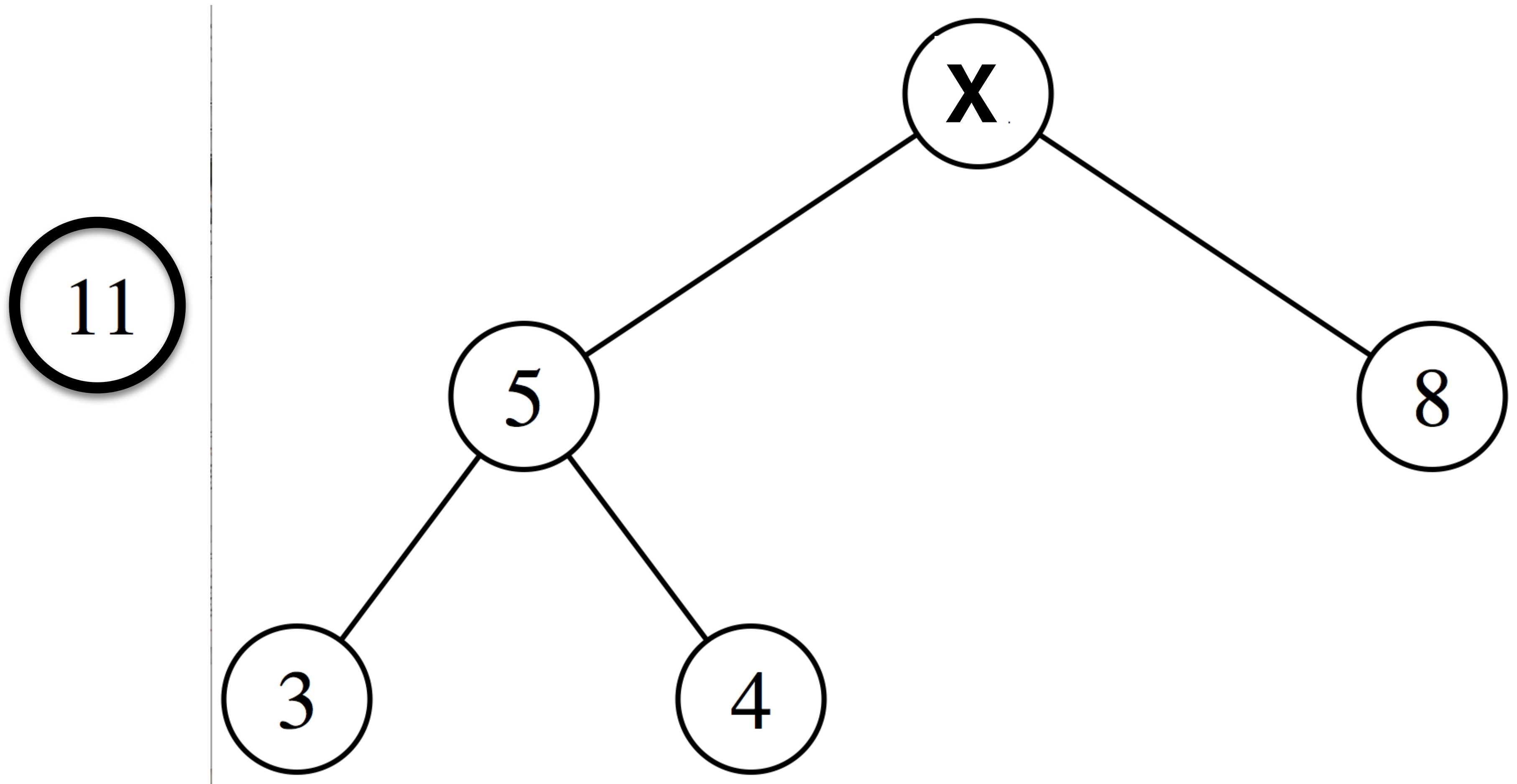


2) put last element at root

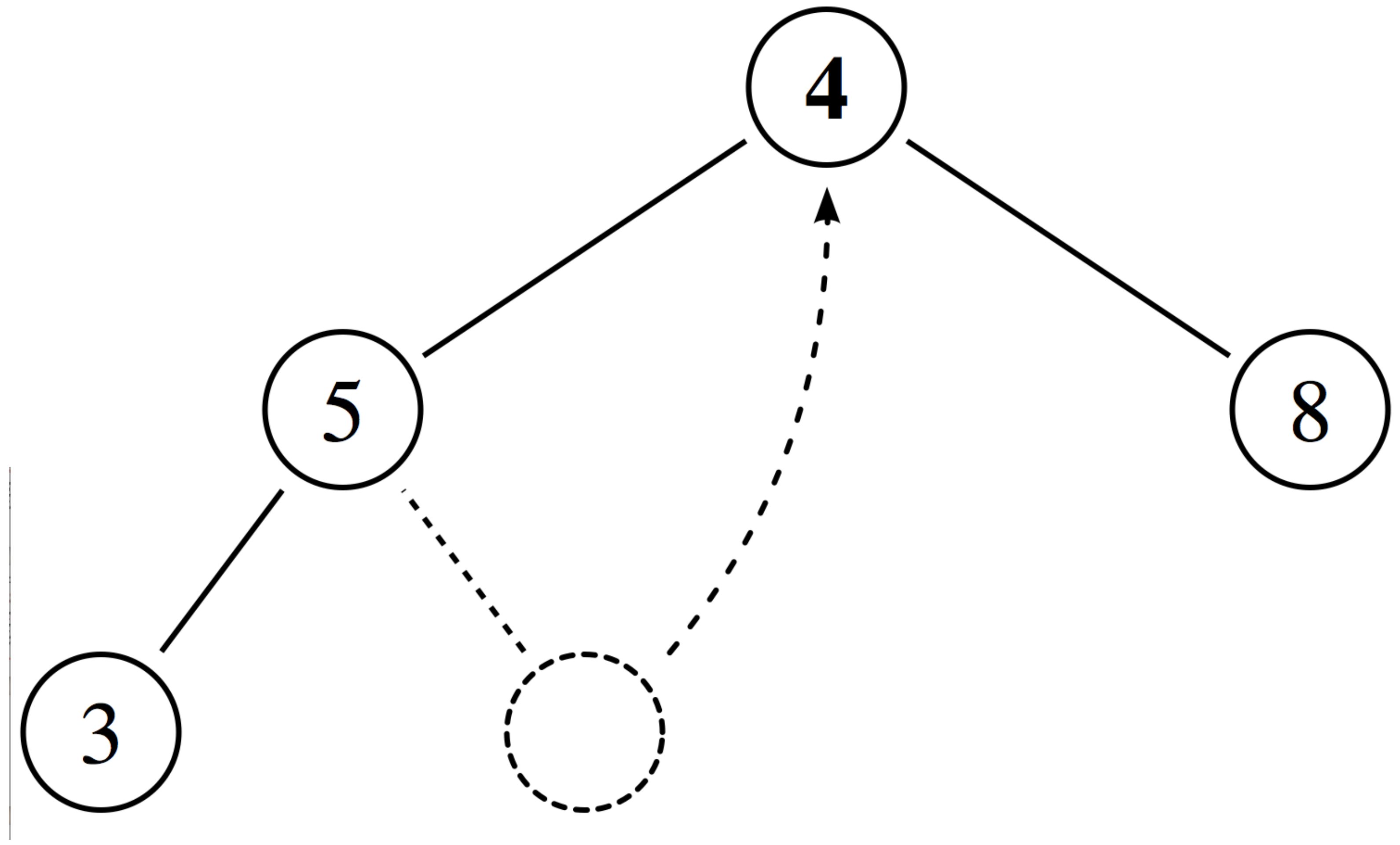


Heap operations: Extract

1) extract root element

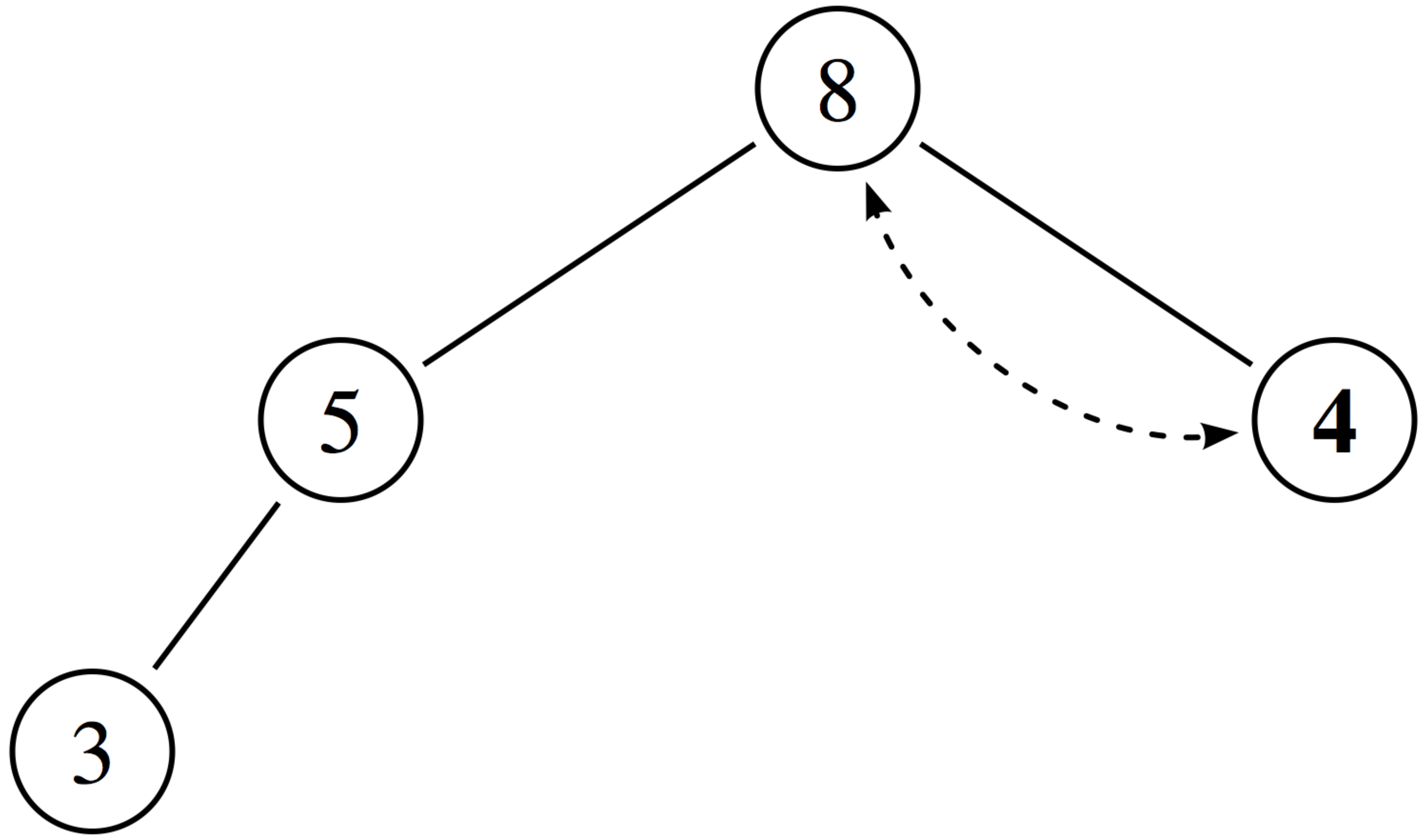


2) put last element at root



3) swap with higher priority child

4) until heaped, do (3)

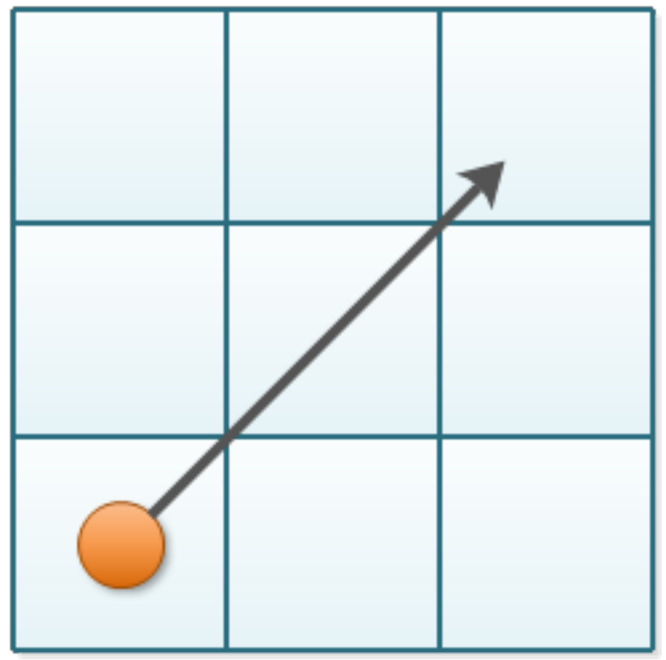


For priority queue, the root of the heap will be the next node to visit

Considerations

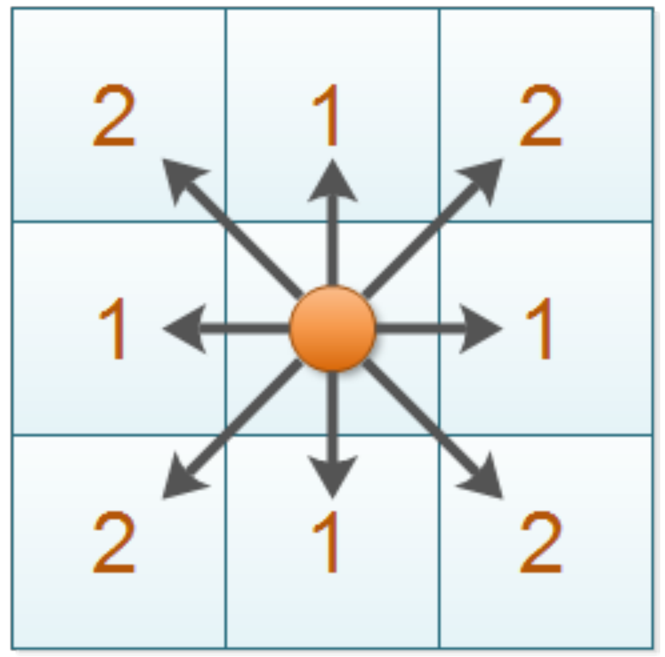
- How many operations are needed for heap insertion and extraction?
- How much better is a min heap than an array wrt. # of operations?
- Can there be duplicate heap elements for the same robot pose?
- How should we measure distance on a uniform grid?
- Is a choice of distance measure both metric and admissible?

Euclidean Distance



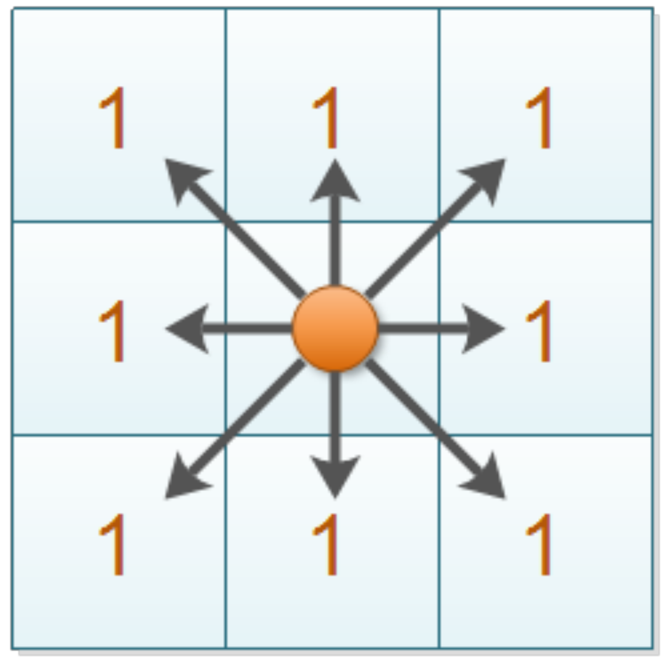
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Manhattan Distance



$$|x_1 - x_2| + |y_1 - y_2|$$

Chebyshev Distance

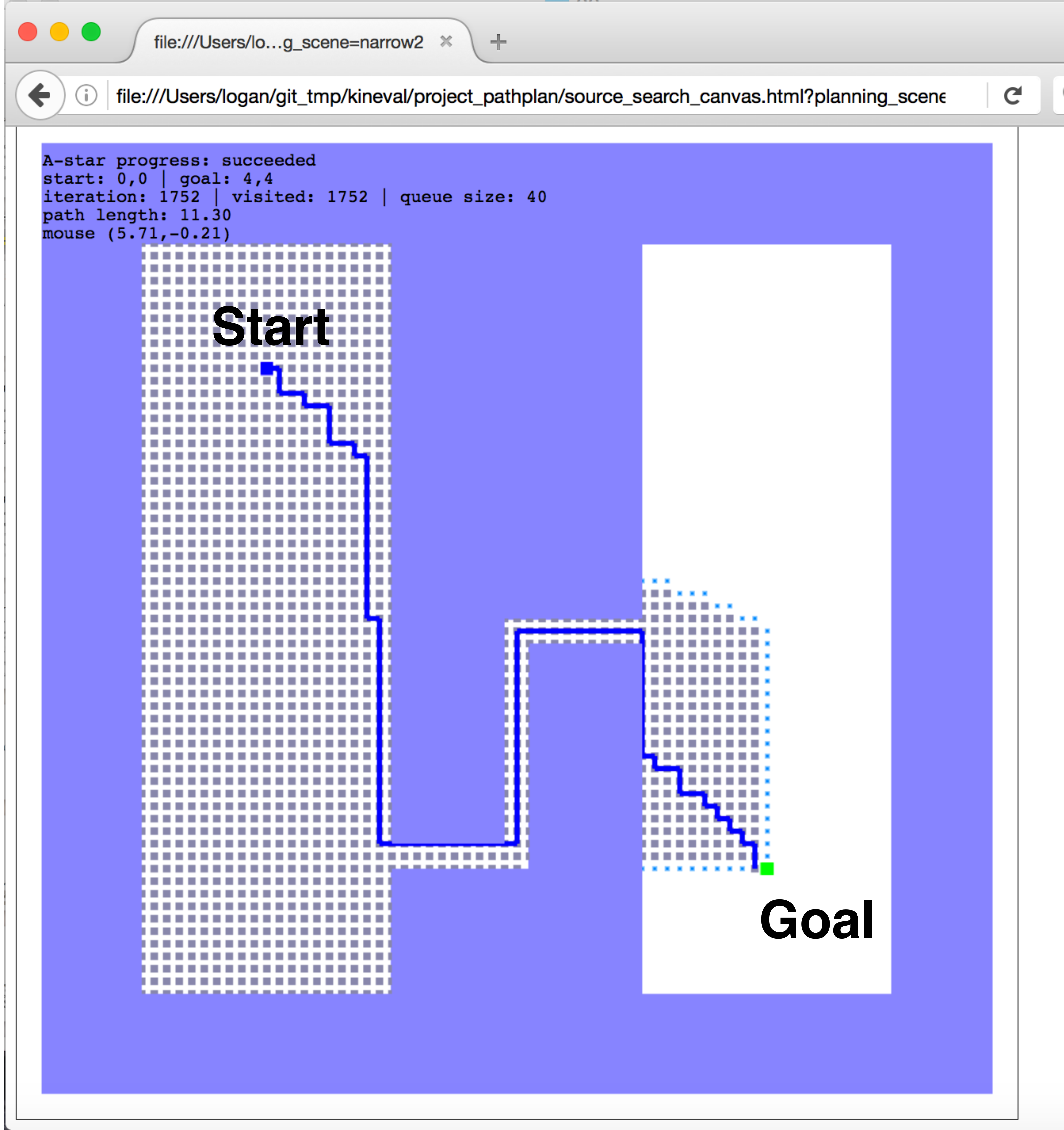


$$\max(|x_1 - x_2|\,, |y_1 - y_2|)$$

https://lyfat.wordpress.com/2012/05/22/euclidean-vs-chebyshev-vs-manhattan-distance/

Project 1:
2D Path Planning

- A-star algorithm for search in a given 2D world
- Heap data structure for priority queue
- Implement in JavaScript/ HTML5 (next lecture)
- Submit through your git repository



```html
<html>
<title>How do we implement this planner?</title>

<body>
<h1>Next lecture:</h1>
<p>JavaScript/HTML5 and git Tutorial</p>

<a href="http://autorob.org">
EECS 367 Introduction to Autonomous Robotics <br>
ROB 320 Robot Operating Systems
</a>

</body>
</html>
```